ATMOS 5140
Lecture 13 – Chapter 12

- Scattering and Absorption by particle

Credit to Dr. Simon Carn at Michigan Tech for several slides used in this lecture.

Radiative Transfer Equation Now with Scattering

$$dI = dI_{ext} + dI_{emit} + dI_{scat}$$

$$dI = -\beta_e I ds + \beta_a B ds + \frac{\beta_s}{4\pi} \int_{4\pi} p(\hat{\Omega}', \hat{\Omega}) I(\hat{\Omega}') d\omega' ds$$

Recall

$$\varpi = \frac{\beta_s}{\beta_e} = \frac{\beta_s}{\beta_a + \beta_s}$$

Now Divide through by

$$d\tau = -\beta_e ds$$

$$\frac{dI(\hat{\Omega})}{d\tau} = I(\hat{\Omega}) - (1 - \varpi)B - \frac{\varpi}{4\pi} \int_{4\pi} p(\hat{\Omega}', \hat{\Omega}) I(\hat{\Omega}') d\omega'$$

Radiative Transfer Equation Now with Scattering

$$\frac{dI(\widehat{\Omega})}{d\tau} = I(\widehat{\Omega}) - (1-\varpi)B - \frac{\varpi}{4\pi}\int_{4\pi} p(\widehat{\Omega}',\widehat{\Omega})I(\widehat{\Omega}')d\omega'$$

Recall

All Sources of Radiation – i.e. Source Function

$J(\widehat{\Omega})$

$$\frac{dI(\widehat{\Omega})}{d\tau} = I(\widehat{\Omega}) - J(\widehat{\Omega})$$

Scattering Phase Function

$$1 = \frac{1}{4\pi}\int_{4\pi} p(\hat{\Omega}', \hat{\Omega})\, d\omega'$$

Recall

Important simplification can be made when particles in atmosphere are spherical or randomly oriented
(does not work for dust or ice)

Scattering phase function depends only on the angle Θ between the original direction $\hat{\Omega}'$ and scattered direction $\hat{\Omega}$

$$\cos\Theta = \hat{\Omega}' \cdot \hat{\Omega}$$

$$1 = \frac{1}{4\pi}\int_{4\pi} p(\cos\Theta)\, d\omega'$$

FINAL Radiative Transfer Equation

$$\frac{dI(\widehat{\Omega})}{d\tau} = I\left(\widehat{\Omega}\right) - (1 - \varpi)B - \frac{\varpi}{4\pi}\int_{4\pi} p(\cos\Theta)\, I\left(\widehat{\Omega}'\right) d\omega'$$

All Sources of Radiation – i.e. Source Function

$$J\left(\widehat{\Omega}\right)$$

$$\frac{dI(\widehat{\Omega})}{d\tau} = I\left(\widehat{\Omega}\right) - J\left(\widehat{\Omega}\right)$$

Radiative Transfer Equation Depends on

1. β_e

Since: $d\tau = -\beta_e ds$

2. $\varpi = \dfrac{\beta_s}{\beta_e}$

3. $p(\cos\Theta)$

VERY IMPORTANT SLIDE

Parameters governing scattering

- (1) The wavelength (λ) of the incident radiation

- (2) The size of the scattering particle, usually expressed as the non-dimensional size parameter, x:

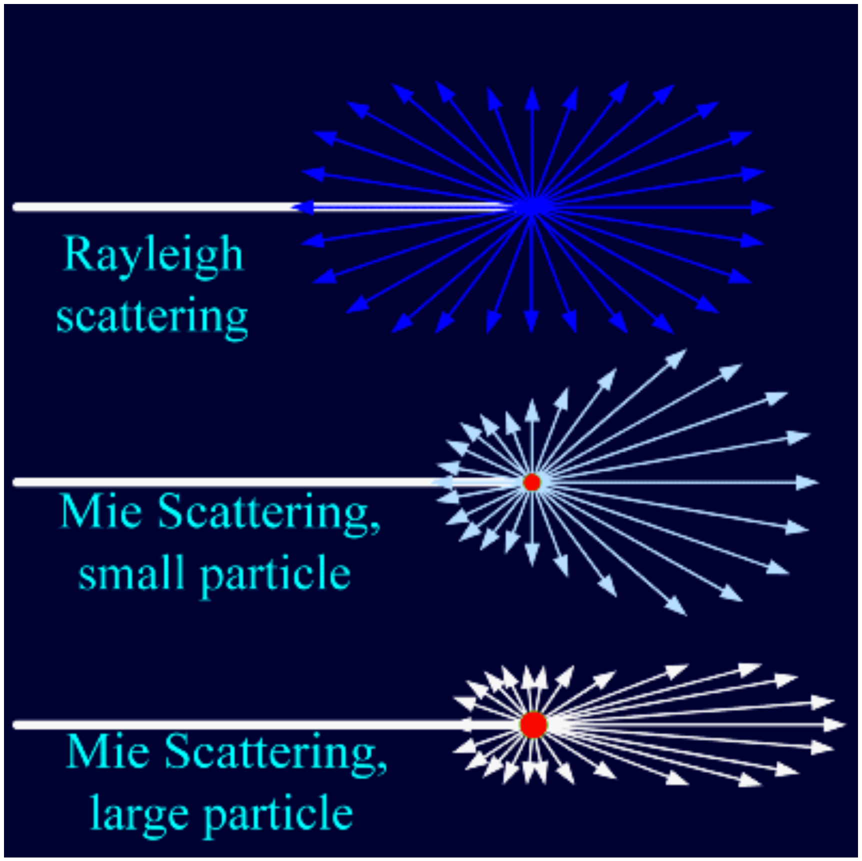
$$x = \frac{2\pi r}{\lambda}$$

- **r** is the radius of a spherical particle, **λ** is wavelength

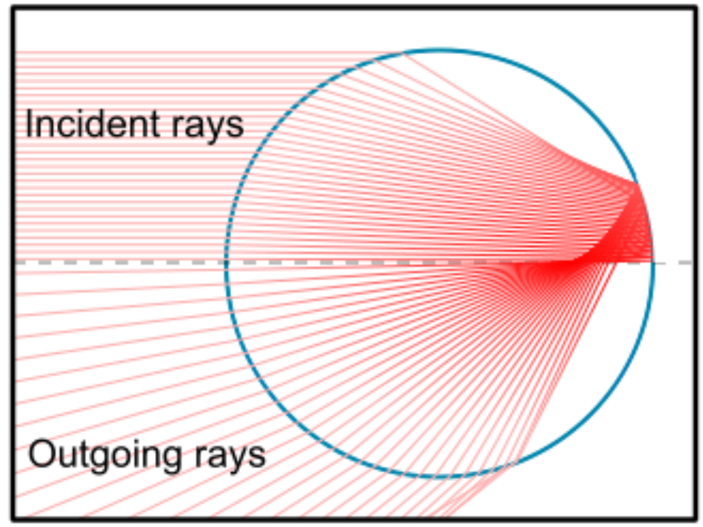
- (3) The particle optical properties relative to the surrounding medium: the complex refractive index

- Scattering regimes:

  - **x << 1** : Rayleigh scattering
  - **x ~ 1** : Mie scattering
  - **x >>1** : Geometric scattering (Ray tracing)

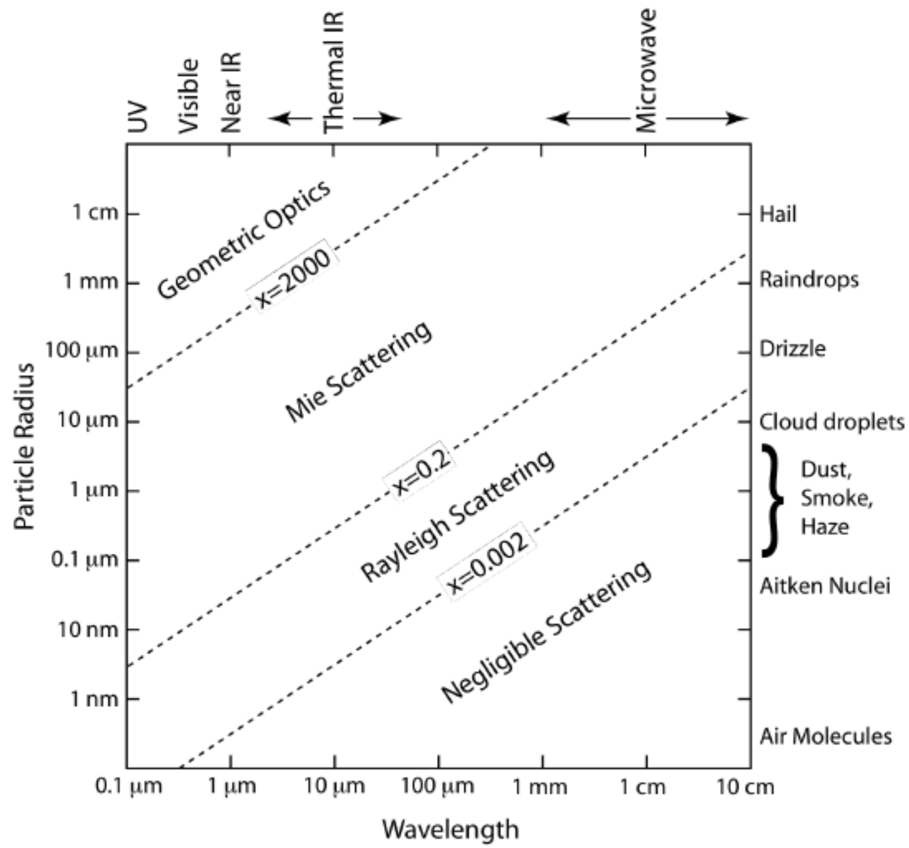


Geometric Optics (ray tracing)



Light rays enter a raindrop from one direction (typically a straight line from the Sun), reflect off the back of the raindrop, and fan out as they leave the raindrop. The light leaving the raindrop is spread over a wide angle, with a maximum intensity at 40.89–42°.

Light scattering regimes

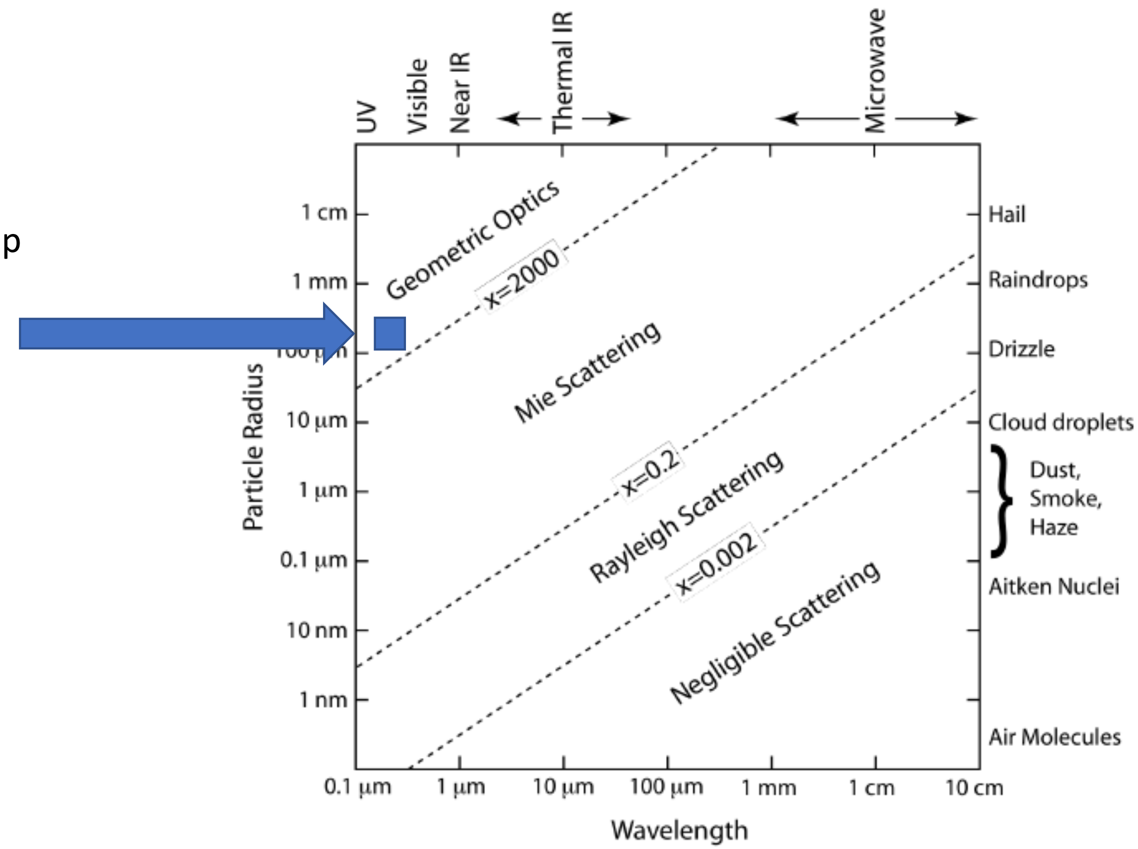


There are many regimes of particle scattering, depending on the particle size, the light wave-length, and the refractive index.

This plot considers only single scattering by spheres. Multiple scattering and scattering by non-spherical objects can get really complex!

Light scattering regimes

Previous Rain Drop Example



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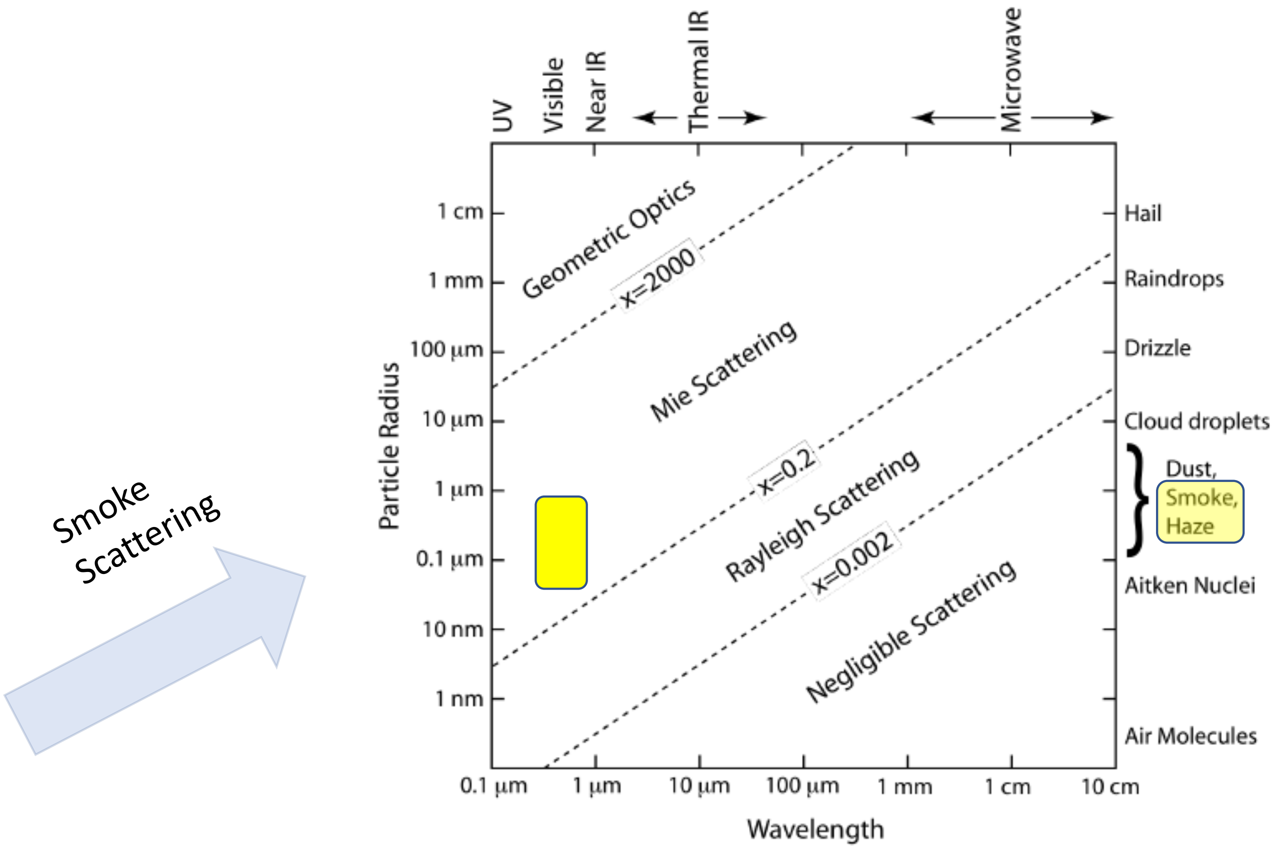
Light scattering regimes



There are many regimes of particle scattering, depending on the particle size, the light wave-length, and the refractive index.

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Maxwell's Equations for plane waves

Review

In a nonvacuum, we can write

$$|\vec{\mathbf{k}}'| + i|\vec{\mathbf{k}}''| = \omega\sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}}\sqrt{\varepsilon_0\mu_0} = \frac{\omega N}{c},$$

where the complex *index of refraction* N is given by

$$\boxed{N \equiv \sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}} = \frac{c}{c'}\,,}$$

Refractive Index

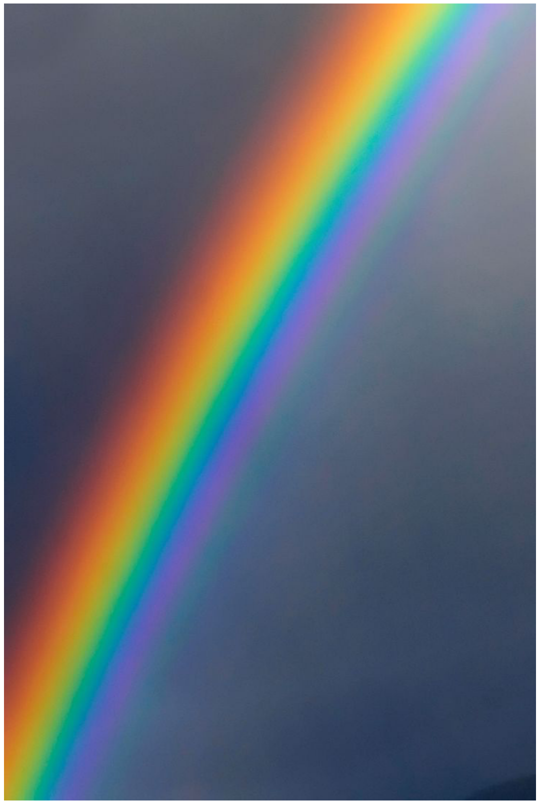
Review

- The refractive index of a material is critical in determining the scattering and absorption of light, with the imaginary part of the refractive index having the greatest effect on absorption.

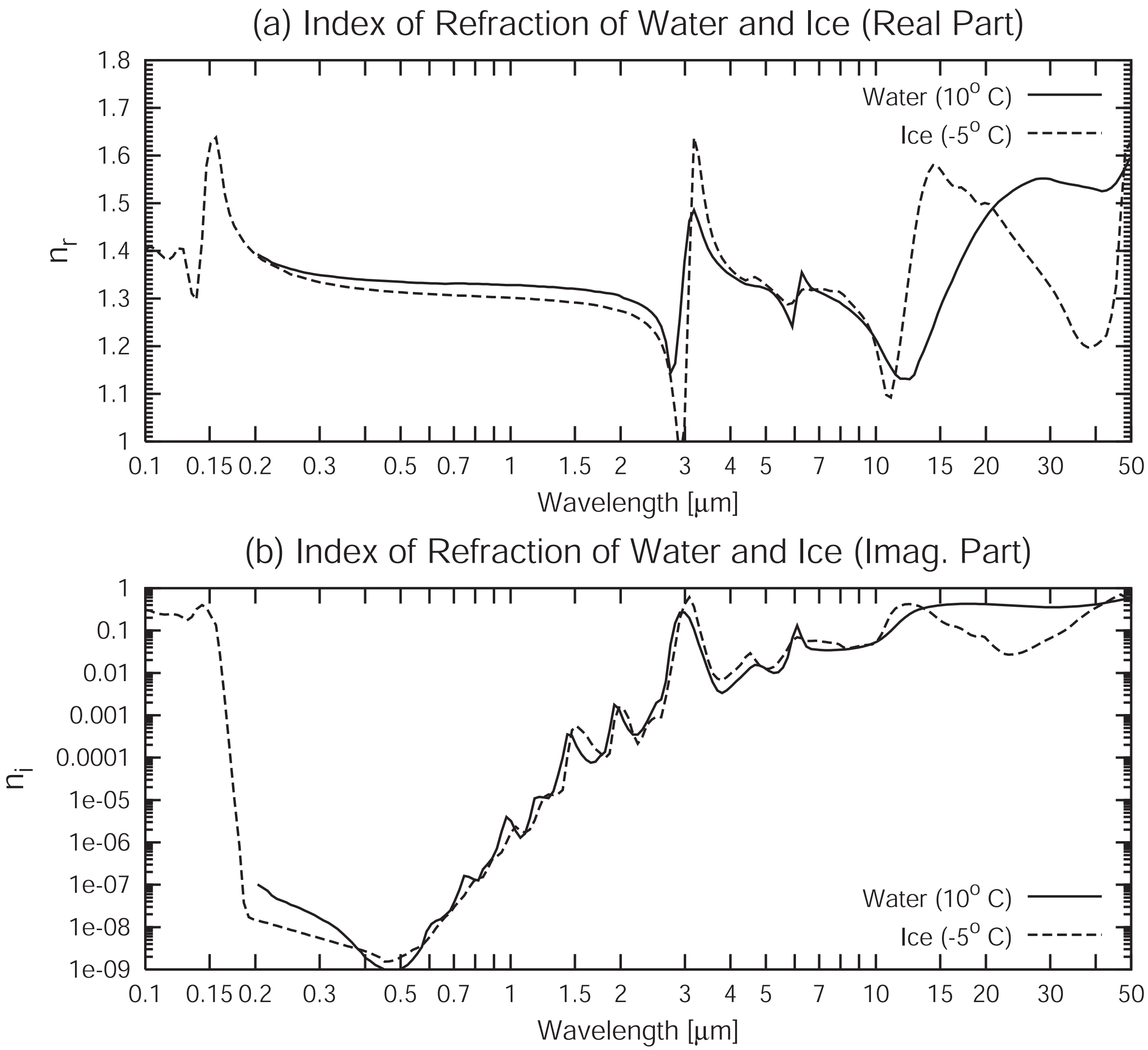
- The refractive index is NOT a constant for any substance but depends strongly on wavelength, and to a lesser degree, temperature &pressure

Rainbow

Review



- Results from variation in the real part of the refractive index with wavelength of rain drops



Review

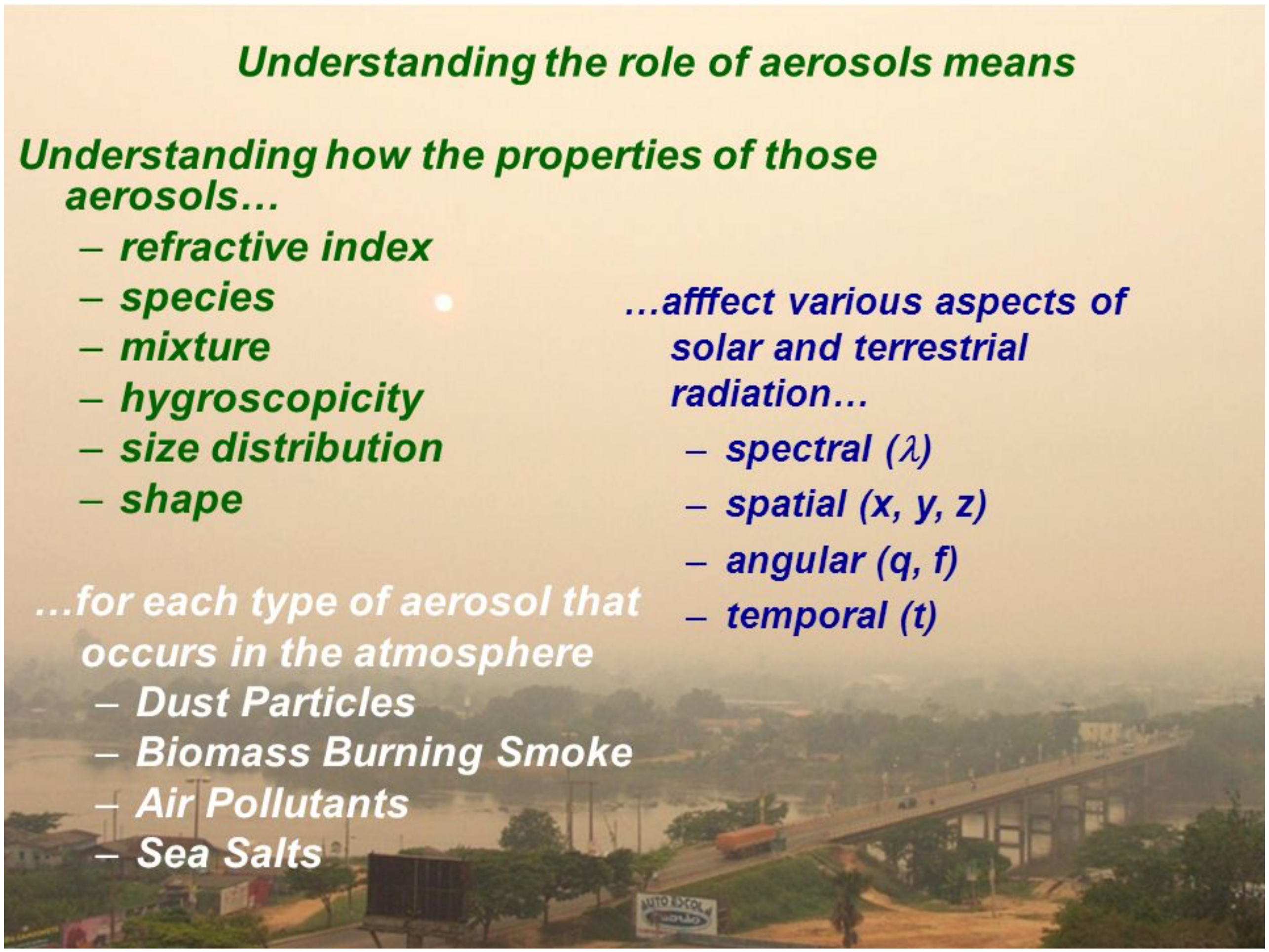
Key Points

$n_r \approx 1.333$ in visible bands

$n_i \approx 1.0003$ in the visible bands
Close to zero absorption

Real world use of Index of Refractions

Review



Shape of Particles

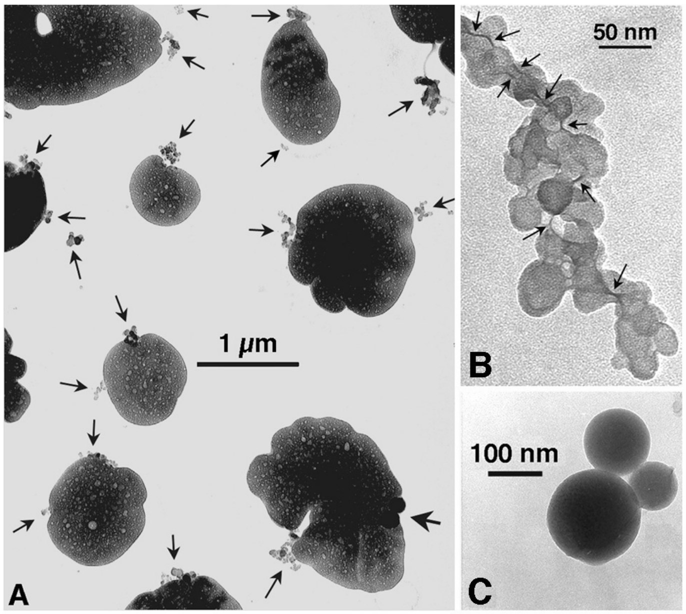
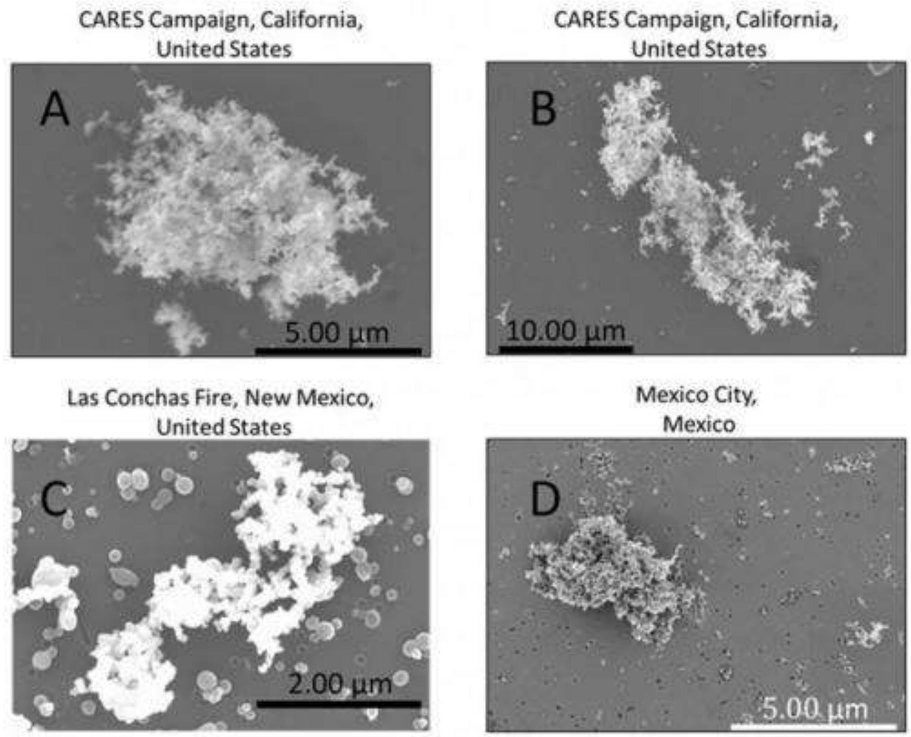


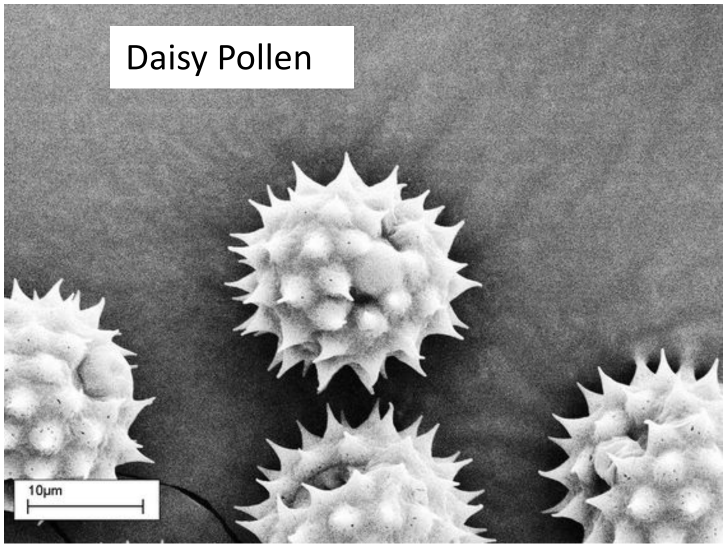
Figure 1.
Pósfai et al.: Soot and sulfate aerosol...

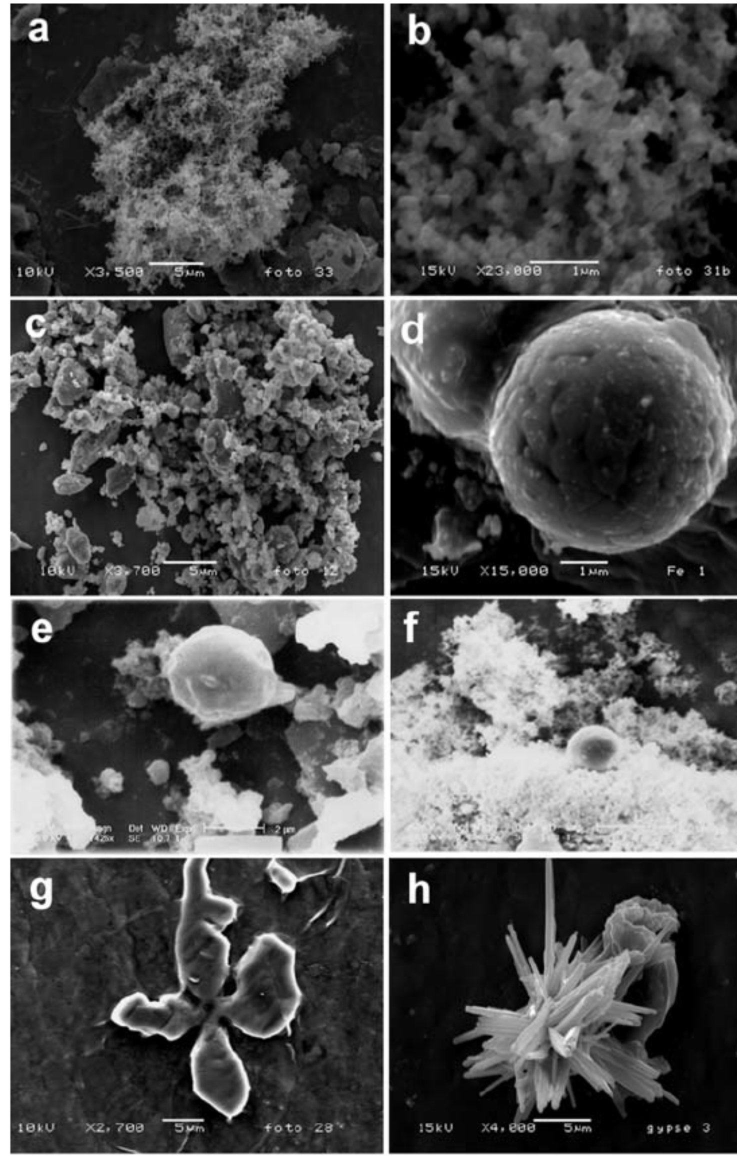


Soot in Wildfire smoke

May play a LARGE ROLE in determining its radiative properties

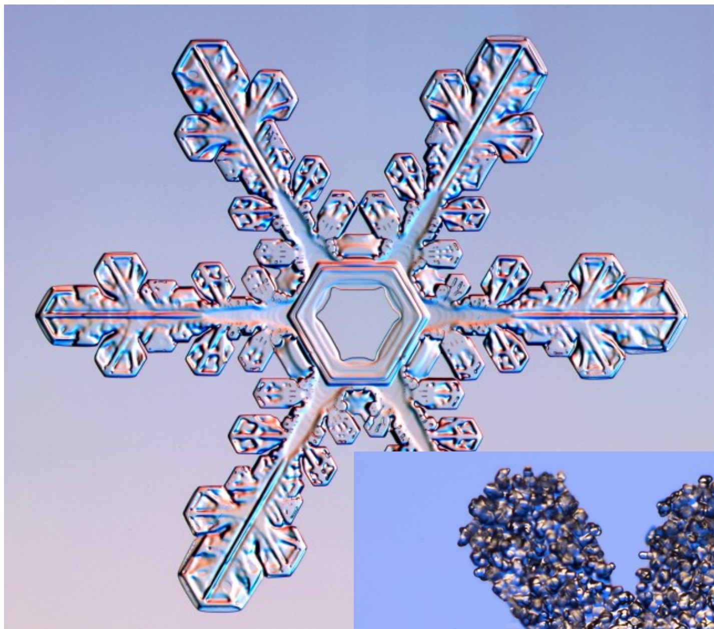
Shape of Particles

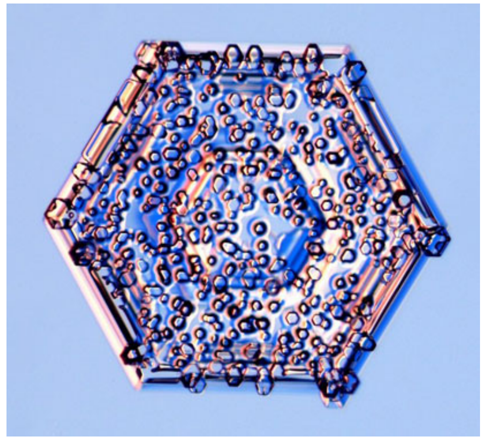


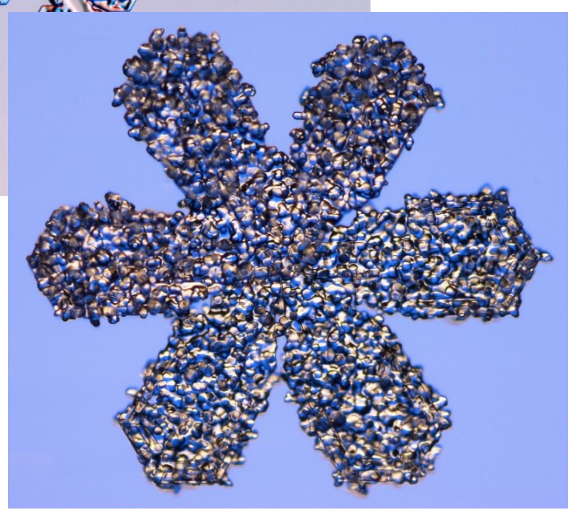


SEM micrographs of atmospheric particles. (a) Smooth spongy anthropogenic particle; (b) Detail of Figure 3a. particle; (c) cluster of particles; (d) spheroid particle of iron; (e) spheroid particle of titanium oxide; (f) spheroid particle of iron onto a spongy particle; (g) NaCl particle showing a “flower” shape; (h) sulfate particle with presence of Na and Ca.

Shape of Particles







Thanks to
Ken Libbrecht –
CalTech

Rayleigh scattering

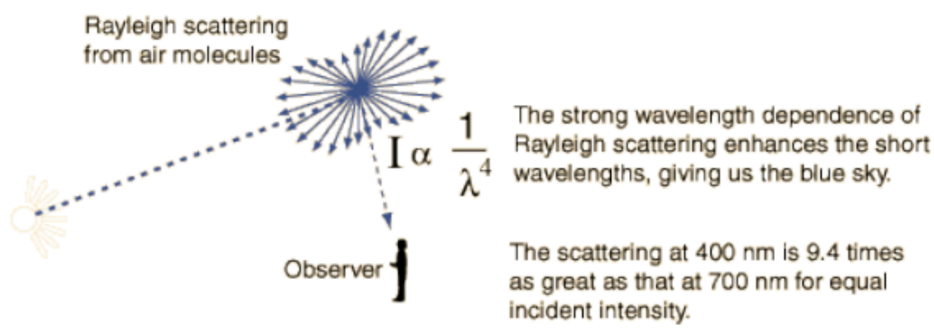
Atmospheric composition: N_2 (78%), O_2 (21%), Ar (1%)

Size of N_2 molecule: 0.31 nm

Size of O_2 molecule: 0.29 nm

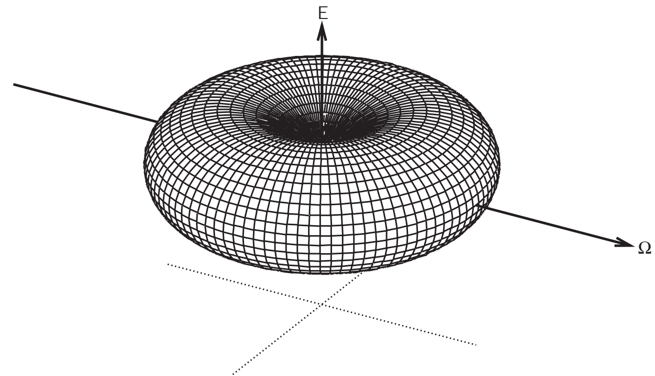
Size of Ar molecule: 0.3 nm

Visible wavelengths ~400-700 nm



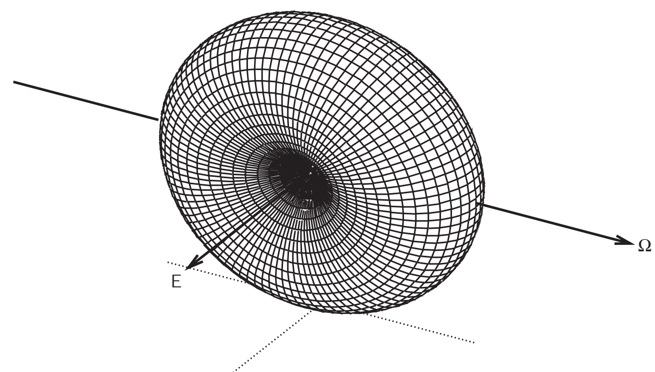
- Scattering of light off air molecules is called Rayleigh Scattering
- Involves particles much smaller than the wavelength of incident light
- Responsible for the blue color of clear sky

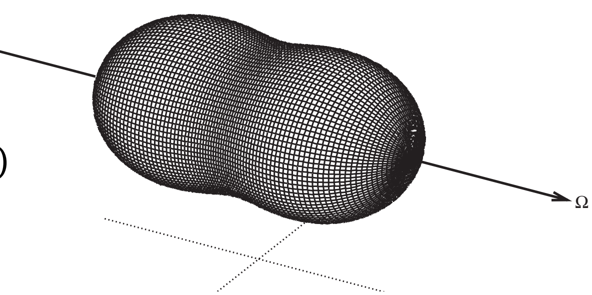
Rayleigh Phase Function



Degree of Polarization

$$P = \frac{1 - \cos^2\Theta}{1 + \cos^2\Theta}$$





$$p(\Theta) = \frac{3}{4}(1 + \cos^2\Theta)$$

Relative Index of Refraction

$$m \equiv \frac{N_1}{N_2}$$

Between the particle and medium

Phase Speed of Wave

Recall real part, $n_r = \Re(N)$
Where imaginary part, $n_i = \Im(N)$

Absorption

Review

- Mass Extinction Coefficient
  - Extinction cross-section per unit mass

$$\beta_e = \rho k_e$$

Density

- Extinction Cross Section
  - Extinction cross section referenced to individual particles

$$\beta_e = N\sigma_e$$

Number of particles per cm^3

- Extinction efficiency
  - Per the cross sectional area of the particle (A)

$$Q_e \equiv \frac{\sigma_e}{A}$$

This also applies to Scattering and Absorption

$$\beta_a = \rho k_a$$

$$\beta_s = \rho k_s$$

$$\beta_a = N\sigma_a$$

$$\beta_s = N\sigma_s$$

$$Q_a \equiv \frac{\sigma_a}{A}$$

$$Q_s \equiv \frac{\sigma_s}{A}$$

Scattering and Absorption Efficiencies

Per the cross sectional area of the particle (A)

Per the size parameter and relative index of refraction

$$Q_a \equiv \frac{\sigma_a}{A} = 4x\,\Im\left\{\frac{m^2-1}{m^2+2}\right\}$$

$$Q_s \equiv \frac{\sigma_s}{A} = \frac{8}{3}x^4\left|\frac{m^2-1}{m^2+2}\right|^2$$

$$Q_e \equiv \frac{\sigma_e}{A} = 4x\,\Im\ \text{*complex function of(m)} + \frac{8}{3}x^4\Re\left\{\left|\frac{m^2-1}{m^2+2}\right|\right\}^2$$

Scattering and Absorption Efficiencies

Per the cross sectional area of the particle (A)

Per the size parameter and relative index of refraction

$Q_a \propto x$ And depends on the imaginary part of N

$Q_s \propto x^4$ And depends on the real & imaginary part of N

$Q_e \propto x$ And depends on the real & imaginary part of N

Scattering and Absorption Efficiencies

When x (size parameter) is very small

$$x = \frac{2\pi r}{\lambda}$$

$$Q_s << Q_a \approx Q_e$$

Rayleigh Regime

$$\varpi = \frac{Q_s}{Q_e} \propto x^3$$

Now, we know that: $Q_s * A \equiv \sigma_s \propto x^4 * A = \left(\frac{2\pi r}{\lambda}\right)^4 * A = \left(\frac{2\pi r}{\lambda}\right)^4 * \pi r^2$

$$\sigma_s \propto \frac{r^6}{x^4}$$

Again Rayleigh Regime !!!

Key Points for Rayleigh regime

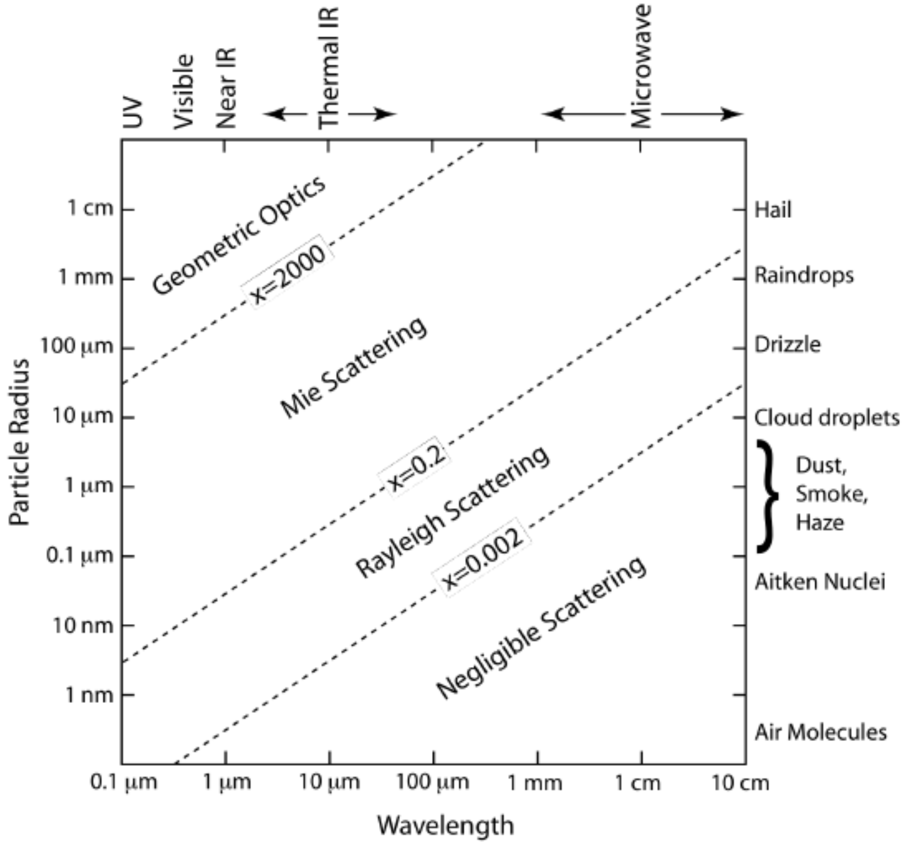
- With 2 different wavelengths ($\lambda_1 < \lambda_2$), then scatter at the shorter wavelength more strongly by a factor of $(\lambda_1/\lambda_2)^4$ (why blue sky)

- With radiation at a fixed wavelength and illuminate two particles of radius ($r_1 < r_2$), the larger particle will scatter the radiation more strongly by a factor $(r_1/r_2)^6$ (relevant to weather radar)

- For sufficiently small x, with complex refractive index, scattering is negligible (relevant to microwave remote sensing of cloud water).

Light scattering regimes

$$x = \frac{2\pi r}{\lambda}$$



There are many regimes of particle scattering, depending on the particle size, the light wave-length, and the refractive index.

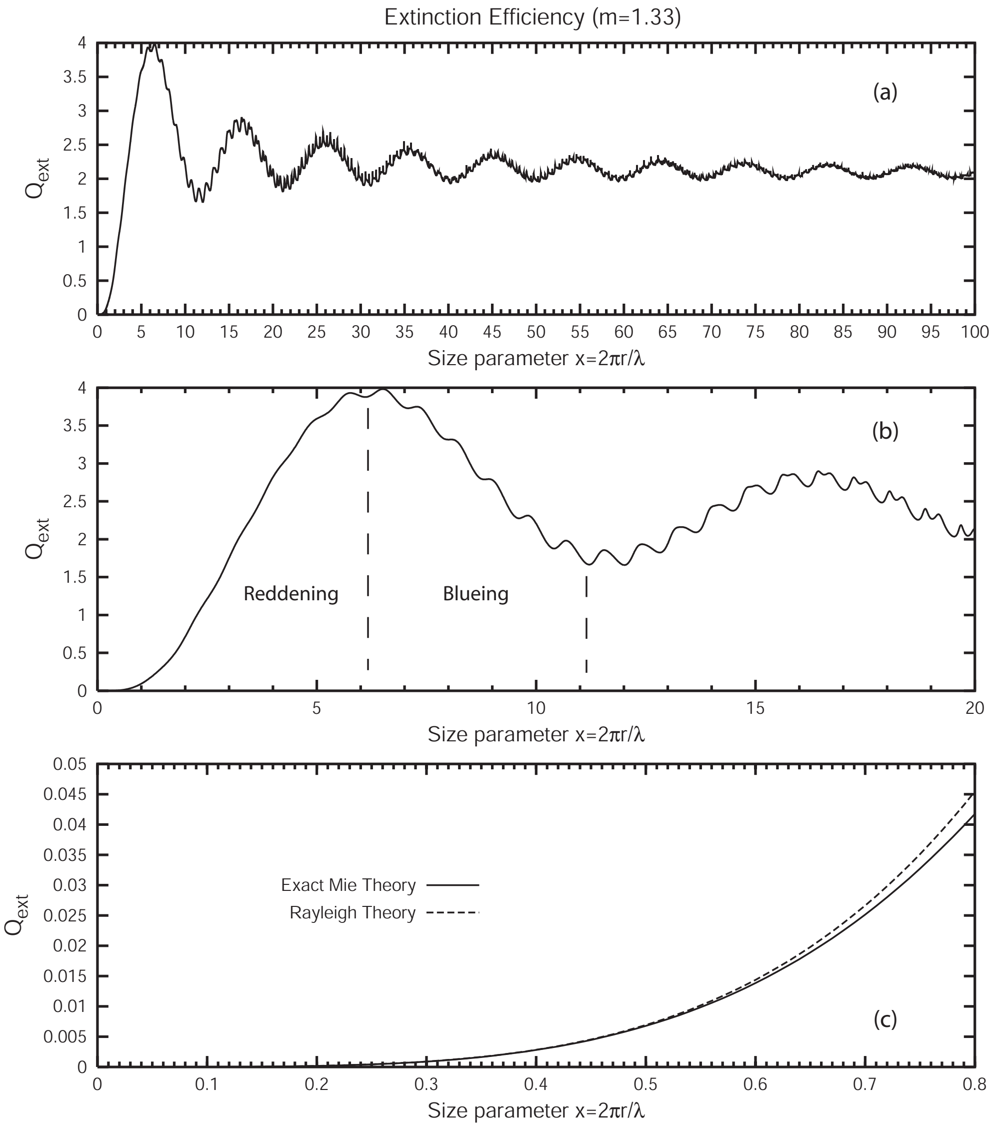
This plot considers only single scattering by spheres. Multiple scattering and scattering by non-spherical objects can get really complex!

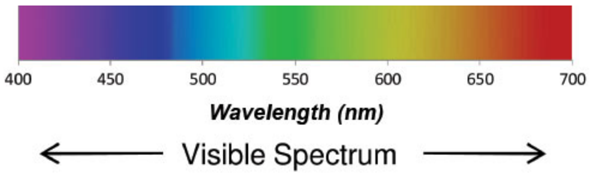
MIE Scattering

Big View

Small View

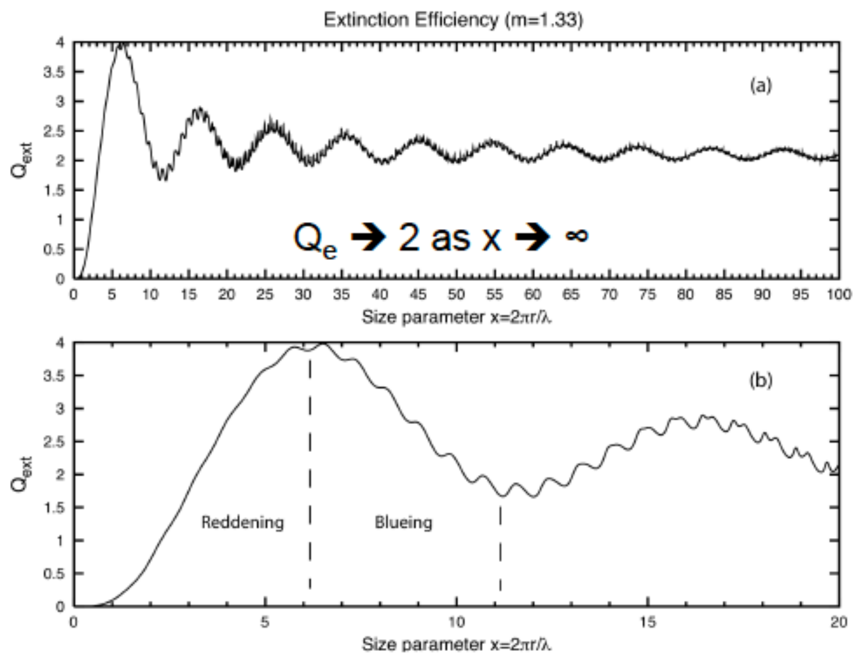
Non-Absorbing Sphere with
Real Index (m) = 1.33





Reddening/Blueing

Non-absorbing sphere with RI (m) = 1.33



$$x = \frac{2\pi r}{\lambda}$$

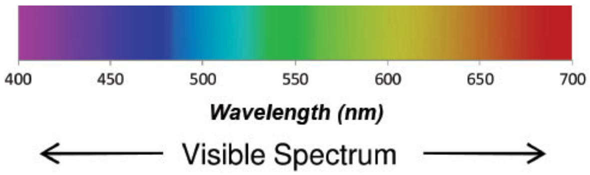
$$Q_e = \frac{\sigma_e}{\pi r^2}$$

$$\beta_e = \sigma_e N$$

Q_e = extinction efficiency factor
σ_e = extinction cross-section

NB. Q_e can be 2 (for cloud droplets at visible wavelengths) or larger!

- Assume r is constant, so variations in x are due to variations in λ
- Hence increasing x implies decreasing λ, and vice versa.
- For 0 < x < 6, shorter wavelengths attenuated more: reddening (e.g., setting sun)
- For 6 < x < 11, longer wavelengths attenuated more: blueing



Reddening/Blueing

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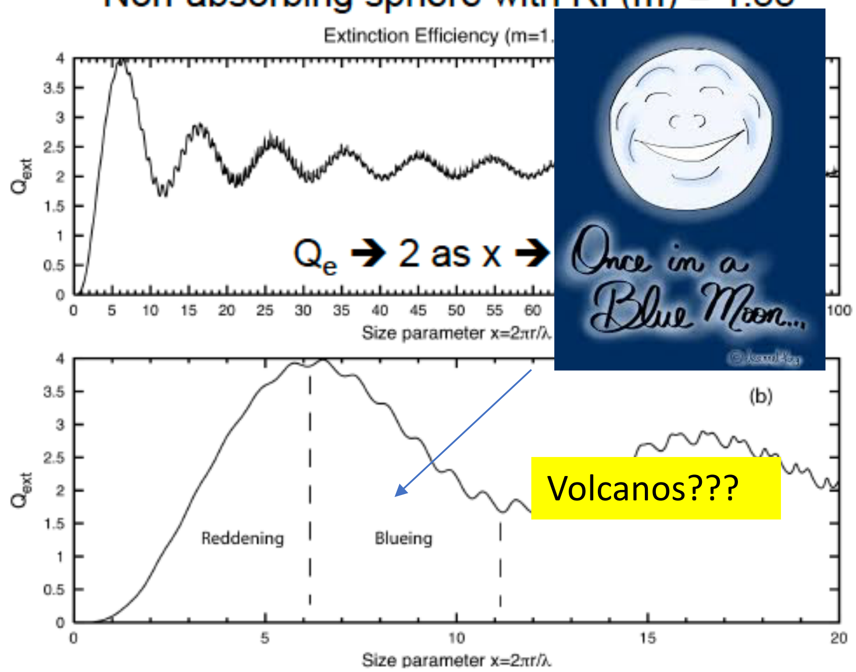

$$Q_e = \frac{\sigma_e}{\pi r^2}$$

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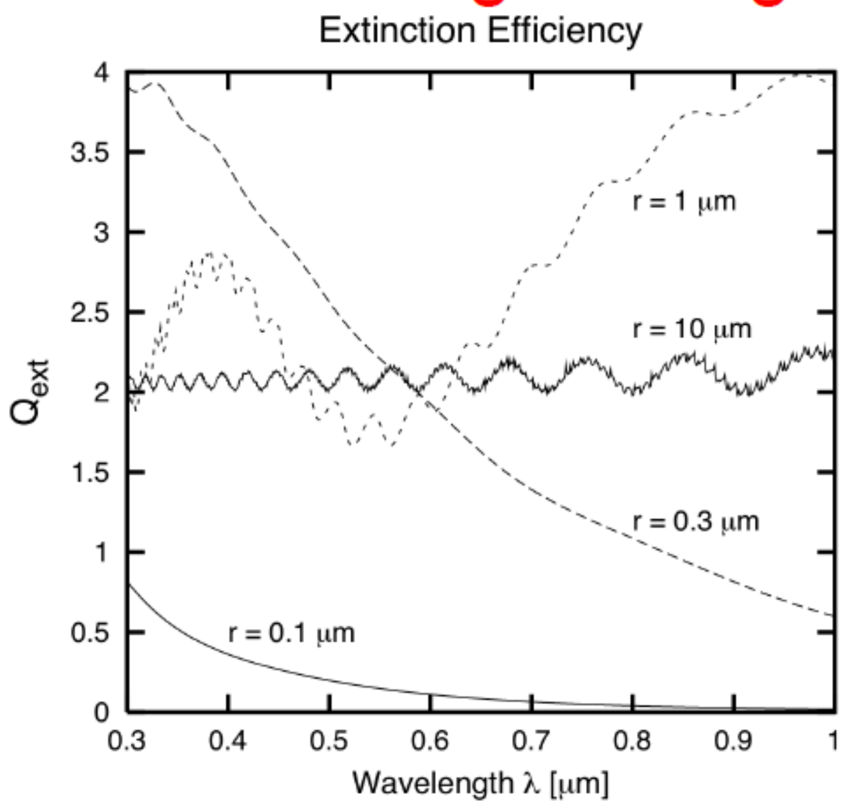
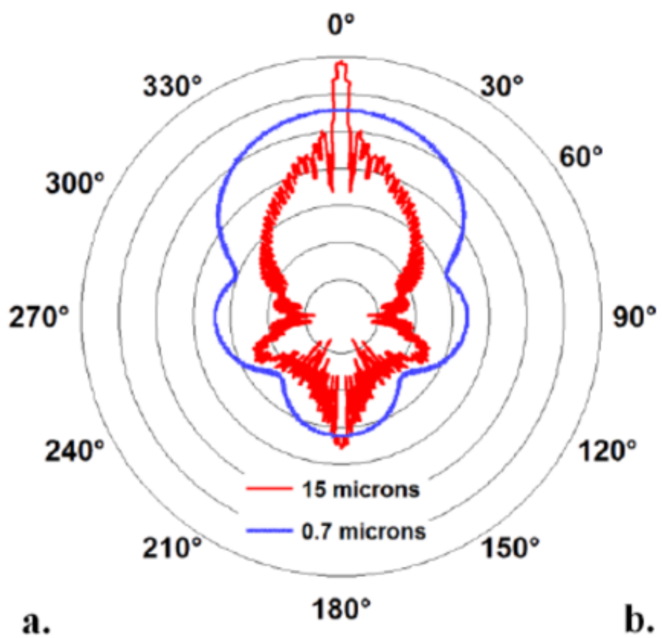
Reddening/Blueing

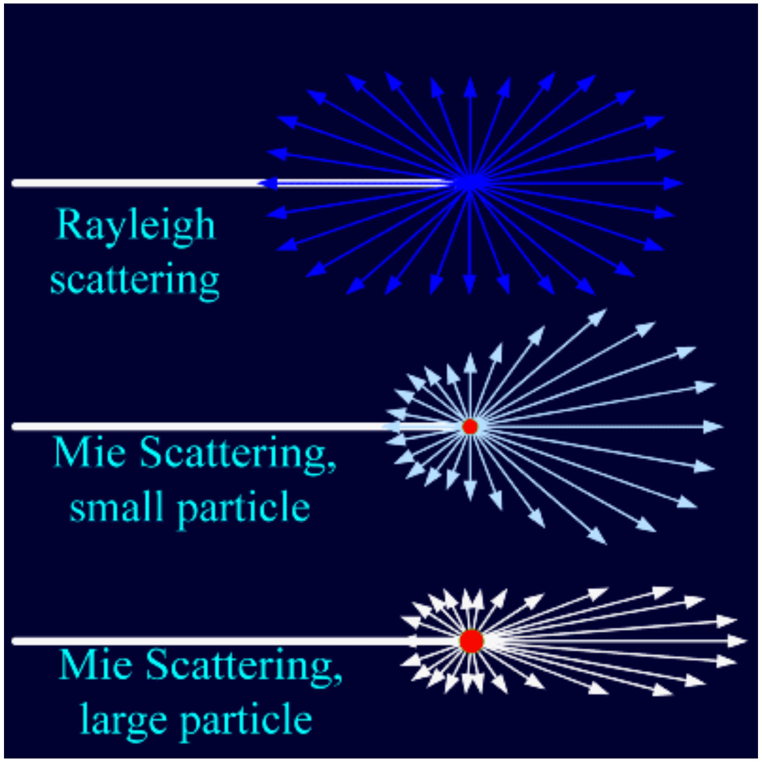


- Extinction efficiency against wavelength for selected water droplet radii
- Haze: 0.1-0.3 μm – classic reddening behavior observed on a hazy day
- Intermediate radius (1 μm) – complex behavior, blue and red light attenuated, with attenuation minimum at 0.5-0.6 μm – would give a green sun at sunset
- For larger radii (10 μm – typical cloud droplet) – no strong wavelength dependence

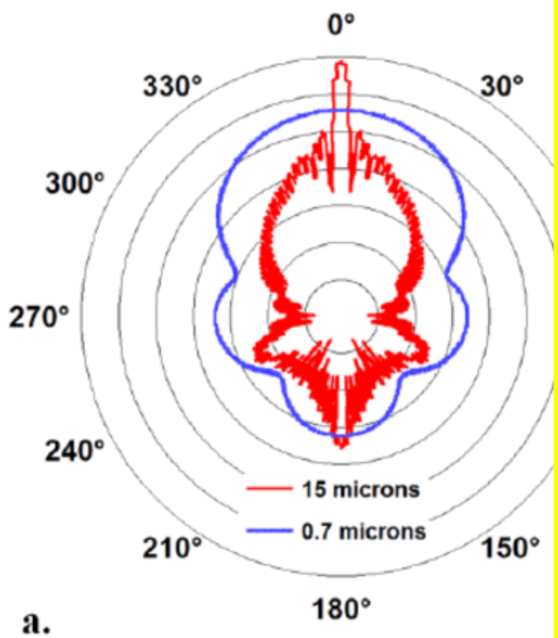
Forward Scattering Increases with Size Parameter



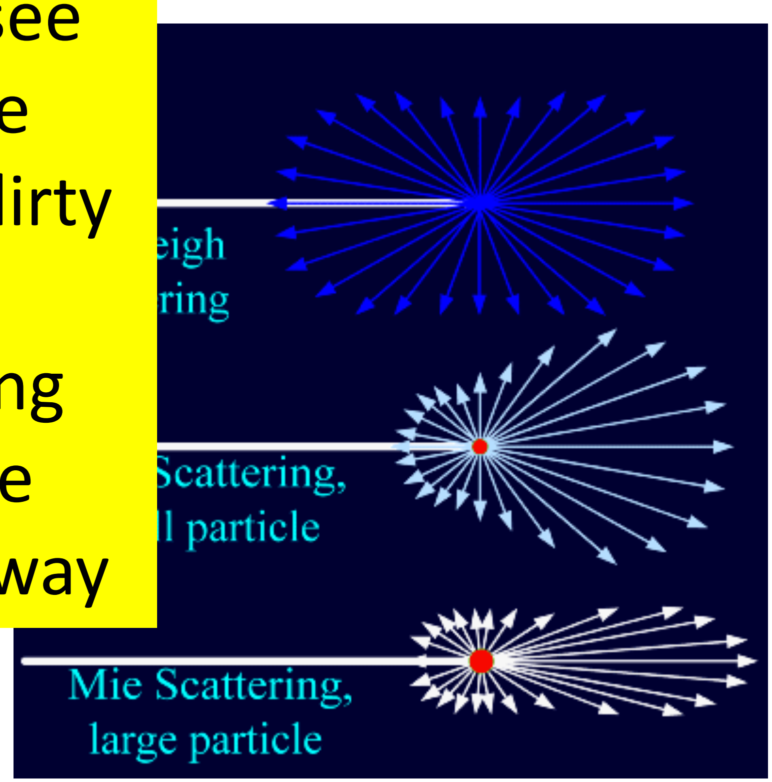
Forward scattering loop becomes narrower and more intense with increasing x



Forward Scattering Increases with Size Parameter

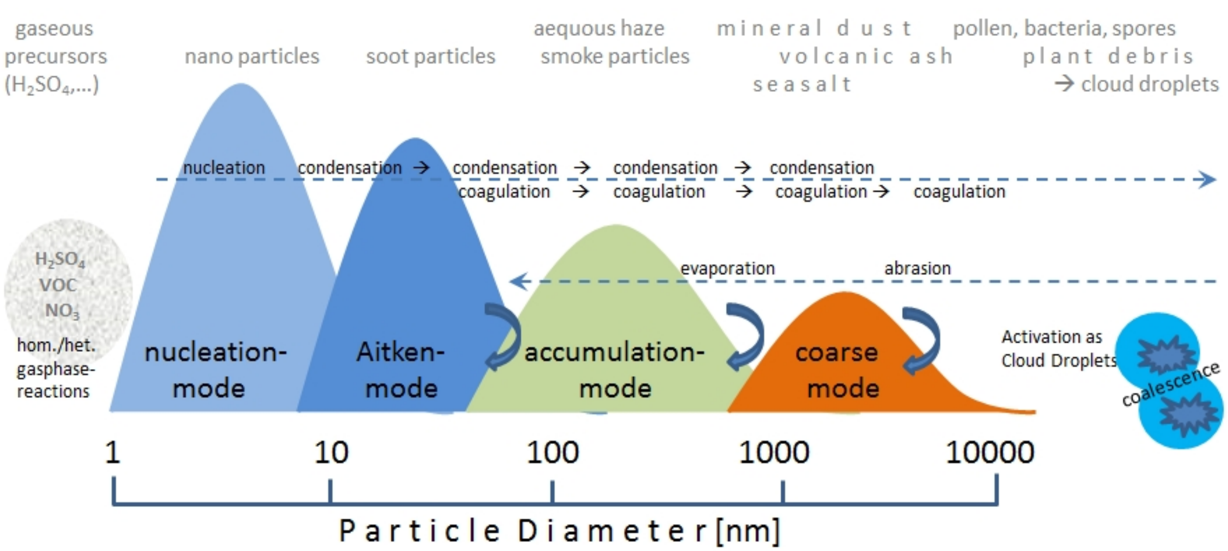


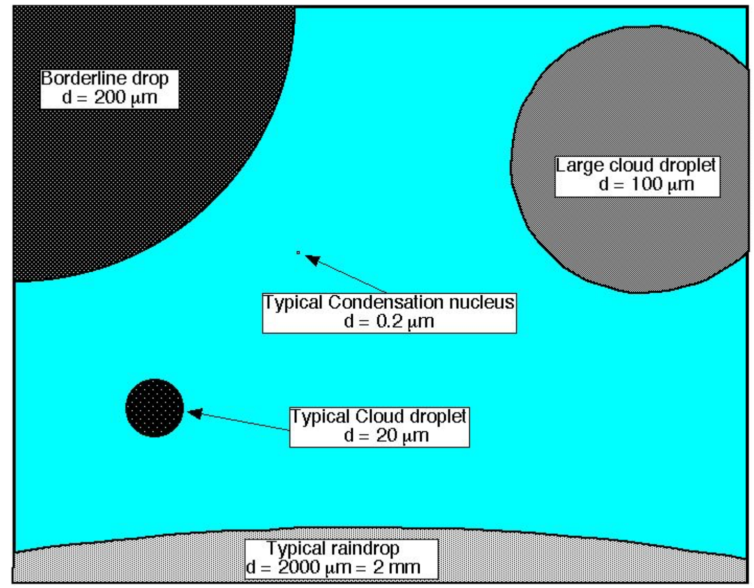
Harder to see through the glare of a dirty windshield when driving towards the sun than away



Forward scattering loop becomes narrower and more intense with increasing x

Distribution of Particles (not just one size)





$$n(r)dr \;=\; \{number\ of\ particles, per\ volume\ of\ air, whose\ raddii\ fall\ in\ the\ range\ (r, r+dr)\}$$

Take Aerosol and Cloud Microphysics next semester will will continue this discussion

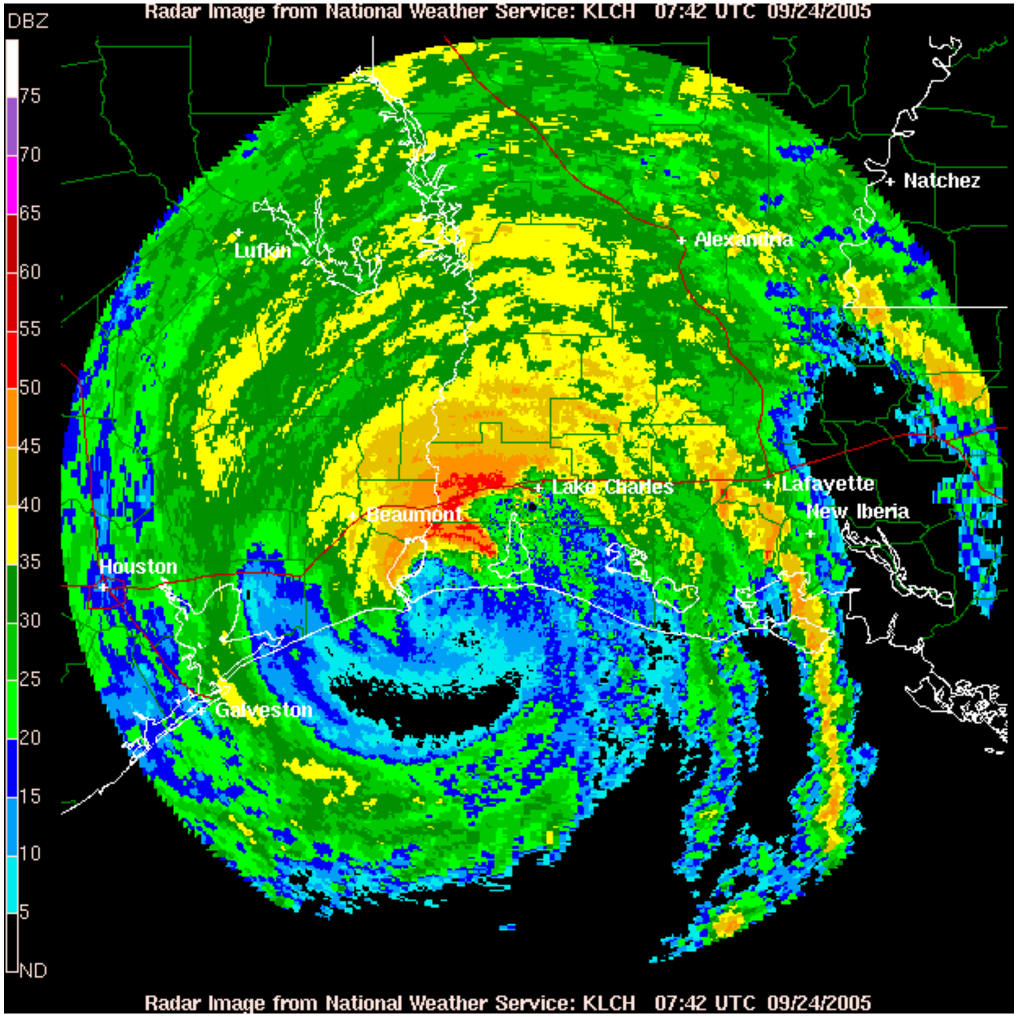
Weather Radar

$$P_r \propto \frac{\eta}{d^2}$$

Backscatter Cross section per unit volume of air

Distance to the radar

Hurricane Rita from Lake Charles NWS



Real World Application
Radar = Microwave Band

Frequency band	Frequency range (GHz)		Wavelength range (cm)
L band	1–2		15–30
S band	2–4	10.71 cm	7.5–15
C band	4–8		3.75–7.5
X band	8–12		2.5–3.75
Ku band	12–18		1.67–2.5
K band	18–27		1.11–1.67
Ka band	27–40		0.75–1.11
V band	40–75		0.4–0.75
W band	75–110		0.27–0.4

NWS
TV
DOW

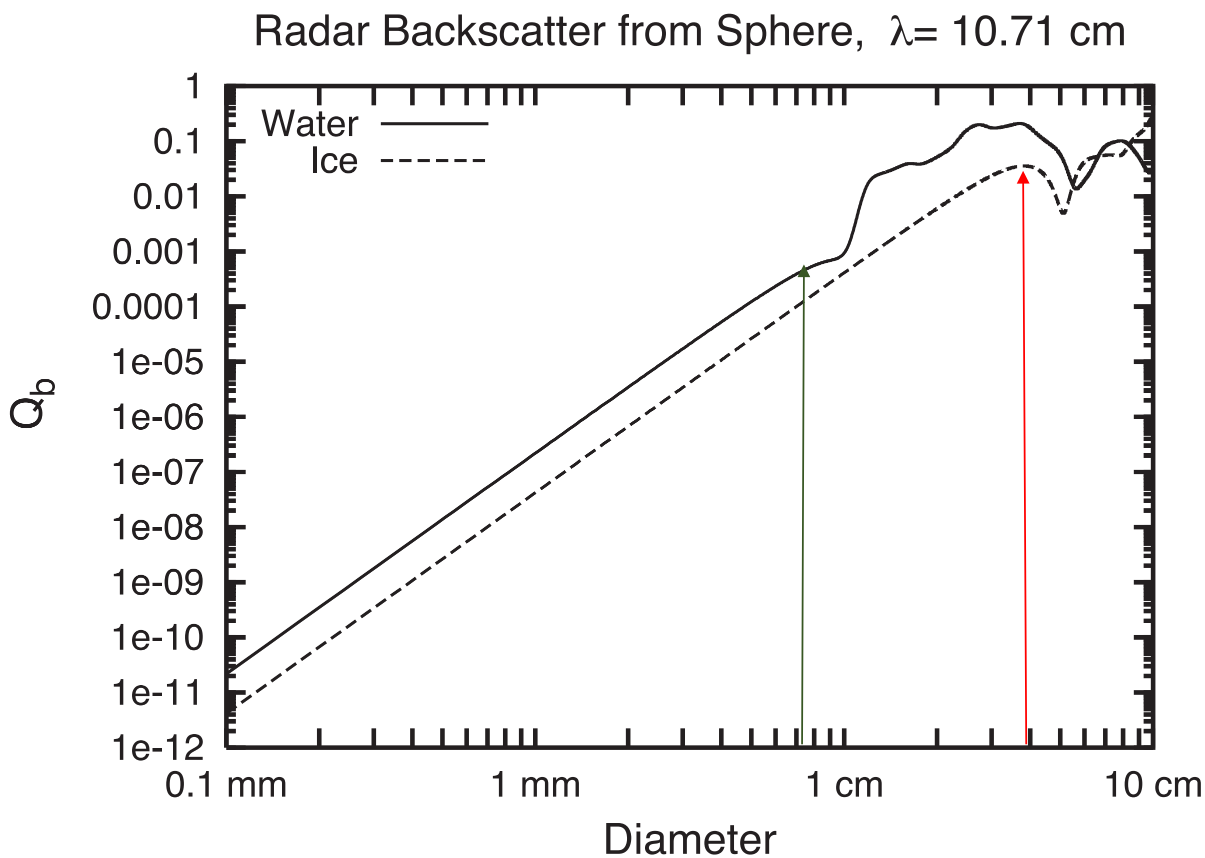
Used for Precipitation (S, C and X bands)

Weather Radar

- Rayleigh Regime
- Backscatter Efficiency

$$Q_b \equiv 4x^4 \left|\frac{m^2 - 1}{m^2 + 2}\right|^2$$

$$x = \frac{2\pi r}{\lambda}$$



Weather Radar

- Rayleigh Regime
- Backscatter Efficiency

$$Q_b \equiv 4x^4 \left|\frac{m^2-1}{m^2+2}\right|^2$$

$$x = \frac{2\pi r}{\lambda}$$

- Backscatter Cross section per unit volume of air

$$\eta = \int_0^\infty Q_b(D)\left[\frac{\pi}{4}D^2\right] n(D)dD$$

Cross Sectional Area of Particles

Number of Particles

POWER

$$P_r \propto \frac{\eta}{d^2} \propto \left|\frac{m^2-1}{m^2+2}\right|^2 \frac{Z}{d^2}$$

Reflectivity Factor

$$Z = \int_0^\infty n(D)D^6 dD$$

$$Z\ [dBZ\} = 10\ log_{10}\ (Z)$$

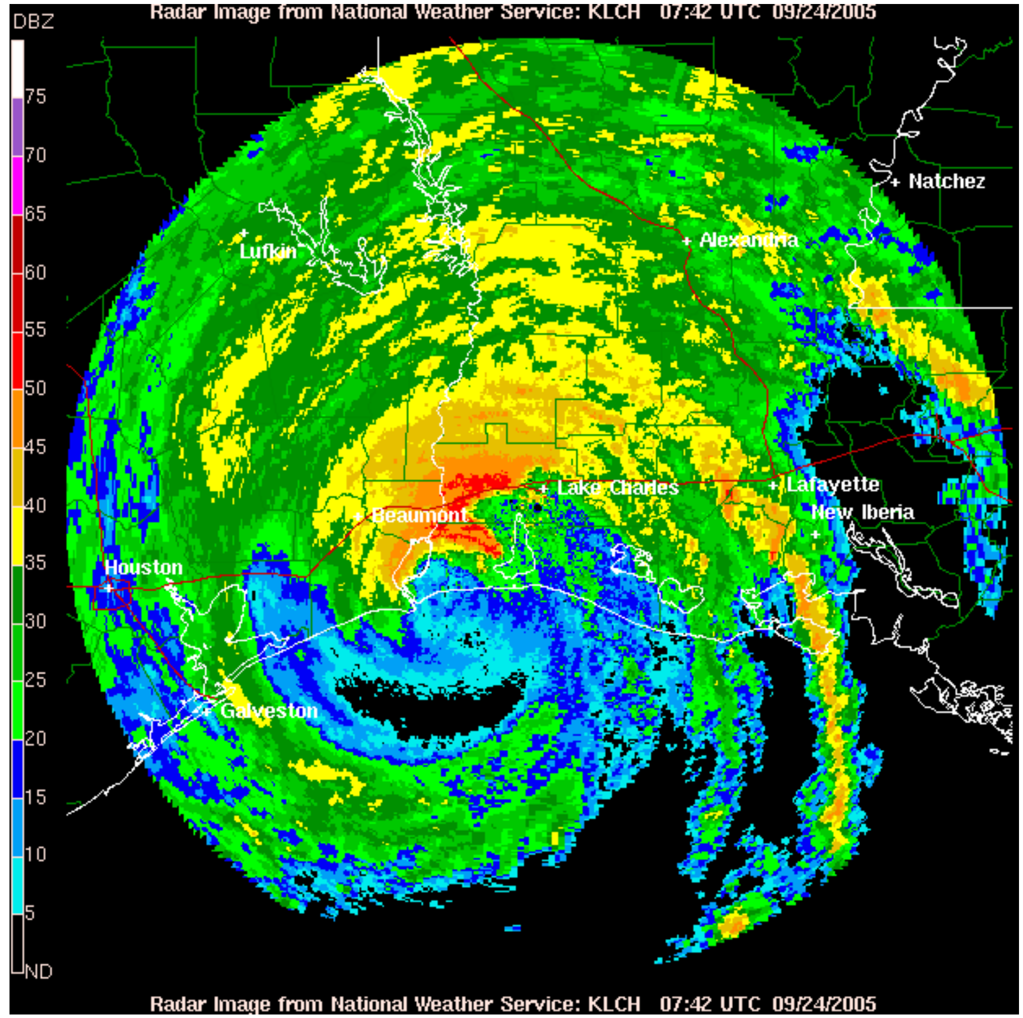
Weather Radar

$$Z\ [dBZ\} = 10\ log_{10}\,(Z)$$

$$Z = \int_0^\infty n(D) D^6 dD$$

BIG Particles Dominate!

Assuming a target is liquid (m value).
Thus, if ice only Z is only really 20%



Radiative Transfer Equation Depends on

VERY IMPORTANT SLIDE

1. β_e

Since: $d\tau = -\beta_e ds$

2. $\varpi = \dfrac{\beta_s}{\beta_e}$

3. $p(\cos\Theta)$

These properties depend upon the size parameter and index of refraction (both depend upon the wavelength of light)