

# ATMOS 5140 Lecture 10 – Chapter 8

- Atmospheric Emission
  - Schwarzschild's Equation
  - Applications

Credit to Dr. Simon Carn at Michigan Tech for several slides used in this lecture.

# Schwarzschild's Equation

• So the net change in radiant intensity is:

$$dI = dI_{abs} + dI_{emit} = \beta_a (B - I) ds$$

$$\frac{dI}{ds} = \beta_a (B - I)$$

NB. All quantities represent a single wavelength!

• Schwarzschild's equation is the most fundamental description of radiative transfer in a *nonscattering* medium (applies to remote sensing in the thermal IR band)

• Radiance along a particular direction either increases or decreases with distance, depending on whether I(s) is less than or greater than B[T(s)], where T(s) is the temperature at point *s*.

Radiative transfer equation for a nonscattering atmosphere

• Some manipulation of Schwarzschild's Equation yields:

$$I(0) = I(\tau')e^{-\tau'} + \int_{0}^{\tau'} Be^{-\tau}d\tau$$

Basis for understanding of radiative transfer in a nonscattering atmosphere

1. I(0) is the radiance observed by a sensor at  $\tau = 0$ 

2. Radiance I at position  $\tau = \tau'$  multiplied by the *transmittance* [ $t(\tau') = e^{-\tau'}$  between the sensor and  $\tau$ `]. For a down-looking satellite sensor, this could represent *emission from the Earth's surface attenuated by transmission along the line-of-sight* 

3. Integrated thermal emission contributions  $Bd\tau$  from each point along the line of sight between the sensor and  $\tau$ , also attenuated by the path transmittances between the sensor and  $\tau$ .

#### Radiative transfer equation for a nonscattering atmosphere



1. I(0) is the radiance observed by a sensor at  $\tau = 0$ 

2. Radiance I at position  $\tau = \tau'$  multiplied by the *transmittance* [ $t(\tau') = e^{-\tau'}$  between the sensor and  $\tau$ `]. For a down-looking satellite sensor, this could represent *emission from the Earth's surface attenuated by transmission along the line-of-sight* 

3. Integrated thermal emission contributions  $Bd\tau$  from each point along the line of sight between the sensor and  $\tau$ , also attenuated by the path transmittances between the sensor and  $\tau$ .

#### Ground-based sensor looking up

• For a sensor located at the surface (z = 0), viewing downward emitted radiation from the atmosphere, the appropriate form of the radiative transfer equation is:

$$I^{\downarrow}(0) = I^{\downarrow}(\infty)t^* + \int_{0}^{\infty} B(z)W^{\downarrow}(z)dz$$

• where  $z = \infty$  is the 'top-of-the-atmosphere' (TOA), t\* is the transmittance from the surface to the TOA, and B(z) is the Planck function applied to the atmospheric temperature profile T(z).

• W(z) is the *emission weighting function*:

$$W^{\downarrow}(z) = -\frac{dt(0,z)}{dz} = \frac{\beta_a(z)}{\mu}t(0,z)$$

### Satellite-based sensor looking down

• For a sensor located in space ( $z = \infty$ ), viewing upward emitted radiation from the atmosphere, the appropriate form of the radiative transfer equation is:

$$I^{\uparrow}(\infty) = I^{\uparrow}(0)t^* + \int_{0}^{\infty} B(z)W^{\uparrow}(z)dz$$

• W(z) is the emission weighting function for upwelling radiation:

$$W^{\uparrow}(z) = \frac{dt(z,\infty)}{dz} = \frac{\beta_a(z)}{\mu}t(z,\infty)$$



ZENITH ATMOSPHERIC TRANSMITTANCE







IR spectra observed by a satellite spectrometer

# Well mixed gases

- E.g. O<sub>2</sub>, CO<sub>2</sub> (throughout the troposphere and stratosphere)
- Present at constant mass ratio to other constituents
- If you know the density profile of the atmosphere then:  $\rho'(z) = w\rho(z)$ , where  $w = mass \ ratio \ of \ well \ mixed \ gas$

$$\beta_a = \rho k_a \frac{R_{ecall}}{R_{ecall}}$$

• So, if you know  $k_a(z)$ , then you can calculate the optical depth  $\tau(z)$ , and thus the emission weighting function  $W^{\uparrow}(z)$ 

### Satellite retrieval of temperature profiles



#### **Requirements:**

Strong absorption band that renders the atmosphere opaque over a range of wavelengths.

Well-mixed constituent throughout troposphere and stratosphere (i.e., constant mixing ratio)

 $\rightarrow$  CO<sub>2</sub> and O<sub>2</sub>

Measurements of radiant intensities for a series of closely spaced wavelengths on the edge of a strong absorption band (e.g.,  $15 \mu m$  $CO_2$ )

## Satellite retrieval of temperature profiles



Actual weighting functions for channels 4-14 of the Advanced Microwave Sounding Unit (AMSU)

Located on the edge of the strong  $O_2$  absorption band at 60 GHz (~5mm)

Essential for weather forecasting!



Active Sensor

