ATMOS 5140
Lecture 12 – Chapter 11

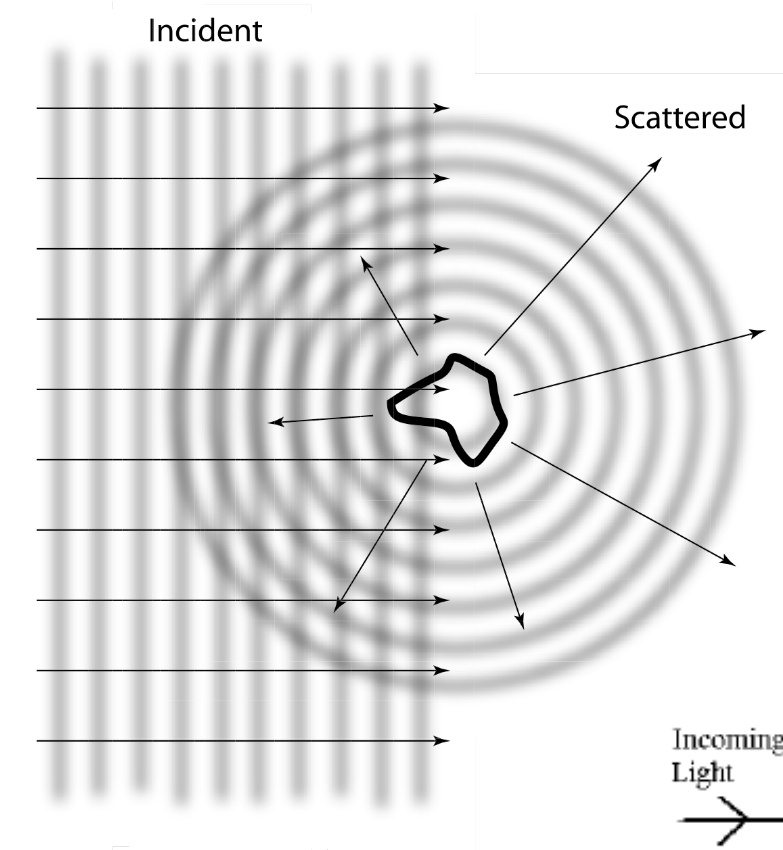
- Radiative Transfer Equation with Scattering
  - Scattering Phase Function
    - Isotropic scattering
    - Asymmetry Parameter
    - Henyey Greenstein Phase Function
  - Single vs. Multiple Scattering
  - Applications
    - Intensity of Sunlight
    - Horizontal Visibility

Scattering fundamentals

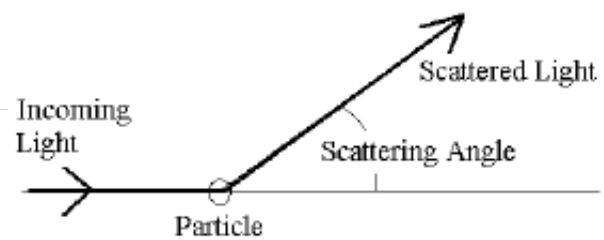
- Scattering can be broadly defined as the redirection of radiation out of the original direction of propagation, usually due to interactions with molecules and particles
- Reflection, refraction, diffraction etc. are actually all just forms of scattering
- Matter is composed of discrete electrical charges(atoms and molecules – dipoles)
- Light is an oscillating EM field – excites charges, which radiate EM waves
- These radiated EM waves are scattered waves, excited by a source external to the scatterer
- The superposition of incident and scattered EM waves is what is observed

Scattering of light by particle

Backward scattering (backscattering)



Forward scattering



Parameters governing scattering

(1) The wavelength (λ) of the incident radiation
(2) The size of the scattering particle
3) The particle optical properties relative to the surrounding medium: the complex refractive index

Types of scattering

Elastic scattering – the wavelength (frequency) of the scattered light is the same as the incident light (Rayleigh and Mie scattering)

Inelastic scattering – the emitted radiation has a wavelength different from that of the incident radiation (Raman scattering, fluorescence)

Quasi-elastic scattering – the wavelength (frequency) of the scattered light shifts (e.g., in moving matter due to Doppler effects)

Extinction, Absorption, and Scattering Coefficients

Review

$$\beta_a = 4\pi n_i/\lambda$$

$$\beta_e = \beta_a + \beta_s$$

Single Scattering Albedo

$$\varpi = \frac{\beta_s}{\beta_e} = \frac{\beta_s}{\beta_a + \beta_s}$$

Atmospheric Emission

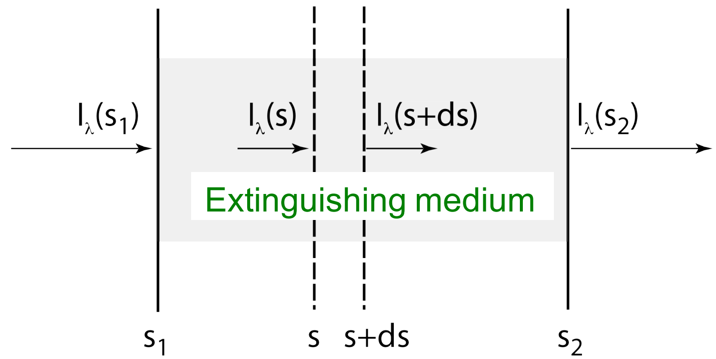
Review

PREVIOUSLY

- Currently will restrict our focus to problems where scattering can be ignored.
- This is a reasonable approach involving the thermal IR, far IR or microwave bands.
- As long as it is not precipitating

Absorption and emission

- Kirchhoff's Law implies a correspondence between absorption and emission, including in the atmosphere



Review

Type equation here.

$$dI_\lambda = I_\lambda(s + ds) - I_\lambda(s) = -I_\lambda(s)\beta_e(s)ds$$

$$dI_{abs} = -\beta_a I\, ds$$ **Assume ω = 0; no scattering**

But Kirchhoff's Law tells us that $$dI_{emit} = \beta_a B\, ds$$ (where $B = B_\lambda(T)$)

Schwarzschild's Equation

Review

• So the net change in radiant intensity is:

$$dI = dI_{abs} + dI_{emit} = \beta_a (B - I)\, ds$$

$$\frac{dI}{ds} = \beta_a (B - I)$$

NB. All quantities represent a single wavelength!

• Schwarzschild's equation is the most fundamental description of radiative transfer in a *nonscattering* medium (applies to remote sensing in the thermal IR band)

• Radiance along a particular direction either increases or decreases with distance, depending on whether *$I(s)$ is less than or greater than $B[T(s)]$, where $T(s)$ is the temperature at point s*.

Radiative Transfer Equation

Now with Scattering

Scattered into beam from other directions

$$dI = dI_{ext} + dI_{emit} + dI_{scat}$$

Depletion due to both absorption and scattering

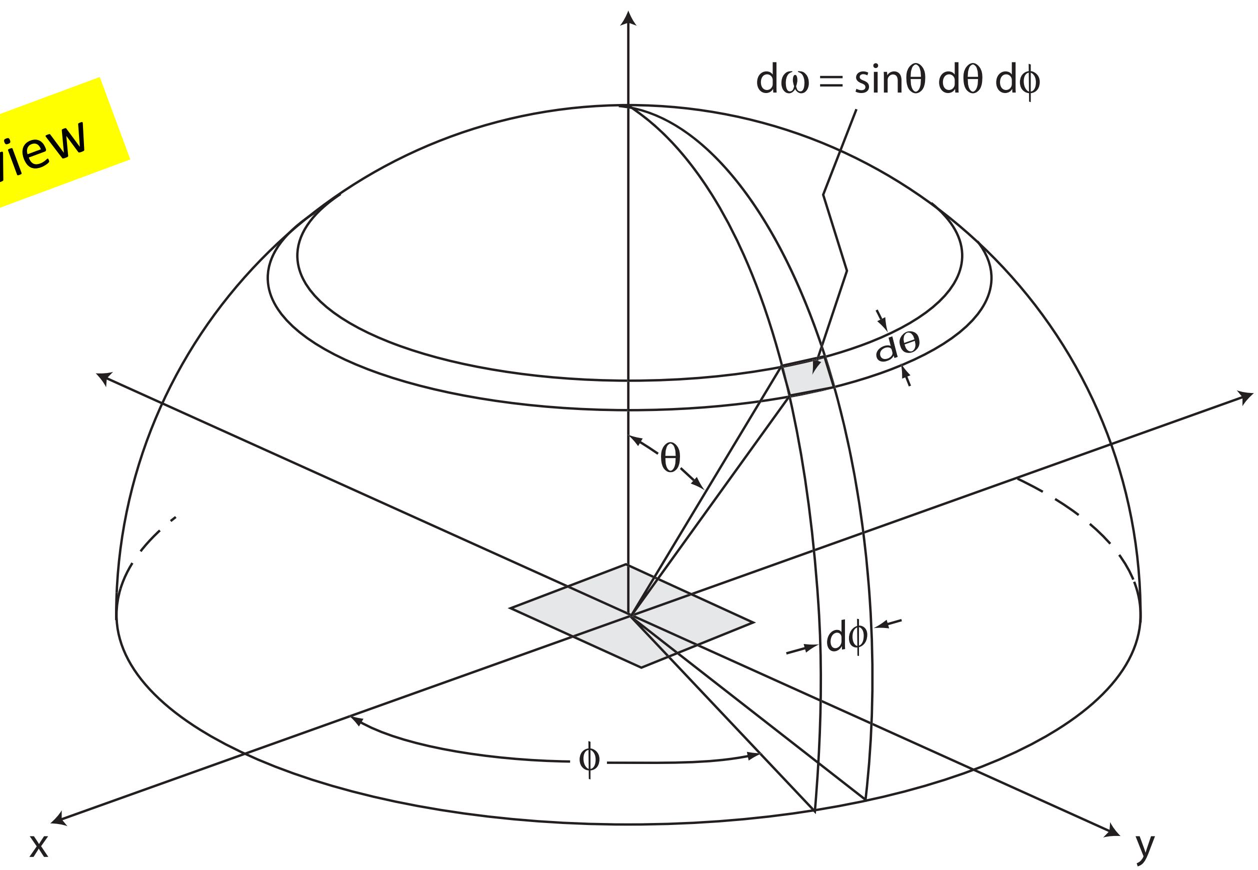
$$dI_{ext} = \beta_e B(T) ds$$

All quantities represent a single wavelength!

Steradian

$$\int_{4\pi} d\omega = \int_0^{2\pi}\int_0^{\pi} \sin\theta \, d\theta \, d\phi = 2\pi \int_0^{\pi} \sin\theta \, d\theta = 4\pi.$$

Review



Radiative Transfer Equation

Now with Scattering

Scattered into beam from other directions

$$dI = dI_{ext} + dI_{emit} + dI_{scat}$$

All quantities represent a single wavelength!

$$dI_{scat} = \frac{\beta_s}{4\pi} \int_{4\pi} p(\hat{\Omega}', \hat{\Omega}) I(\hat{\Omega}') \, d\omega' ds$$

Where radiation from any direction $\hat{\Omega}'$*,*
can contribute scattered radiation in the direction of interest $\hat{\Omega}$

$p(\hat{\Omega}', \hat{\Omega})$ = phase function, which is required to satisfy the normalization across a sphere

$$1 = \frac{1}{4\pi} \int_{4\pi} p(\hat{\Omega}', \hat{\Omega}) d\omega'$$

Radiative Transfer Equation Now with Scattering

$$dI = dI_{ext} + dI_{emit} + dI_{scat}$$

$$dI = -\beta_e I ds + \beta_a B ds + \frac{\beta_s}{4\pi} \int_{4\pi} p\left(\hat{\Omega}', \hat{\Omega}\right) I\left(\hat{\Omega}'\right) d\omega' ds$$

Radiative Transfer Equation Now with Scattering

$$dI = dI_{ext} + dI_{emit} + dI_{scat}$$

$$dI = -\beta_e I ds + \beta_a B ds + \frac{\beta_s}{4\pi}\int_{4\pi} p(\hat{\Omega}', \hat{\Omega}) I(\hat{\Omega}') d\omega' ds$$

Recall

$$\varpi = \frac{\beta_s}{\beta_e} = \frac{\beta_s}{\beta_a + \beta_s}$$

Now Divide through by

$$d\tau = -\beta_e ds$$

$$\frac{dI(\hat{\Omega})}{d\tau} = I(\hat{\Omega}) - (1 - \varpi)B - \frac{\varpi}{4\pi}\int_{4\pi} p(\hat{\Omega}', \hat{\Omega}) I(\hat{\Omega}') d\omega'$$

Radiative Transfer Equation Now with Scattering

$$\frac{dI(\hat{\Omega})}{d\tau} = I(\hat{\Omega}) - (1-\varpi)B - \frac{\varpi}{4\pi}\int_{4\pi} p(\hat{\Omega}', \hat{\Omega}) I(\hat{\Omega}') d\omega'$$

All Sources of Radiation – i.e. Source Function

$J(\hat{\Omega})$

$$\frac{dI(\hat{\Omega})}{d\tau} = I(\hat{\Omega}) - J(\hat{\Omega})$$

Radiative Transfer Equation Now with Scattering

$$\frac{dI(\hat{\Omega})}{d\tau} = I(\hat{\Omega}) - (1-\varpi)B - \frac{\varpi}{4\pi}\int_{4\pi} p(\hat{\Omega}', \hat{\Omega}) I(\hat{\Omega}') d\omega'$$

Recall

$$\varpi = \frac{\beta_s}{\beta_e} = \frac{\beta_s}{\beta_a + \beta_s}$$

When:

$\varpi = 0$

i.e. No Scattering

$$\frac{dI(\hat{\Omega})}{d\tau} = I(\hat{\Omega}) - (1-\varpi)B ~~- \frac{\varpi}{4\pi}\int_{4\pi} p(\hat{\Omega}', \hat{\Omega}) I(\hat{\Omega}') d\omega'~~$$

Radiative Transfer Equation Now with Scattering

$$\frac{dI(\hat{\Omega})}{d\tau} = I(\hat{\Omega}) - (1-\varpi)B - \frac{\varpi}{4\pi}\int_{4\pi} p(\hat{\Omega}',\hat{\Omega})I(\hat{\Omega}')d\omega'$$

Recall

$$\varpi = \frac{\beta_s}{\beta_e} = \frac{\beta_s}{\beta_a + \beta_s}$$

When:

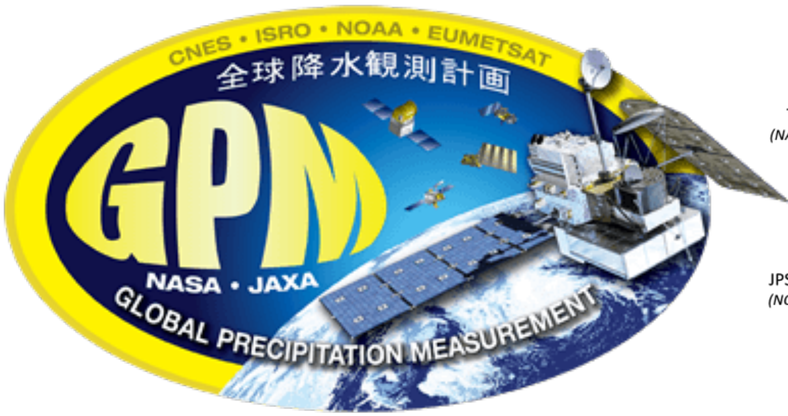
ϖ = 1

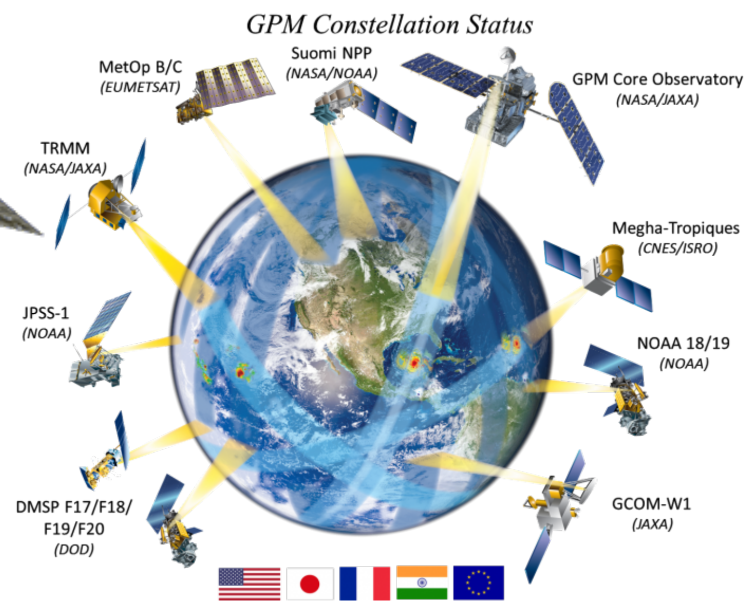
i.e. No Absorption

$$\frac{dI(\hat{\Omega})}{d\tau} = I(\hat{\Omega}) - \cancel{(1-\varpi)B} - \frac{\varpi}{4\pi}\int_{4\pi} p(\hat{\Omega}',\hat{\Omega})I(\hat{\Omega}')d\omega'$$

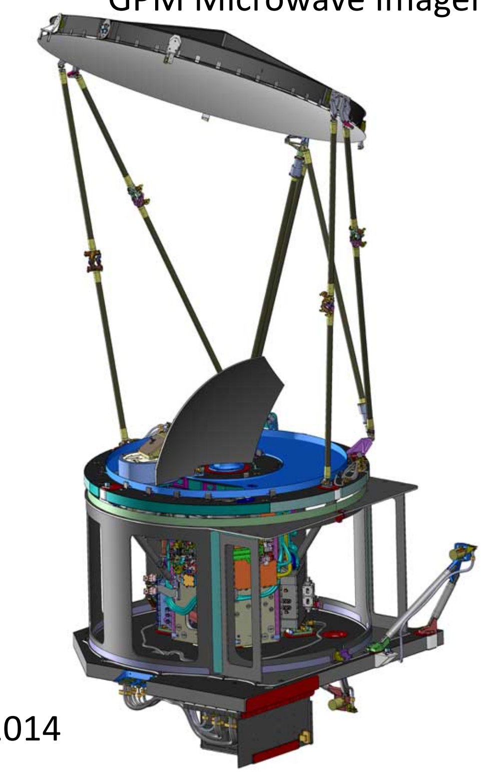
There is only a relatively small class of applications where you need to consider both scatting and emission at the same time!!

1. Microwave remote sensing of precipitation
2. Remote sensing of clouds near 4um wavelength



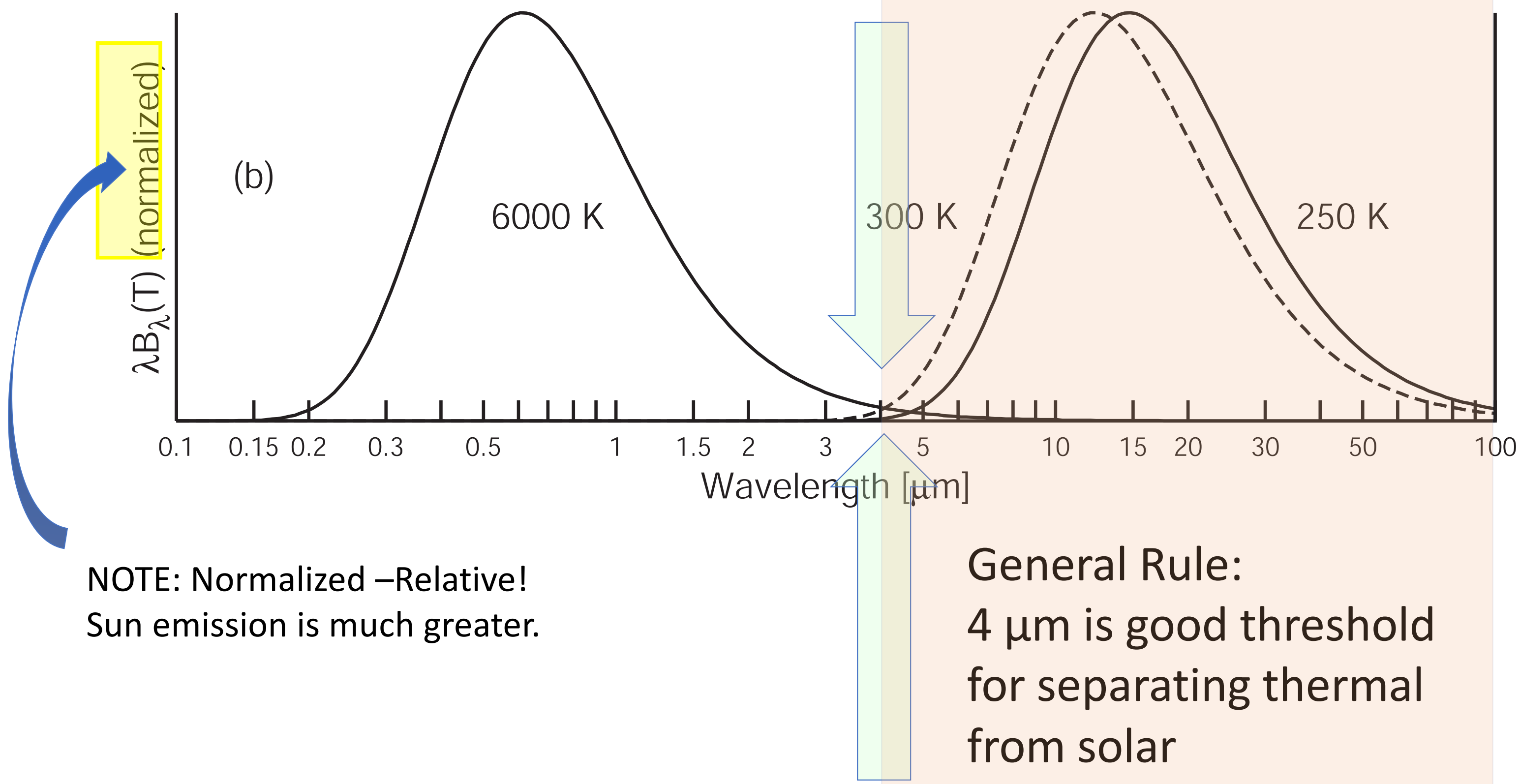


GPM Microwave Imager



Launched 2014

When Does Thermal Emission Matter?



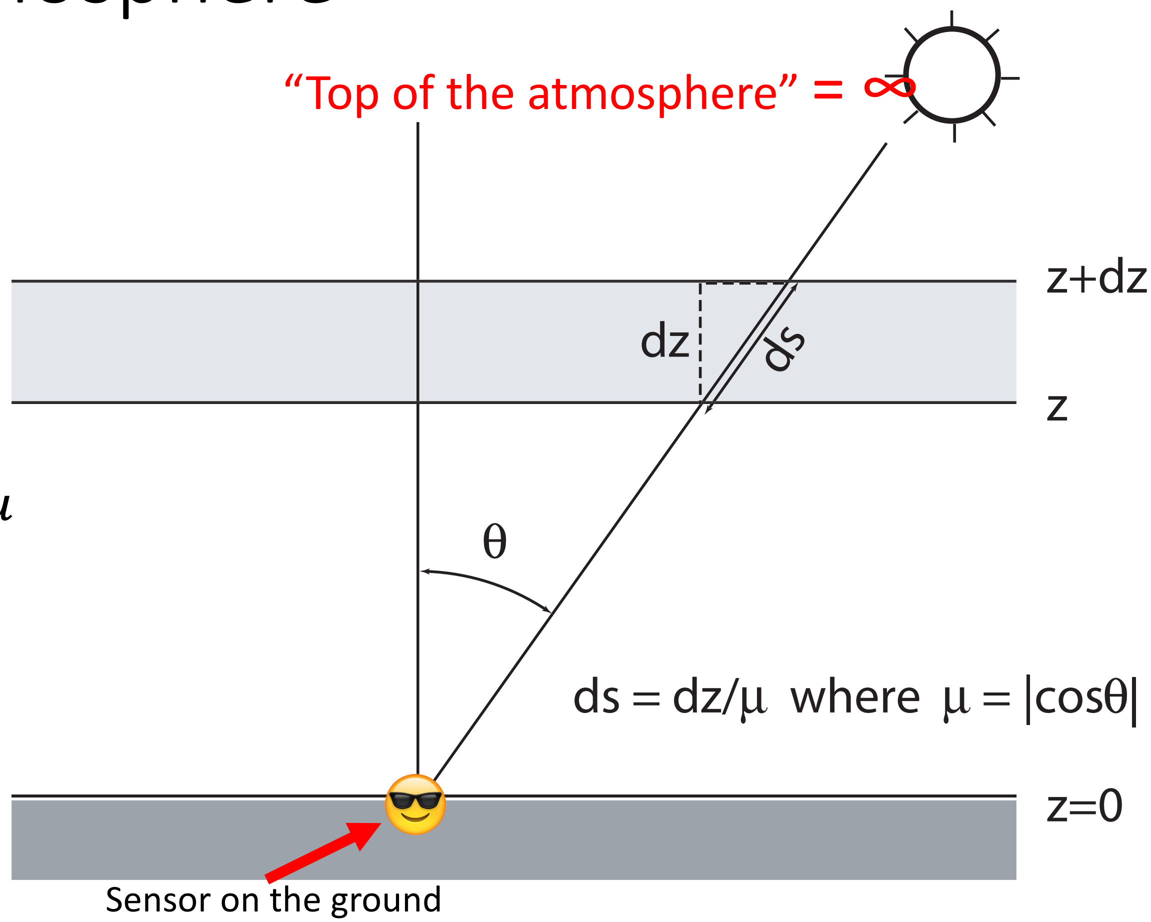
NOTE: Normalized –Relative!
Sun emission is much greater.

General Rule:
4 µm is good threshold for separating thermal from solar

Plane Parallel Atmosphere

Most analytic solution and approximations to the radiative transfer equation with scattering have been derived for the plane parallel case

$$t^*(0,\infty) \equiv \frac{I_\lambda(0)}{I_\lambda(\infty)} = e^{-\tau^*/\mu}$$



Scattering Phase Function

$$dI_{scat} = \frac{\beta_s}{4\pi} \int_{4\pi} p(\hat{\Omega}', \hat{\Omega}) I(\hat{\Omega}')\, d\omega' ds$$

$$Where\ radiation\ from\ any\ direction\ \hat{\Omega}',$$
$$can\ contribute\ scattered\ radiation\ in\ the\ direction\ of\ interest\ \hat{\Omega}$$

$p(\hat{\Omega}', \hat{\Omega})$ = phase function, which is required to satisfy the normalization across a sphere

$$1 = \frac{1}{4\pi} \int_{4\pi} p(\hat{\Omega}', \hat{\Omega}) d\omega'$$

Functional dependence of the phase function on $(\hat{\Omega}', \hat{\Omega})$ can be quite complicated depending on the size and shape of the particles responsible for scattering

Scattering Phase Function

$$1 = \frac{1}{4\pi}\int_{4\pi} p(\hat{\Omega}', \hat{\Omega})\, d\omega'$$

Important simplification can be made when particles in atmosphere are spherical or randomly oriented
(does not work for dust or ice)

Scattering phase function depends only on the angle Θ between the original direction $\hat{\Omega}'$ and scattered direction $\hat{\Omega}$

$$\cos\Theta = \hat{\Omega}' \cdot \hat{\Omega}$$

$$1 = \frac{1}{4\pi}\int_{4\pi} p(\cos\Theta)\, d\omega'$$

Scattering Phase Function

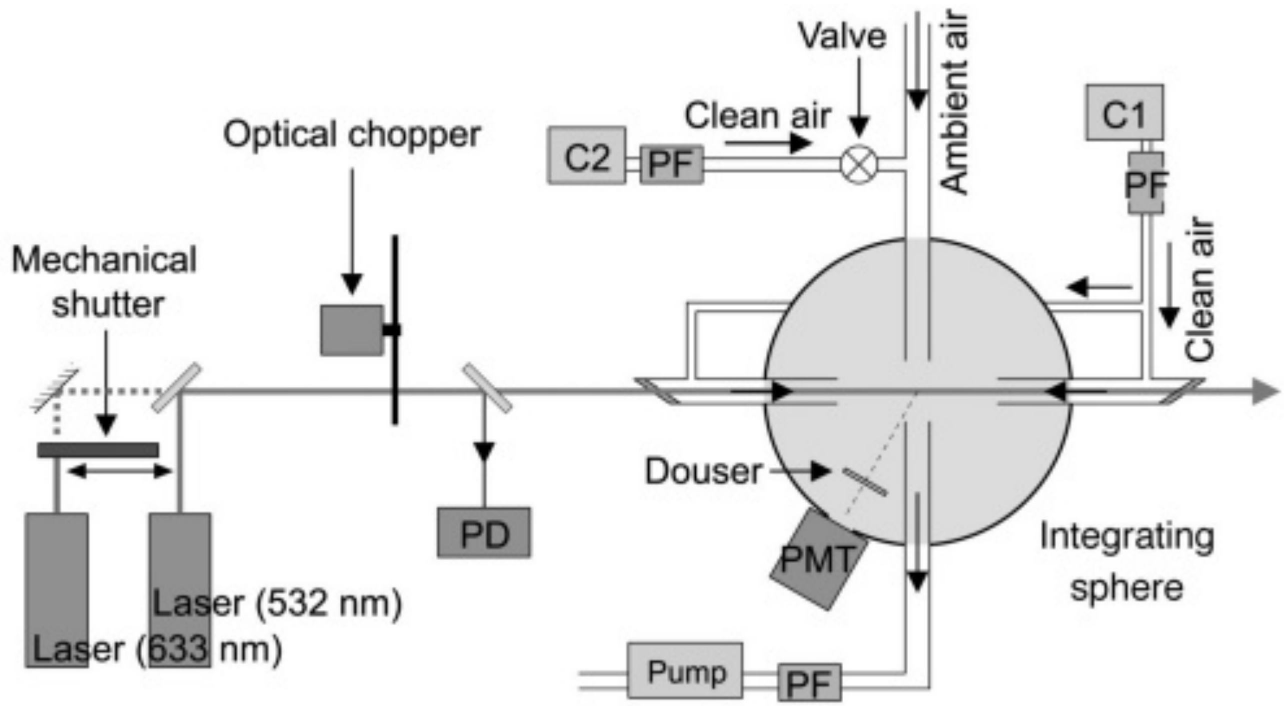
$$1 = \frac{1}{4\pi} \int_{4\pi} p(\cos\Theta)\, d\omega'$$

$$1 = \frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi} p(\cos\Theta) \sin\Theta \, d\Theta \, d\phi$$

$$1 = \frac{1}{2} \int_{-1}^{1} p(\cos\Theta)\, d\cos\Theta$$

This simplified notation will be used from this point forward

Measure the Scattering Phase Function



Isotropic Scattering

$$1 = p(\cos\Theta)$$

- Simplest possible scattering phase function (constant)
- Case that all directions $\hat{\Omega}$ are equally likely for a photon that has just been scattered.
- New direction of photon no way predictable from the direction it was traveling prior to being scattered.
- **Scattering by real particle in the atmosphere is never even approximately isotropic,** yet it is useful for theoretical studies

For Isotropic Scattering

Radiative Transfer Equation Now with Scattering

$$\frac{dI(\hat{\Omega})}{d\tau} = I(\hat{\Omega}) - (1-\varpi)B - \frac{\varpi}{4\pi}\int_{4\pi} p(\hat{\Omega}', \hat{\Omega}) I(\hat{\Omega}') d\omega'$$

Becomes

$$\frac{dI(\hat{\Omega})}{d\tau} = I(\hat{\Omega}) - (1-\varpi)B - \frac{\varpi}{4\pi}\int_{4\pi} I(\hat{\Omega}') d\omega'$$

$$Gain\ independence\ from\ both\ \hat{\Omega}', \hat{\Omega}$$

Asymmetry Parameter

$$g \equiv \frac{1}{4\pi} \int_{4\pi} p(\cos\Theta) \cos\Theta \, d\omega$$

- Used to consider the flux, rather than the intensity
- Relative proportion of photons that are scattered in the forward versus backward direction
- May be interpreted as the average value of $\cos\Theta$ for a large number of scatted photons

$$-1 \le g \le 1$$

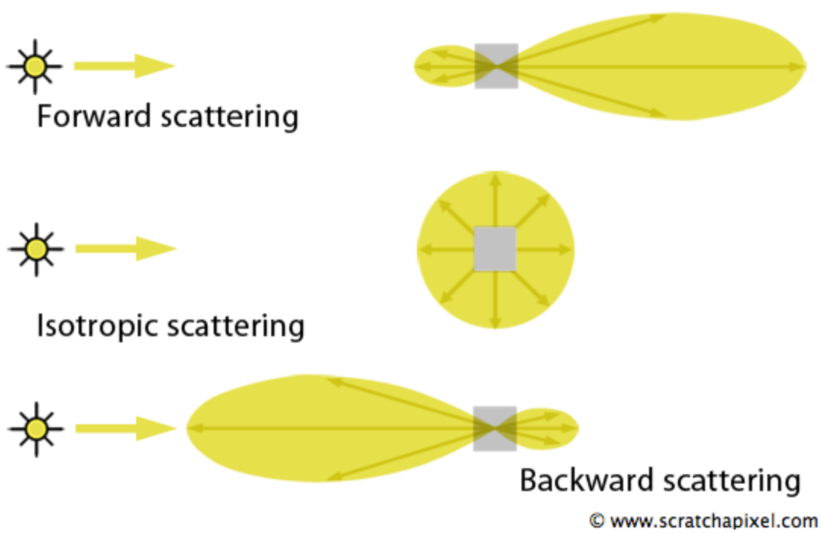
Asymmetry Parameter

$$g \equiv \frac{1}{4\pi} \int_{4\pi} p(\cos\Theta) \cos\Theta \, d\omega$$

- Relative proportion of photons that are scattered in the forward versus backward direction

$$-1 \leq g \leq 1$$

When g>0
Forward scattering



When g<0
Backward scattering

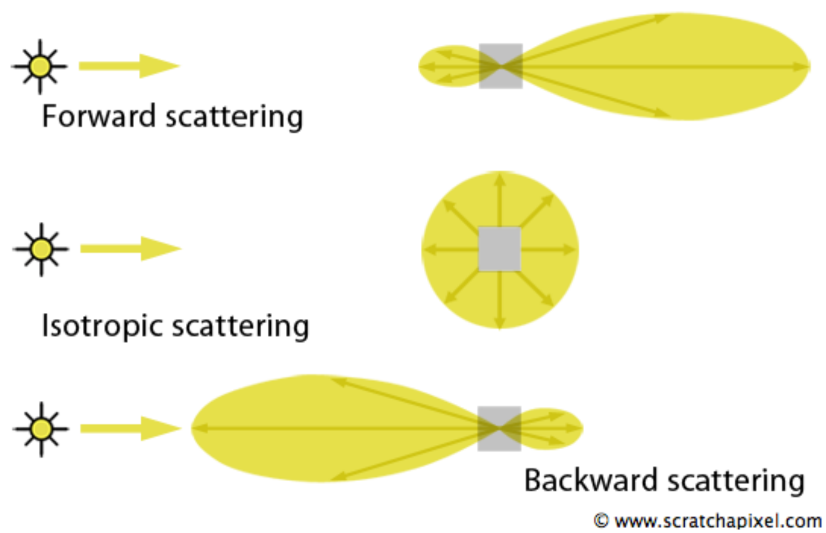
Asymmetry Parameter

$$g \equiv \frac{1}{4\pi} \int_{4\pi} p(\cos\Theta)\cos\Theta \, d\omega$$

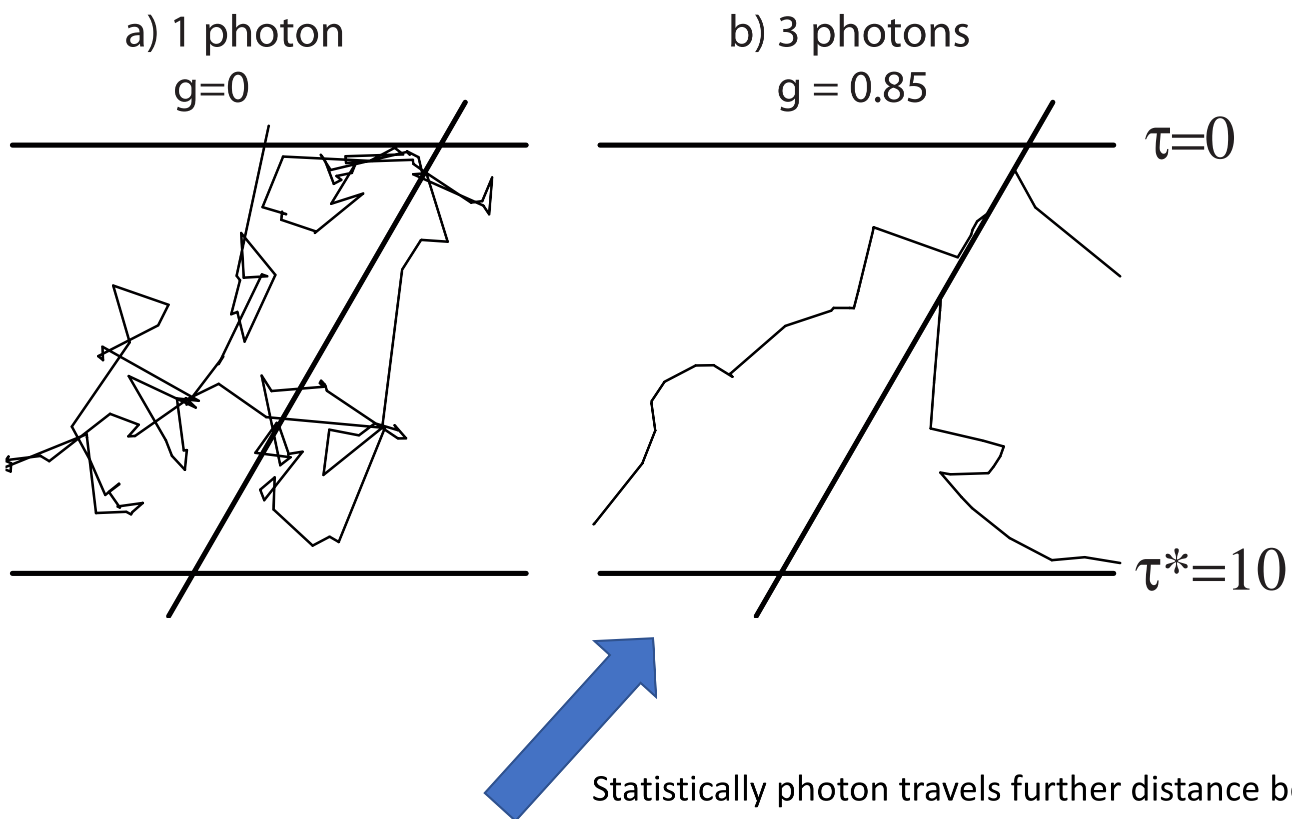
What does g= for isotropic scattering?

$$-1 \leq g \leq 1$$

When g>0 Forward scattering



When g<0 Backward scattering



- Cloud droplets are strongly forward scattering at solar wavelengths.
- g falls in the range of 0.8-0.9

Statistically photon travels further distance before experiencing a sharp reversal in course.

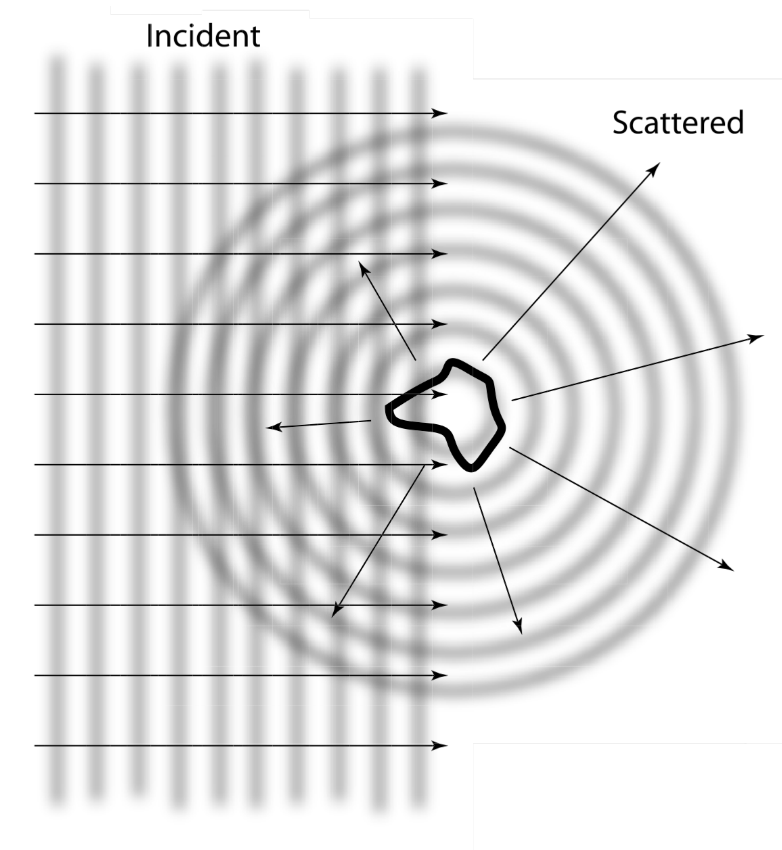
More likely to reach cloud base!

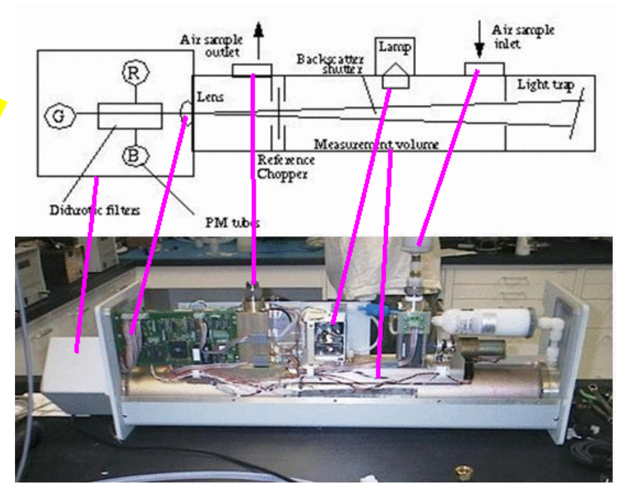
Recall

- Scattering by real particle in the atmosphere is never even approximately isotropic
- Often COMPLICATED!
- Can be simplified by knowing g (asymmetry parameter)

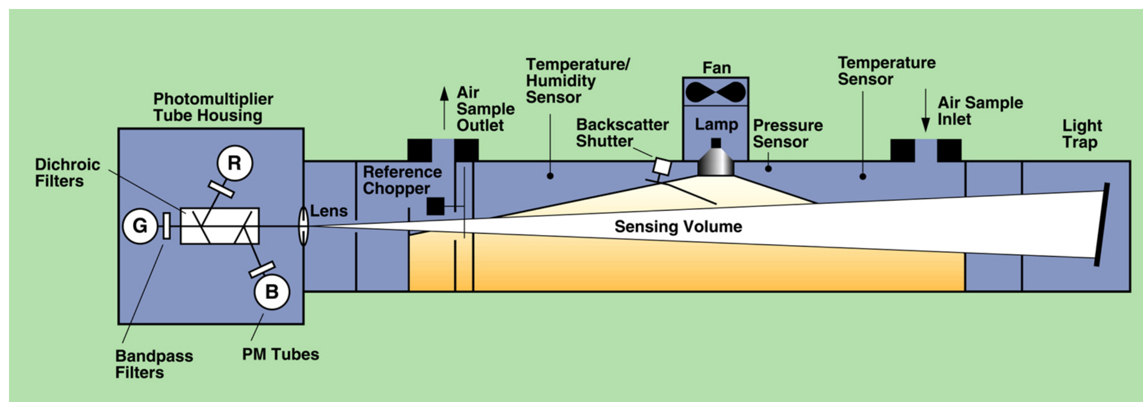
Scattering of light by particle

Review





Measure with a Nephelometer



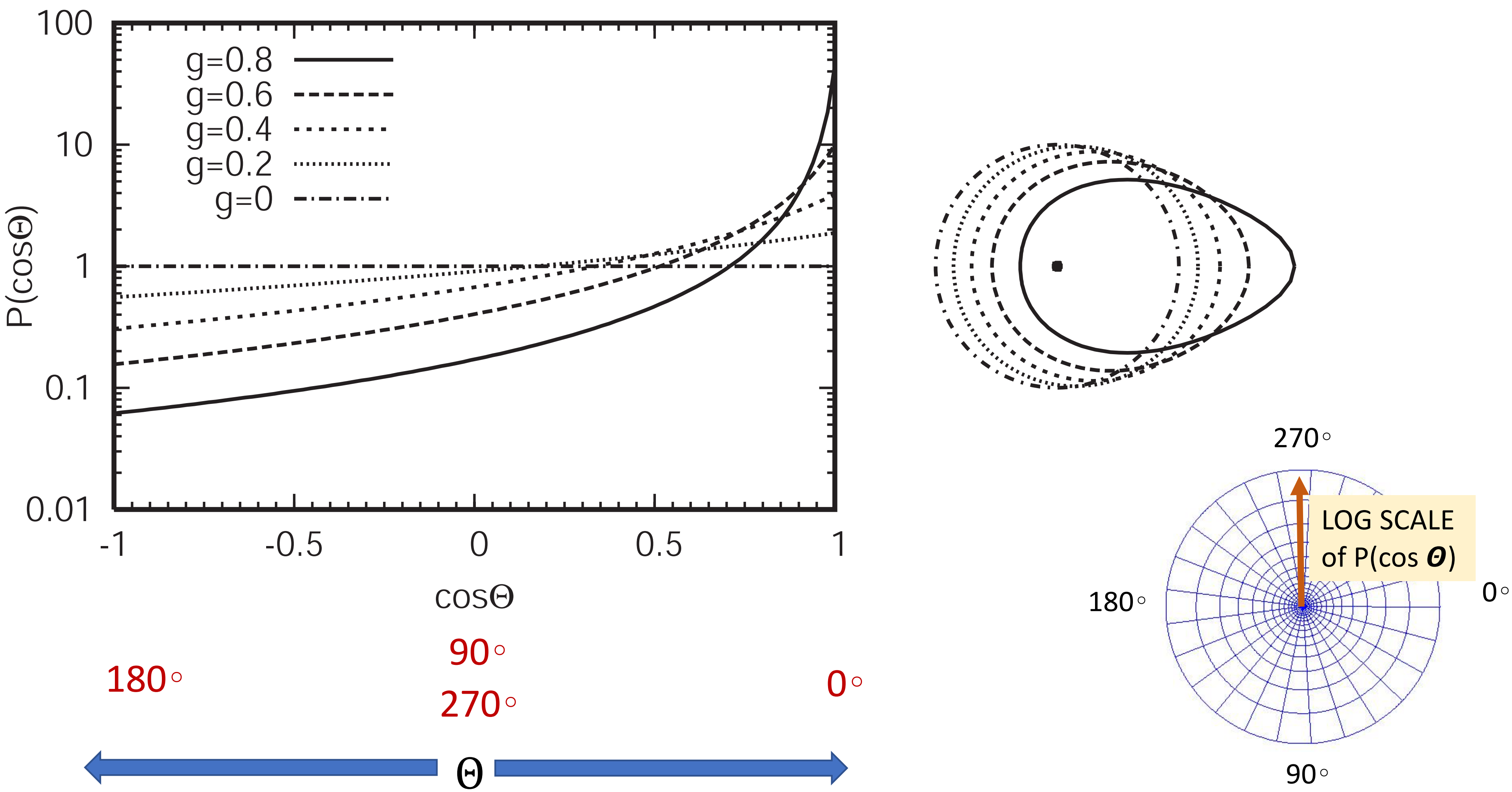
Recall

- Scattering by real particle in the atmosphere is never even approximately isotropic
- Often COMPLICATED!
- Can be simplified by knowing g (asymmetry parameter)

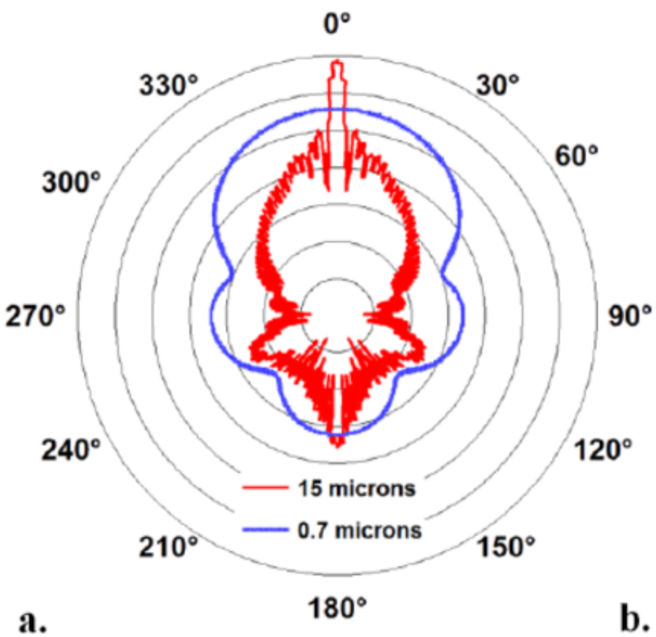
Henyey Greenstein Phase Function

$$p_{HG}(\cos\Theta) = \frac{1-g^2}{\left(1+g^2-2g\cos\Theta\right)^{3/2}}$$

Henyey-Greenstein Phase Function



REAL Scattering Phase Function



b.

Particle size	$d = 0.7\ \mu m$	$d = 15\ \mu m$
Wavelength: λ [nm]	800	800
Refractive index particles: n_p	1.58+0.0*i*	1.58+0.0*i*
Refractive index medium: n_m	1.33+0.0*i*	1.33+0.0*i*
Anisotropy factor: g	0.845	0.920
Extinction cross-section: σ_e [mm²]	3.390 10^{-7}	4.006 10^{-4}
Number density at *OD* 10: N [#/mm³]	2.950 10^{6}	2496
Number density at *OD* 14: N [#/mm³]	4.130 10^{6}	3494
Scattering parameter: $X = (\pi \cdot d / \lambda)$	2.75	58.9

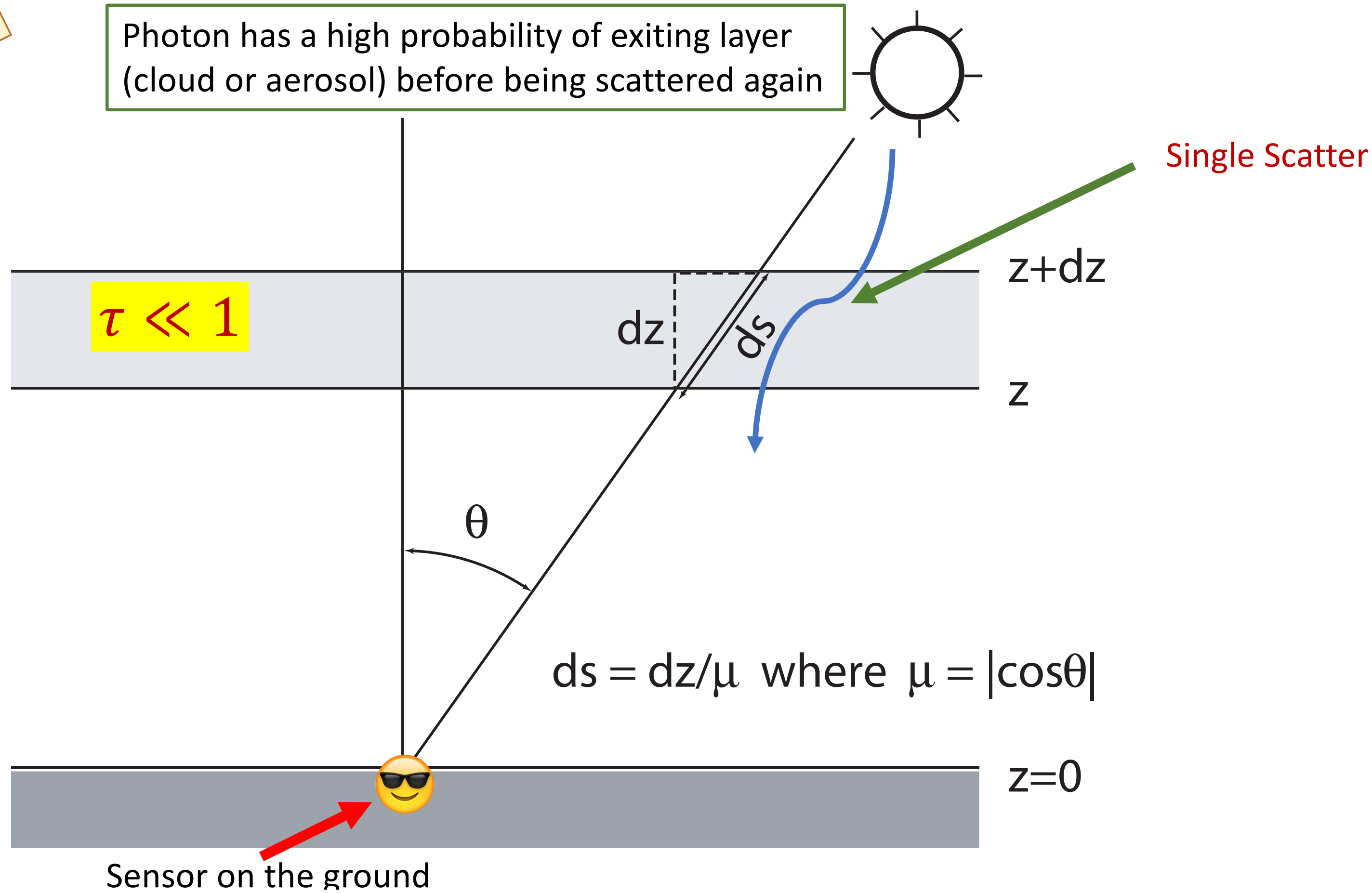
a) Polar plot - logarithmic scale - of the angular Lorenz-Mie scattering phase functions for the two different sizes of polystyrene microspheres. The phase function for 0.7 µm (blue line) with broad-lobed, relative homogeneous angular distribution is shown superimposed on the 15 µm phase function (red line) which exhibits a strong forward scattering peak. Note that the phase functions shown here are circularly symmetric about the central (vertical) axis. The corresponding optical characteristics of the scattering particles and medium are shown in (b).

Single Scattering vs. Multiple Scattering

General Cases where Single Scattering Dominates

1. Layer is optically thin (i.e. $\tau \ll 1$)

Photon has a high probability of exiting layer (cloud or aerosol) before being scattered again

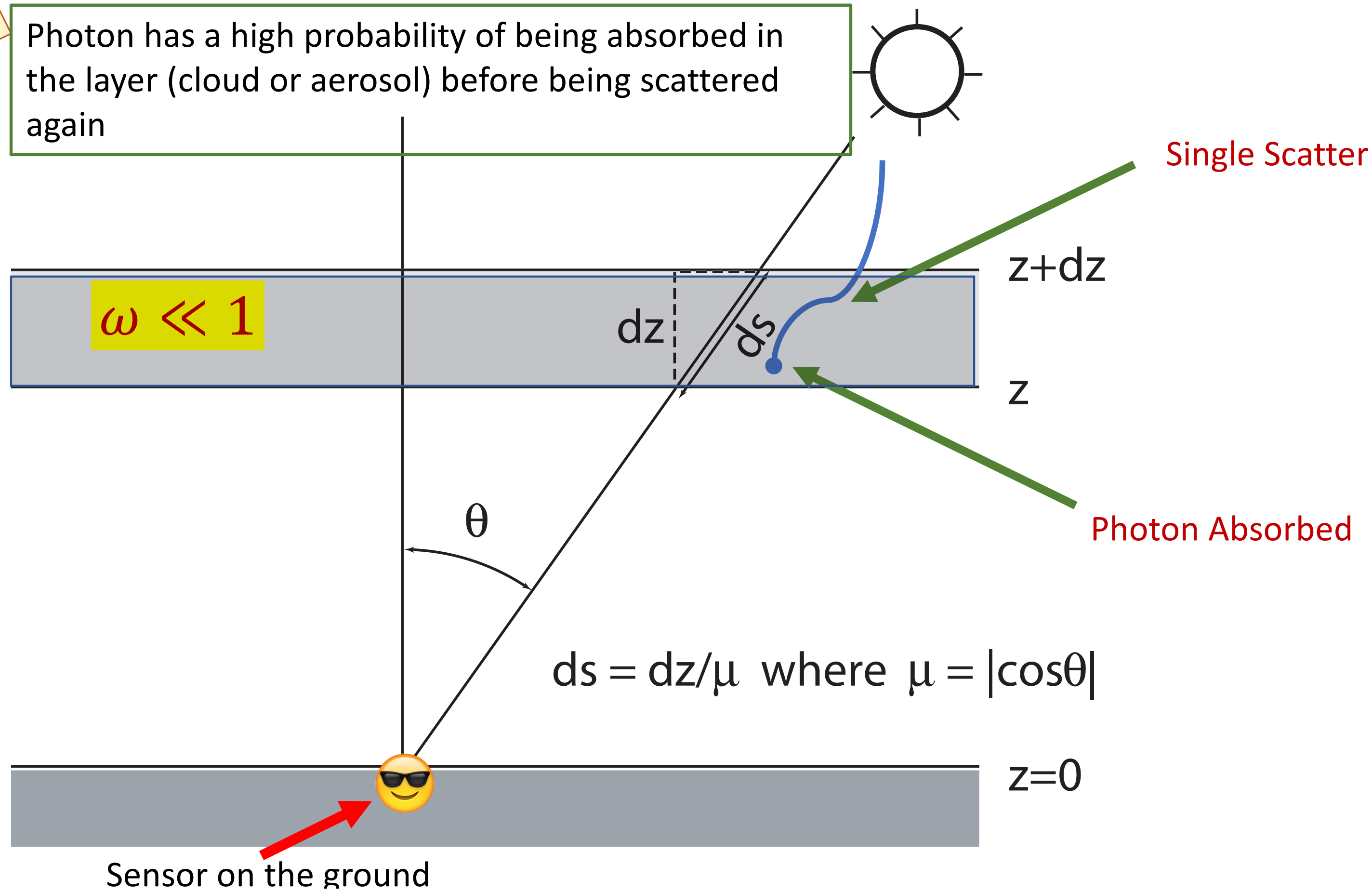


Single Scattering vs. Multiple Scattering

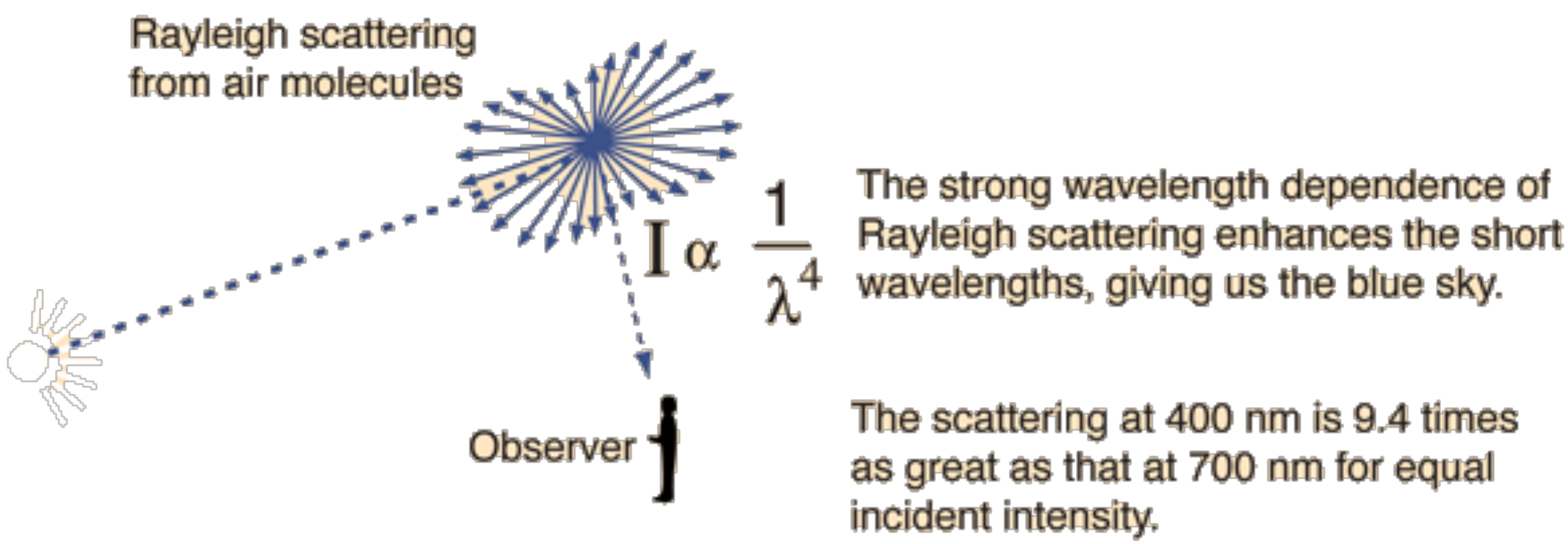
General Cases where Single Scattering Dominates

2. Layer is strongly absorbing ($\omega \ll 1$)

Photon has a high probability of being absorbed in the layer (cloud or aerosol) before being scattered again



Applications

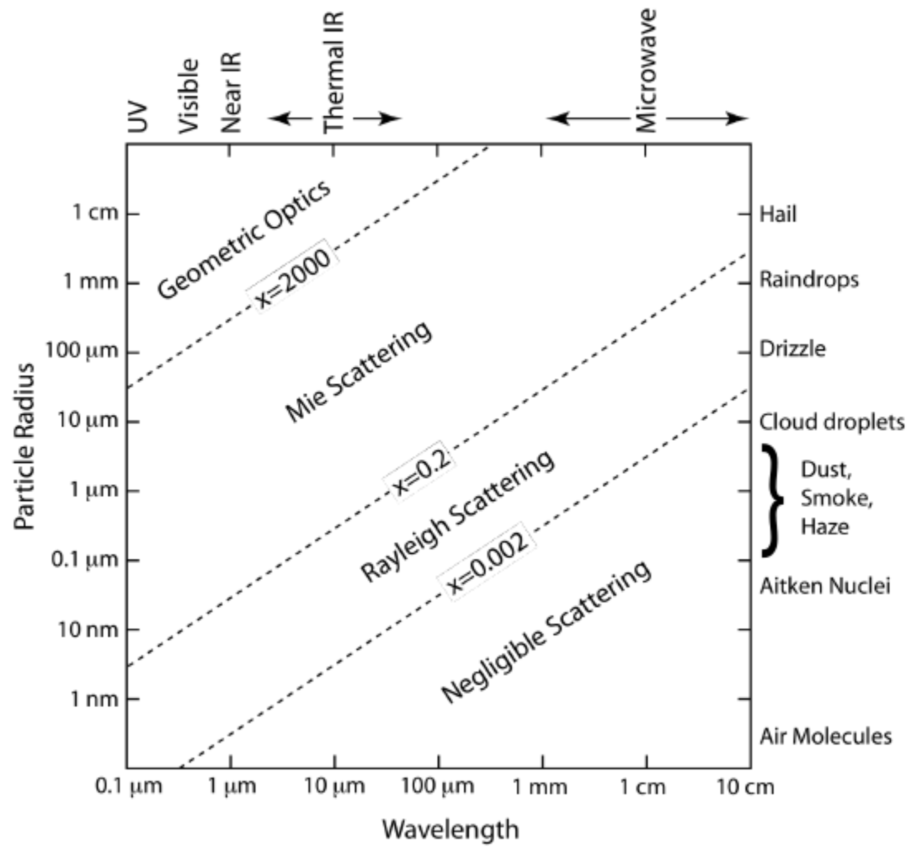


Why is sun yellow?

http://www.iflscience.com/physics/why-sky-blue-and-sun-yellow-ls-currently-working/

Light scattering regimes

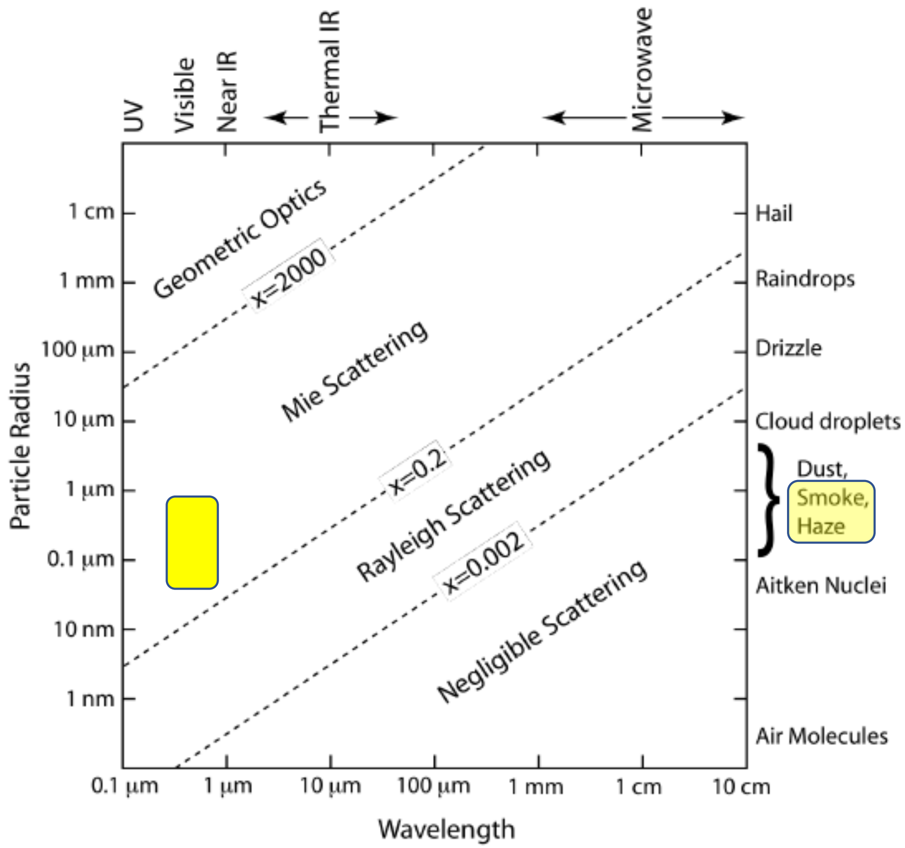
Particle Scattering

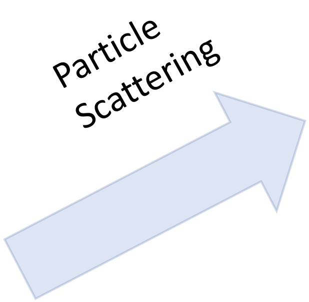


There are many regimes of particle scattering, depending on the particle size, the light wave-length, and the refractive index.

This plot considers only single scattering by spheres. Multiple scattering and scattering by non-spherical objects can get really complex!

Light scattering regimes





There are many regimes of particle scattering, depending on the particle size, the light wave-length, and the refractive index.

This plot considers only single scattering by spheres. Multiple scattering and scattering by non-spherical objects can get really complex!

Visibility Regulations in Intermountain West

The goal of the EPA Regional Haze Program is to achieve natural background visibility conditions (pristine) in all Class I areas by 2064.

Lost visibility

Colorado's four national parks have natural visibility of about 175 miles but combined have an average 47 miles of distance lost because of haze.

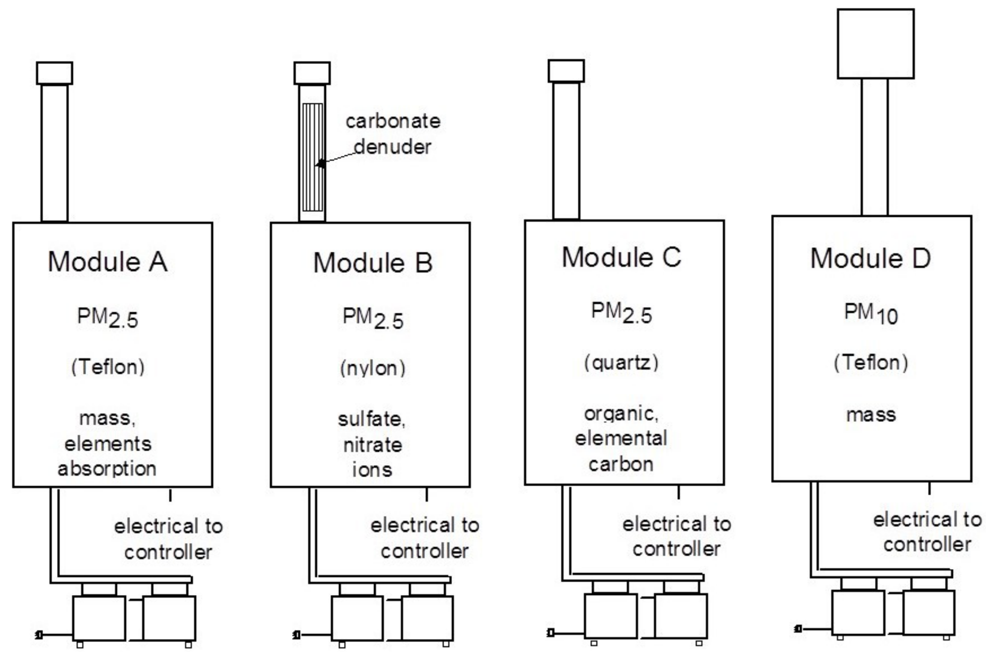
	Miles of natural visibility	Mileage short because of haze
1. Great Sand Dunes	172	55
2. Mesa Verde	174	50
3. Rocky Mountain	175	46
4. Black Canyon	176	38

Sources: State and federal data, National Parks Conservation Association

The Denver Post

Visibility Regulations in Intermountain West

The goal of the EPA Regional Haze Program is to achieve natural background visibility conditions (pristine) in all Class I areas by 2064.

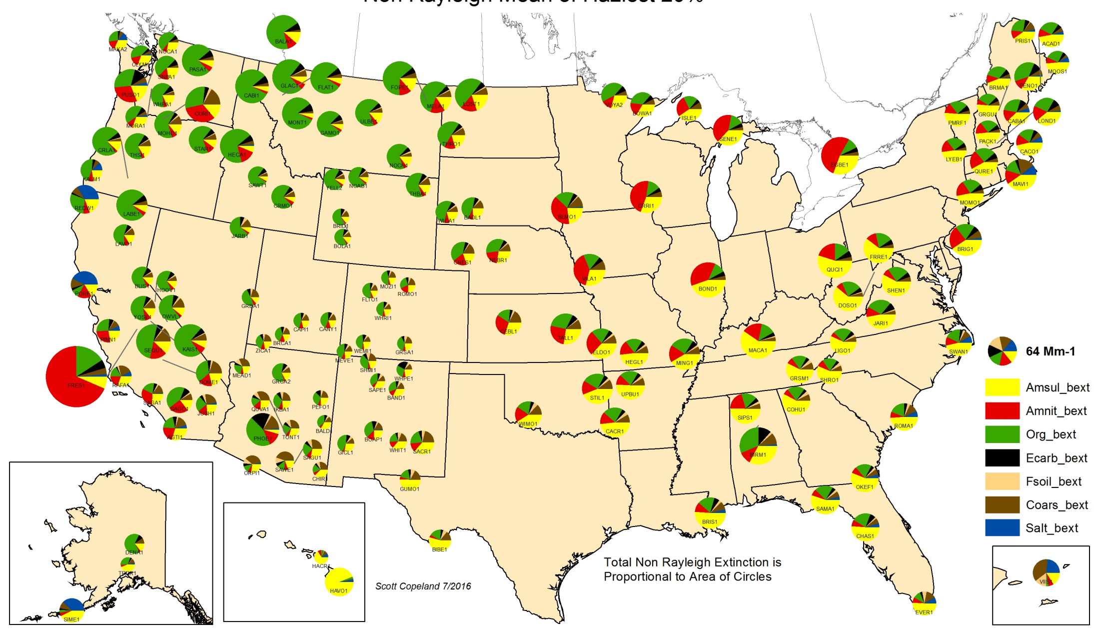


Visibility Regulations in Intermountain West

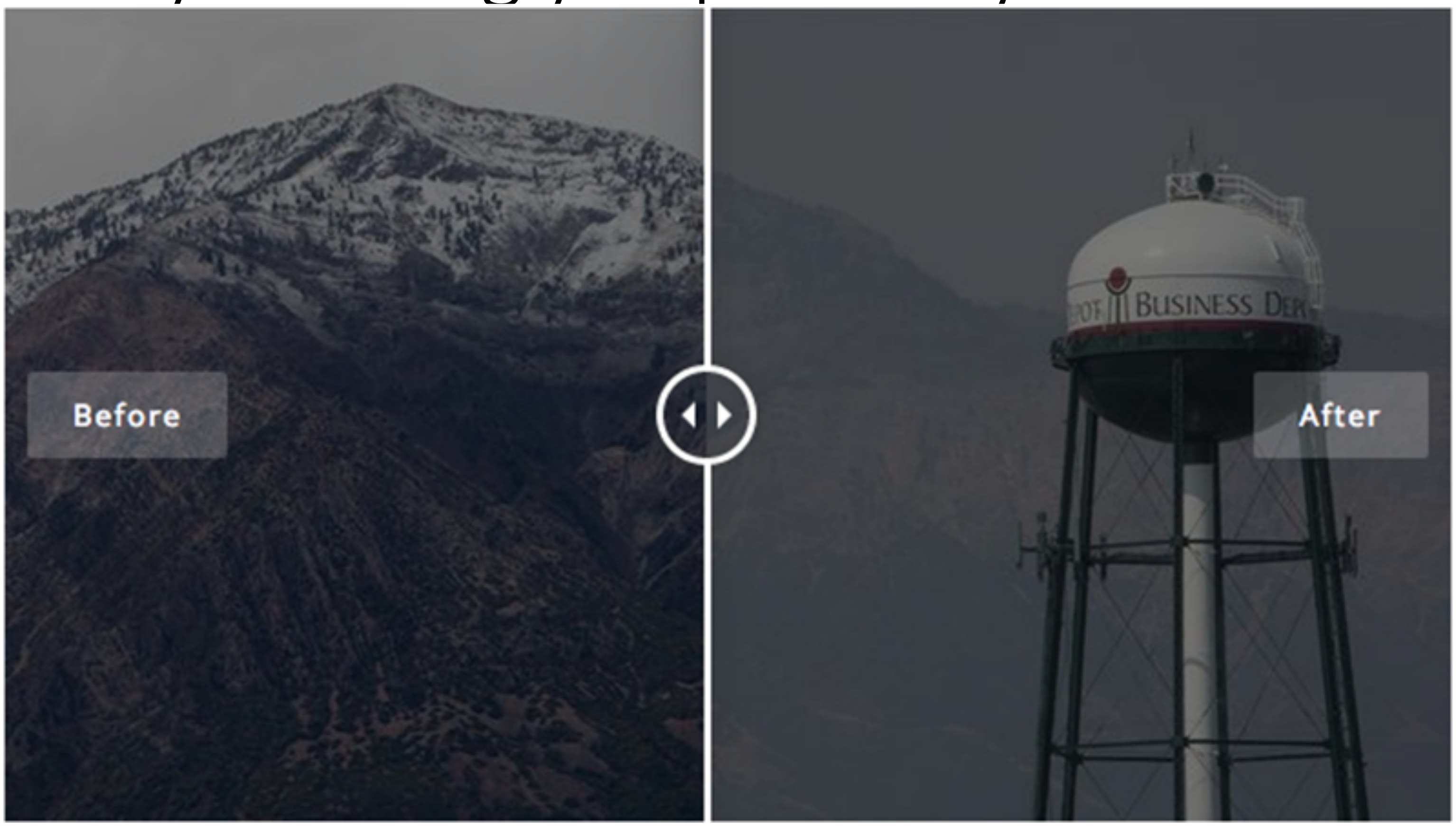
The goal of the EPA Regional Haze Program is to achieve natural background visibility conditions (pristine) in all Class I areas by 2064.

IMPROVE Data - 2015 Second IMPROVE Algorithm

Non Rayleigh Mean of Haziest 20%



Visibility is strongly impaired by fire



Northern Utah: Before and after smoke impact in August 2015

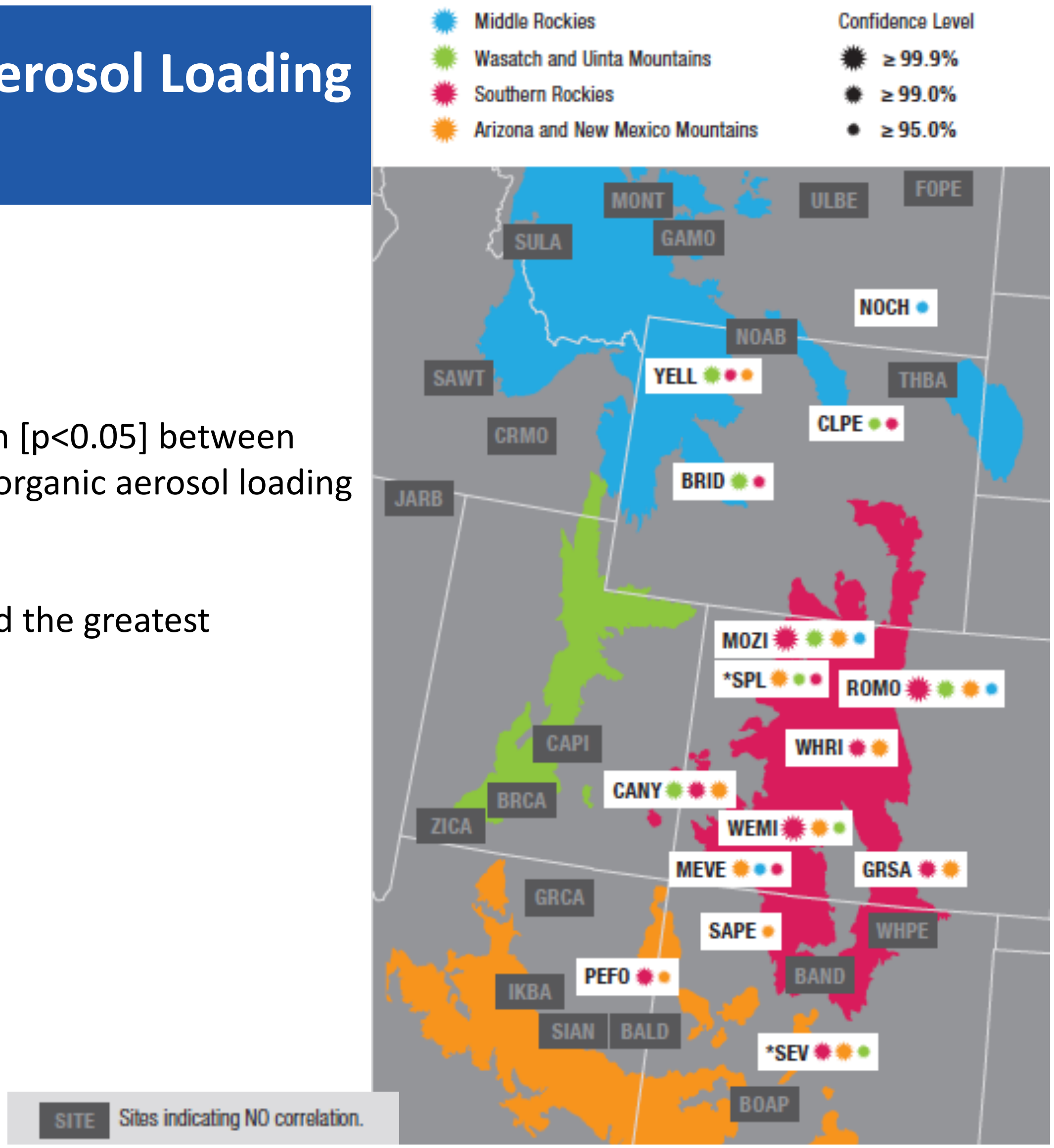
Link between Aerosol Loading and Drought

12 IMPROVE sites
2 AOD sites

Demonstrated a correlation [p<0.05] between surface level summertime organic aerosol loading and aridity.

Organic aerosol loading had the greatest correlation with aridity.

Southern Rockies Area

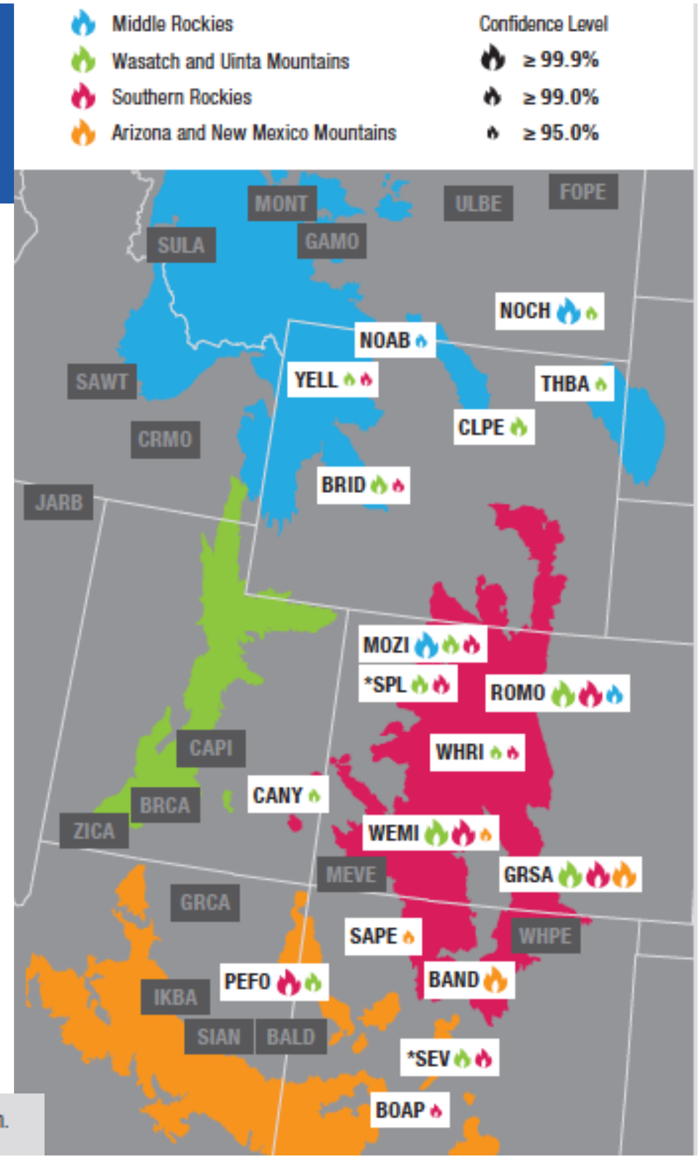


Link between Aerosol Loading and Fire Area

16 IMPROVE sites
2 AOD sites

Demonstrated a correlation [$p<0.05$] between surface level summertime organic aerosol loading and fire area burned.

Southern Rockies Area
Middle Rockies Area



Your favorite views may be disappearing thanks to wildfire haze

January 19, 2017 By Julie Kailus

GRINDTV

Hazier days in the high country, Western U.S. due to drought and forest fires, scientists find

By **BRUCE FINLEY** | bfinley@denverpost.com
PUBLISHED: January 8, 2017 at 7:12 pm | UPD

THE DENVER POST