THE UNIVERSITY OF UTAH

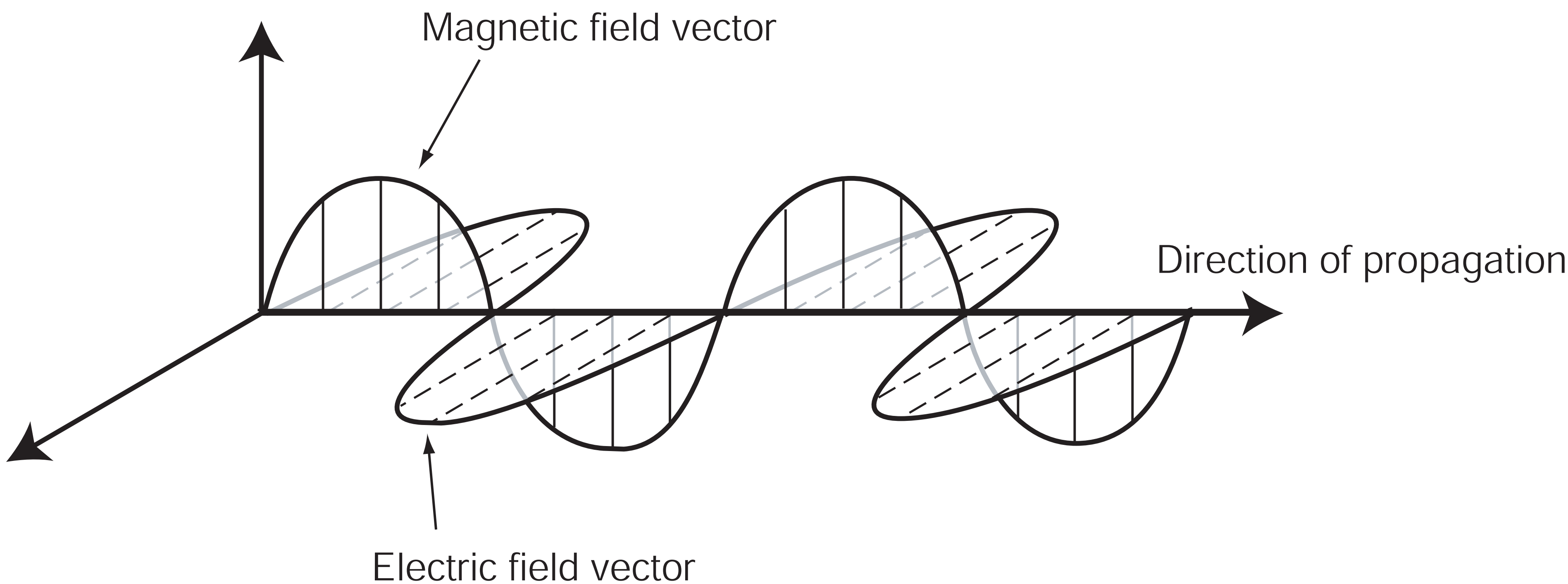
ATMOS 5140

Lecture 3 – Chapter 2

- More on Maxwell's Equations
- Quantum Properties of Radiation
- Flux and Intensity
  - Solid Angle

Maxwell’s Equations

- Changing electric field induces a magnetic field
- Changing magnetic field induces an electric field
- Does not need material medium (vacuum works)
- Quantitative Interplay described by Maxwell’s Equations



Magnetic field vector
Direction of propagation
Electric field vector

Maxwell’s Equations

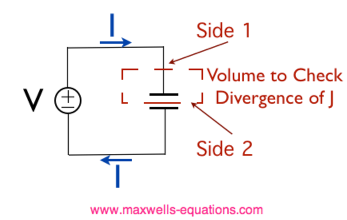
D = Electric displacement
E = Electric field
B = Magnetic induction
H = Magnetic field
J = Electric Current
ρ_V = density of electric charge

1. $\nabla \cdot \mathbf{D} = \rho_V$ — Gauss’s Law of Electric Field

2. $\nabla \cdot \mathbf{B} = 0$ — Gauss’s Law of Magnetic Field

3. $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ — Faraday’s Law

4. $\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}$ — Ampere’s Law and Maxwell’s addition of displacement current density



Maxwell's Equations

D = Electric displacement
E = Electric field
B = Magnetic induction
H = Magnetic field
J = Electric Current

$$\mathbf{D} = \varepsilon \mathbf{E} \qquad \mathbf{B} = \mu \mathbf{H} \qquad \mathbf{J} = \sigma \mathbf{E}$$

$\varepsilon = permittivity\ of\ space$
$\mu = magnetic\ permeability\ of\ the\ medium$
$\sigma = conductivity\ of\ medium$

Maxwell's Equations

D = Electric displacement
E = Electric field
B = Magnetic induction
H = Magnetic field
J = Electric Current
ρ_V = density of electric charge

1. $\nabla \cdot \mathbf{D} = \rho_V$
2. $\nabla \cdot \mathbf{B} = 0$
3. $\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$
4. $\nabla \times \mathbf{H} = \dfrac{\partial \mathbf{D}}{\partial t} + \mathbf{J}$

1. $\nabla \cdot \mathbf{E} = \dfrac{\rho_V}{\varepsilon}$
2. $\nabla \cdot \mathbf{H} = 0$

www.maxwells-equations.com

3. $\nabla \times \mathbf{E} = -\mu \dfrac{\partial \mathbf{H}}{\partial t}$
4. $\nabla \times \mathbf{H} = \varepsilon \dfrac{\partial \mathbf{E}}{\partial t} + \sigma \mathbf{E}$

Maxwell's Equations for plane waves

- Sinusoidal plane-wave solutions are particular solutions to the electromagnetic wave equation.
- The general solution of the electromagnetic wave equation in homogeneous, linear, time-independent media can be written as a linear superposition of plane-waves of different frequencies and polarizations.

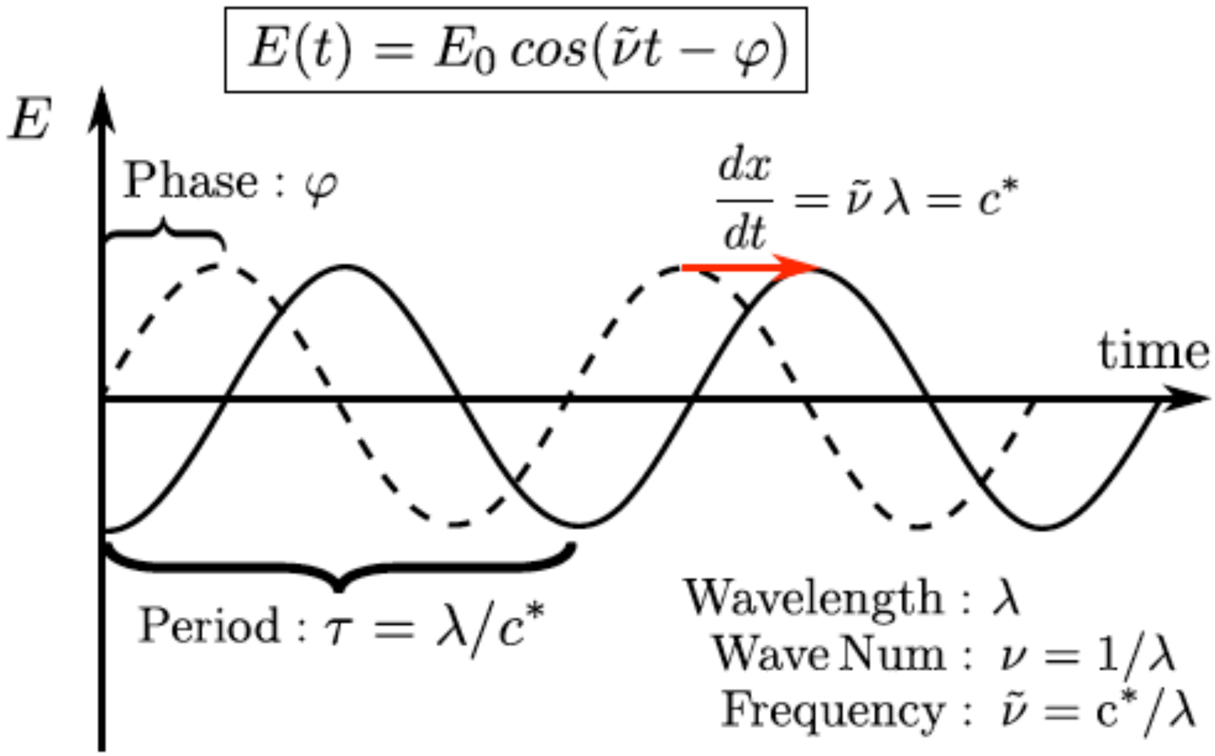


Figure 1.2: *Electromagnetic wave quantities*

Figure from Tim Garrett

Maxwell's Equations for plane waves

The fundamental equations that describe electromagnetic radiation are Maxwell's equations. The solutions to the equations are sinusoidal of form

$$\vec{E} = \vec{E}_0 \exp\left(i\vec{k}\cdot\vec{x} - i\omega t\right) \tag{1.1}$$

$$\vec{H} = \vec{H}_0 \exp\left(i\vec{k}\cdot\vec{x} - i\omega t\right) \tag{1.2}$$

where E is the electric field and H is the magnetic field, the real component of $\vec{k}$ is the wavenumber $2\pi/\lambda$, $\vec{x}$ is the direction of wave propagation and ω is the angular frequency of the radiation $2\pi\nu$.

Maxwell's Equations for plane waves

Complex Vectors (real and imaginary parts)
If you need a refresher: watch Khan video

https://www.youtube.com/watch?v=kpywdu1afas

$$\vec{E} = \vec{E}_0 \exp\left(i\vec{k}\cdot\vec{x} - i\omega t\right) \tag{1.1}$$

$$\vec{H} = \vec{H}_0 \exp\left(i\vec{k}\cdot\vec{x} - i\omega t\right) \tag{1.2}$$

where E is the electric field and H is the magnetic field, the real component of $\vec{k}$ is the wavenumber $2\pi/\lambda$, $\vec{x}$ is the direction of wave propagation and ω is the angular frequency of the radiation $2\pi\nu$.

Maxwell’s Equations for plane waves

Constant complex vectors

Complex Vectors (real and imaginary parts)

$$\vec{E} = \vec{E}_0 \exp\left(i\vec{k}\cdot\vec{x} - i\omega t\right) \quad (1.1)$$

Constant complex **wave** vectors *(here only consider real part)*

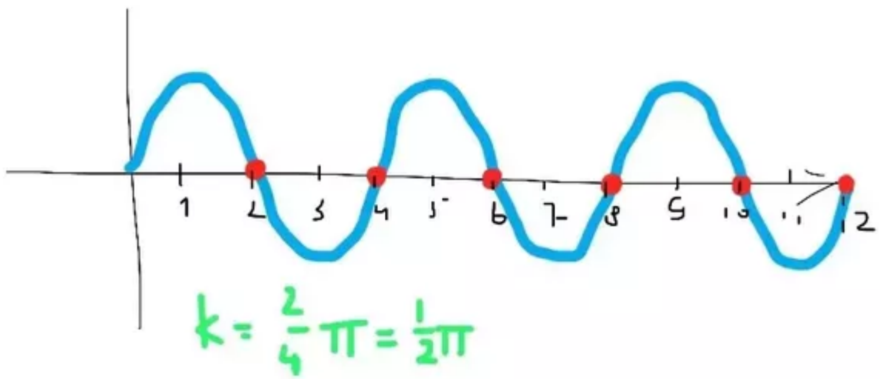
$$\vec{H} = \vec{H}_0 \exp\left(i\vec{k}\cdot\vec{x} - i\omega t\right) \quad (1.2)$$

where E is the electric field and H is the magnetic field, the real component of $\vec{k}$ is the wavenumber $2\pi/\lambda$, $\vec{x}$ is the direction of wave propagation and ω is the angular frequency of the radiation $2\pi\nu$.

Wave Vector

- The wave vector, k, counts the wavenumber in a particular direction.
- Spatial frequency of a wave
- For example, the wavenumber of the blue line is the number of nodes (red dots) you see per unit distance times π.
- Count 2 nodes in 4 units of distance, or 3 nodes in 6 units of distance ½ nodes per unit distance.
- Wavenumber is the number of radians per unit of distance

  k=1/2π



Maxwell's Equations for plane waves

$$\vec{\mathbf{k}} = \vec{\mathbf{k}}' + i\vec{\mathbf{k}}''$$

If $\vec{k}''$ = 0, then the amplitude of the wave is constant
Then the medium is non absorbing!

$$\vec{\mathbf{E}}_c = \vec{\mathbf{E}}_0 \exp(-\vec{\mathbf{k}}'' \cdot \vec{\mathbf{x}}) \exp[i(\vec{\mathbf{k}}' \cdot \vec{\mathbf{x}} - \omega t)],$$

$$\vec{\mathbf{H}}_c = \vec{\mathbf{H}}_0 \exp(-\vec{\mathbf{k}}'' \cdot \vec{\mathbf{x}}) \exp[i(\vec{\mathbf{k}}' \cdot \vec{\mathbf{x}} - \omega t)]$$

Maxwell's Equations for plane waves

- Index of refraction (also called refractive index)- N

Maxwell's Equations for plane waves

PHASE SPEED

The *phase speed* of the wave is given by

$$v = \frac{\omega}{|\vec{\mathbf{k}}'|}$$

$$|\vec{\mathbf{k}}'| + i|\vec{\mathbf{k}}''| = \omega\sqrt{\varepsilon\mu}.$$

μ is the magnetic *permeability*

ε = permittivity of the medium

frequency ω

In a vacuum:

$\vec{k}'' = 0$

$$c \equiv 1/\sqrt{\varepsilon_0\mu_0}.$$

Maxwell's Equations for plane waves

In a nonvacuum, we can write

$$|\vec{\mathbf{k}}'| + i|\vec{\mathbf{k}}''| = \omega\sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}}\sqrt{\varepsilon_0\mu_0} = \frac{\omega N}{c},$$

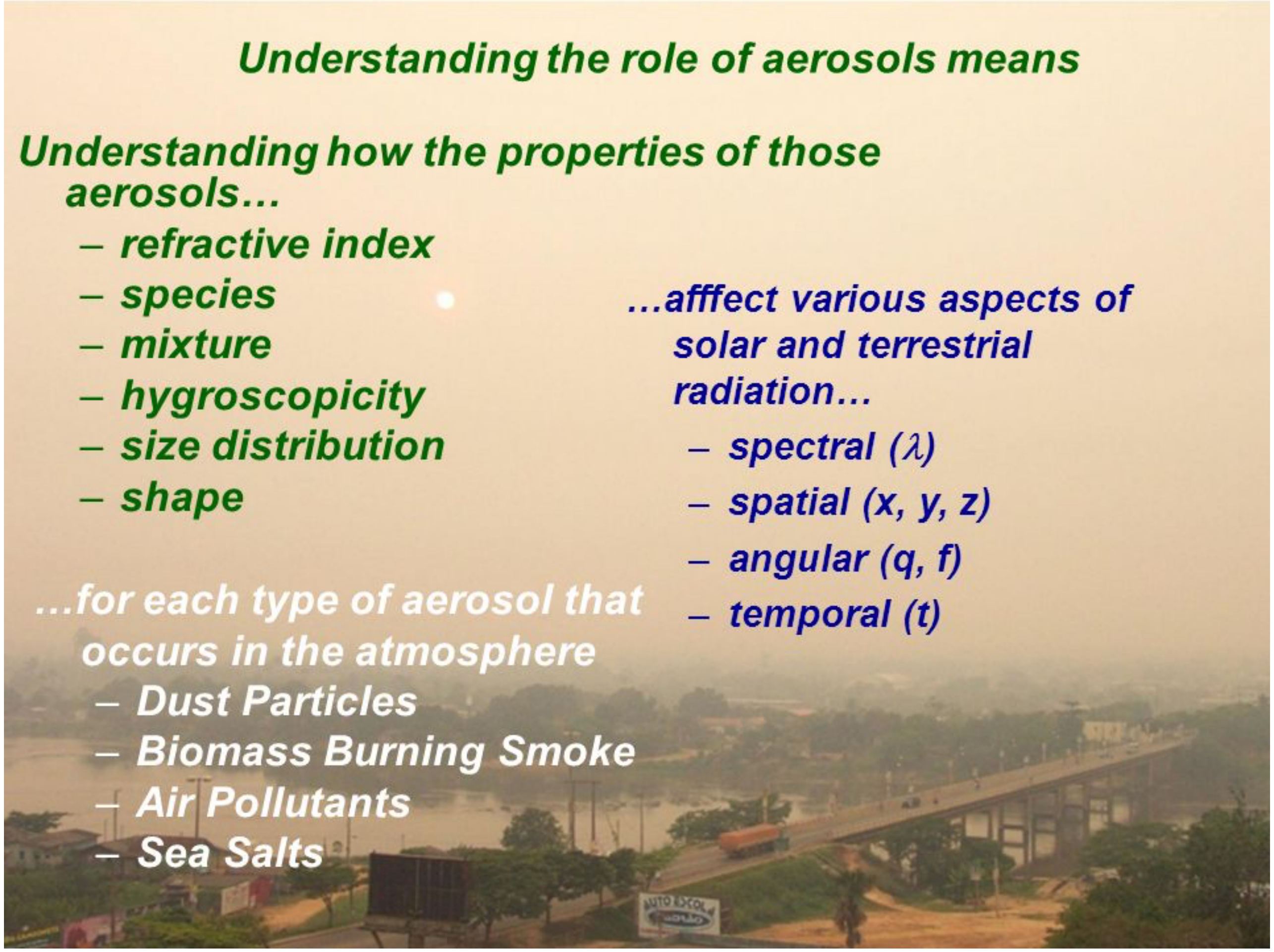
where the complex *index of refraction* N is given by

$$\boxed{N \equiv \sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}} = \frac{c}{c'}},$$

Refractive Index

- The refractive index of a material is critical in determining the scattering and absorption of light, with the imaginary part of the refractive index having the greatest effect on absorption.

Real world use of Index of Refractions



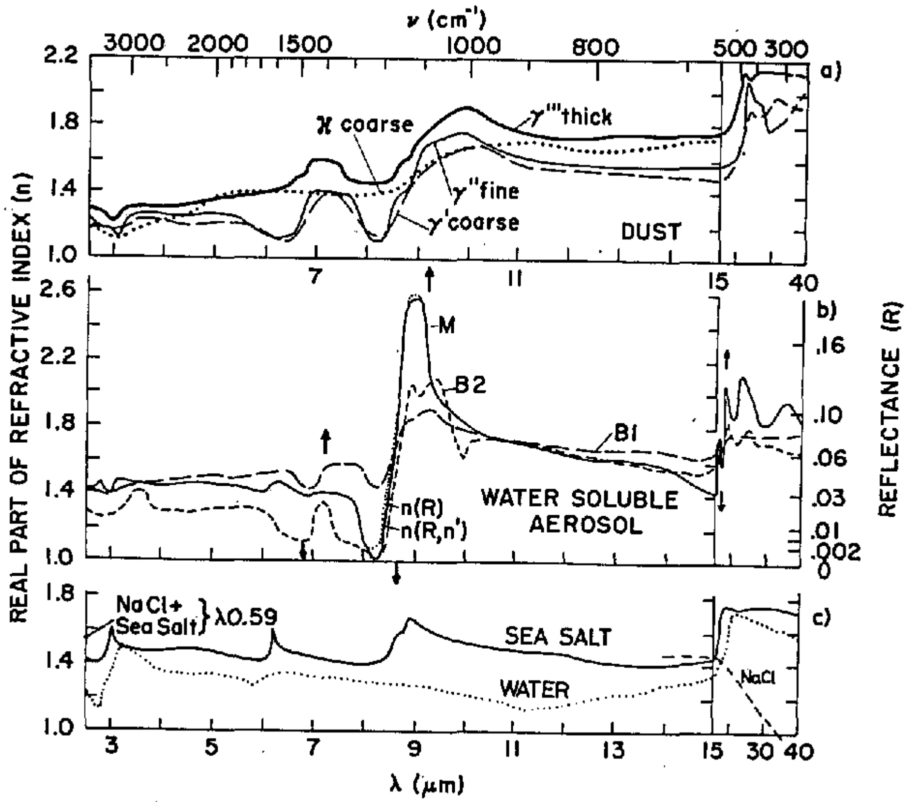


Fig. 3

Citation

Frederic E. Volz, "Infrared Refractive Index of Atmospheric Aerosol Substances," Appl. Opt. **11,** 755-759 (1972);
https://www.osapublishing.org/ao/abstract.cfm?uri=ao-11-4-755

Image © 1972 Optical Society of America and may be used for noncommercial purposes only. Report a copyright concern regarding this image.

Easier to measure Refractive Index in the IR than in the visible

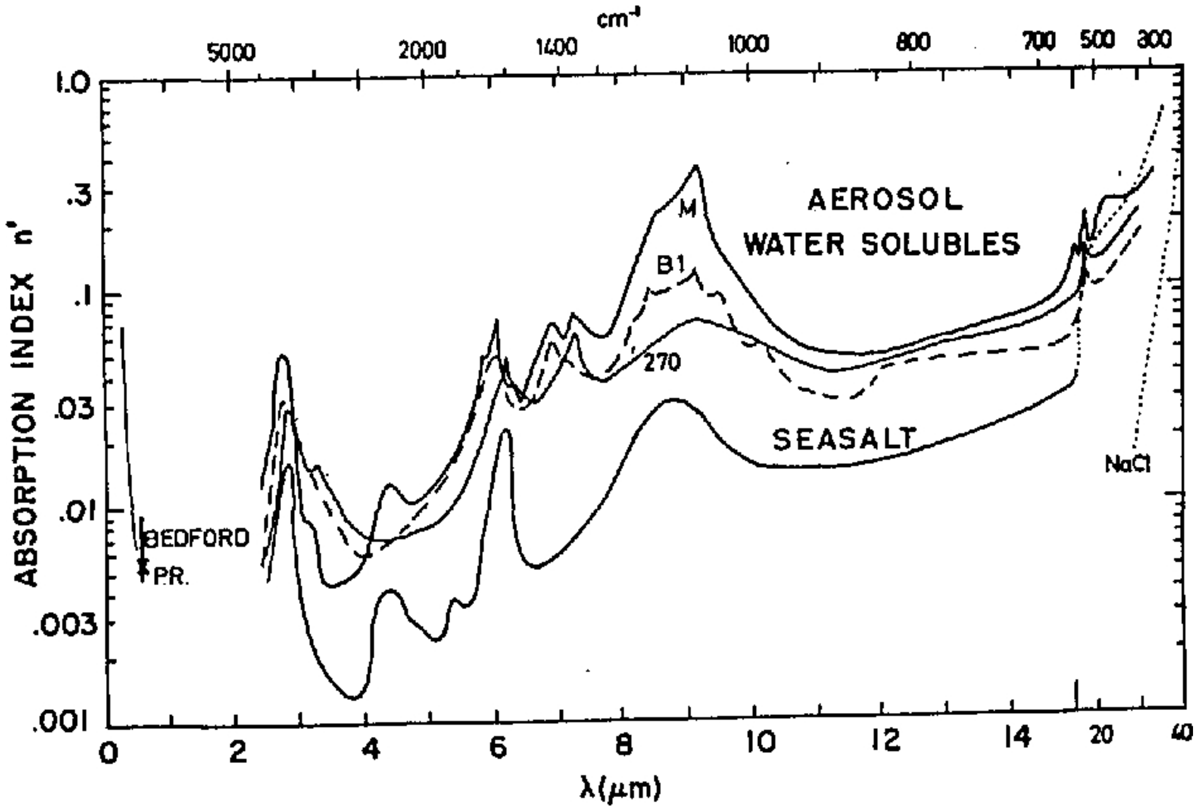


Fig. 2

Citation

Frederic E. Volz, "Infrared Refractive Index of Atmospheric Aerosol Substances," Appl. Opt. **11,** 755-759 (1972);
https://www.osapublishing.org/ao/abstract.cfm?uri=ao-11-4-755

Image © 1972 Optical Society of America and may be used for noncommercial purposes only. Report a copyright concern regarding this image.

OSA The Optical Society

A variety of values for the refractive index of BC has been used in global climate models including the Optical Properties of Aerosols and Clouds (OPAC) value of $1.74 - 0.44i$ [*Hess et al*., **1998**]. As reviewed by *Bond and Bergstrom* [**2006**], reported values of the refractive index of light absorbing carbon vary widely; the real part, *n*, appears to vary from that of water to that of diamond, and the imaginary part, *k*, varies from that of negligibly absorbing material to that of graphite. *Bond and Bergstrom* [**2006**] hypothesize that strongly absorbing carbon with a single refractive index exists and that some of the variation in reported values results from void fractions in the material. Based on agreement between measured real and imaginary parts of the refractive index of light absorbing carbon, *Bond and Bergstrom* [**2006**] recommended a value of $1.95 - 0.79i$ at 550 nm. They caution, however, that this value may not represent void-free carbon. *Stier et al*. [**2007**] found that this value led to better agreement with observed atmospheric absorption, compared with the OPAC value. *Moteki et al*. [**2010**] found that the refractive index of ambient BC in the Tokyo urban area was about $2.26 - 1.26i$ at 1064 nm. The OPAC assumption for imaginary refractive index is not taken from combustion-generated particles, is lower than either of the latter two recommendations, and would lead to a MAC prediction about 30% lower if all other factors were equal.

Maxwell's Equations for plane waves

- Index of refraction - N
- **Flux – F (W/m^2)**

Flux - Poynting vector

- The instantaneous flux density of the electric field in the direction of the wave is

$$\vec{S} = \vec{E} \times \vec{H}$$

- For a harmonic wave (average over complete cycle)

$$F = \frac{1}{2} c \varepsilon_0 E^2$$

$\varepsilon =$ permittivity of the medium

Maxwell's Equations for plane waves

- Index of refraction - N
- Flux – F (W/m^2)
- Absorption coefficient – β_a

Absorption

Consider scalar amplitude of the wave

$$E = |\vec{\mathbf{E}}_0 \exp(-\vec{\mathbf{k}}'' \cdot \vec{\mathbf{x}})|$$

Now go back to our definition of flux

$$F = F_0[\exp(-\vec{\mathbf{k}}'' \cdot \vec{\mathbf{x}})]^2 = F_0 \exp(-2\vec{\mathbf{k}}'' \cdot \vec{\mathbf{x}}).$$

Recall that our imaginary part of wave vector is responsible for absorption

$$|\vec{\mathbf{k}}''| = \frac{\omega}{c}\mathrm{Im}\{N\} = \frac{\omega n_i}{c} = \frac{2\pi \nu n_i}{c}$$

$$F = F_0 e^{-\beta_a x}$$

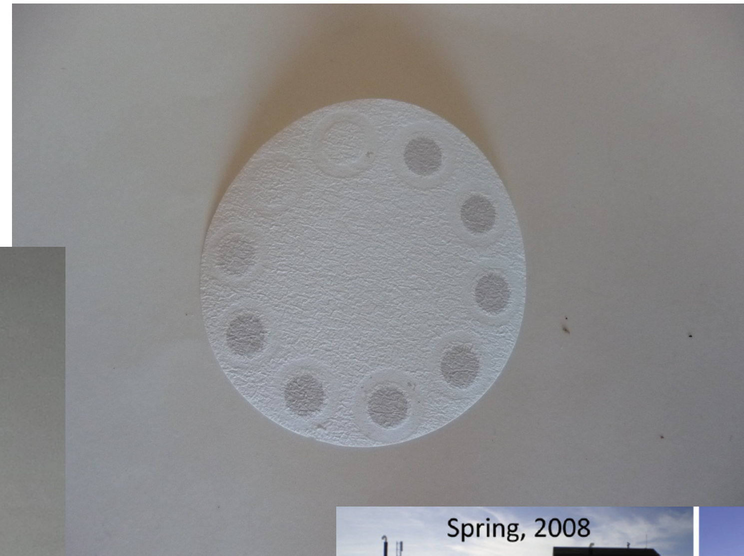
Absorption

$$\beta_a = 4\pi n_i/\lambda$$

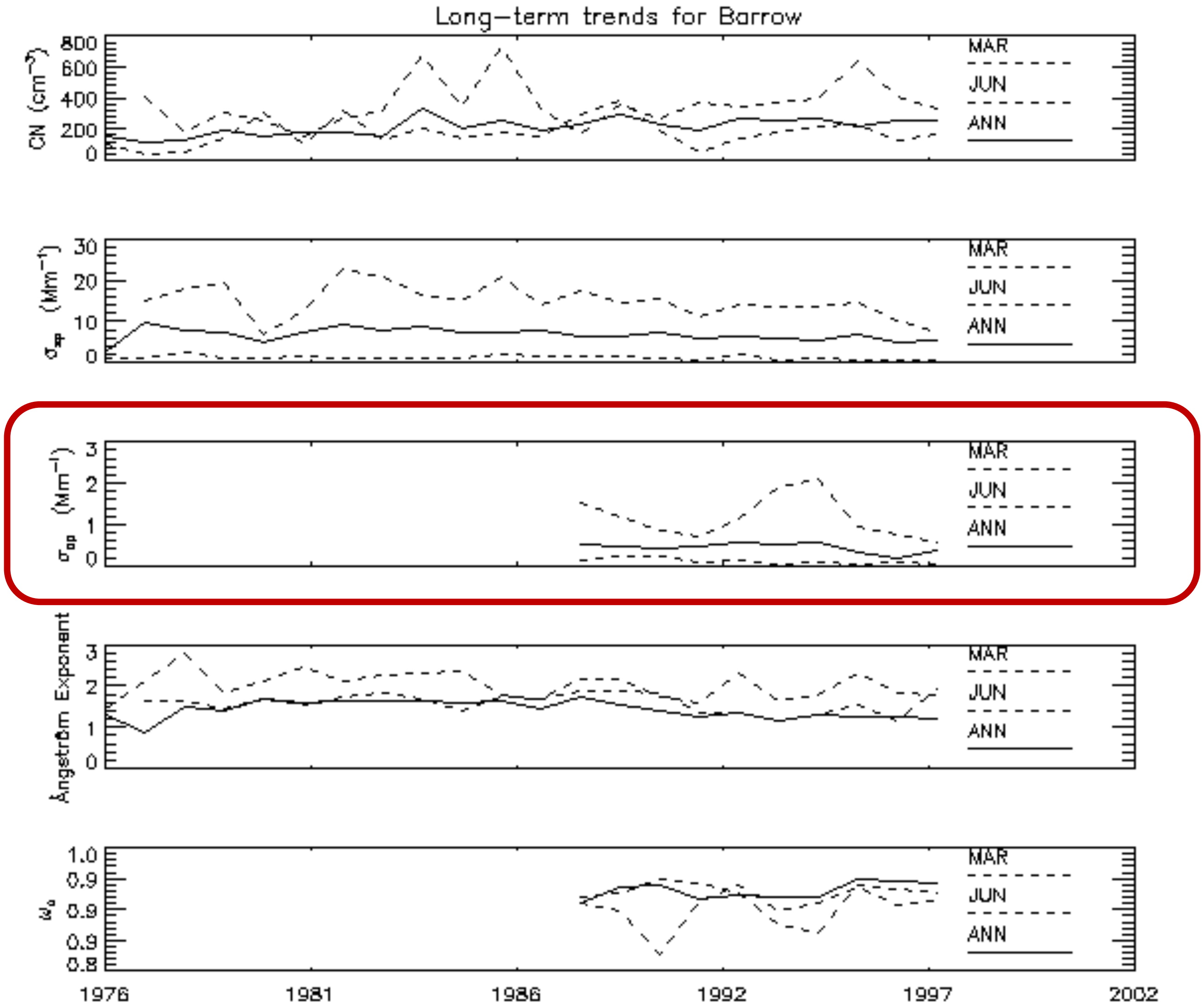
$\frac{1}{\beta_a}$ = distance required for the wave's energy to be attenuated to e^{-1} (about 37%)

Absorption


Continuous
Light
Absorption
Photometer
STATUS
FLOW RATE
VALVE
NOAA
Tour Line: 303-497-3333
http://www.noaa.gov/

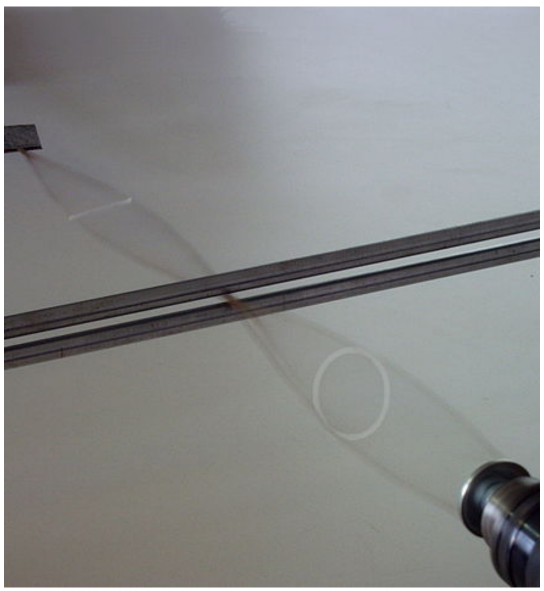



Spring, 2008
Summer, 2009
Fall, 2009
Winter, 2010



Long−term trends for Barrow
CN (cm⁻³)
σsp (Mm⁻¹)
σap (Mm⁻¹)
Ångstrøm Exponent
ω₀
MAR
JUN
ANN
1976
1981
1986
1992
1997
2002

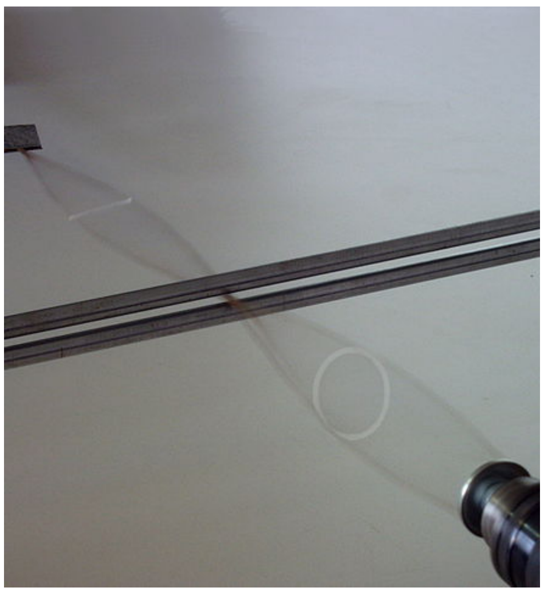
Polarization

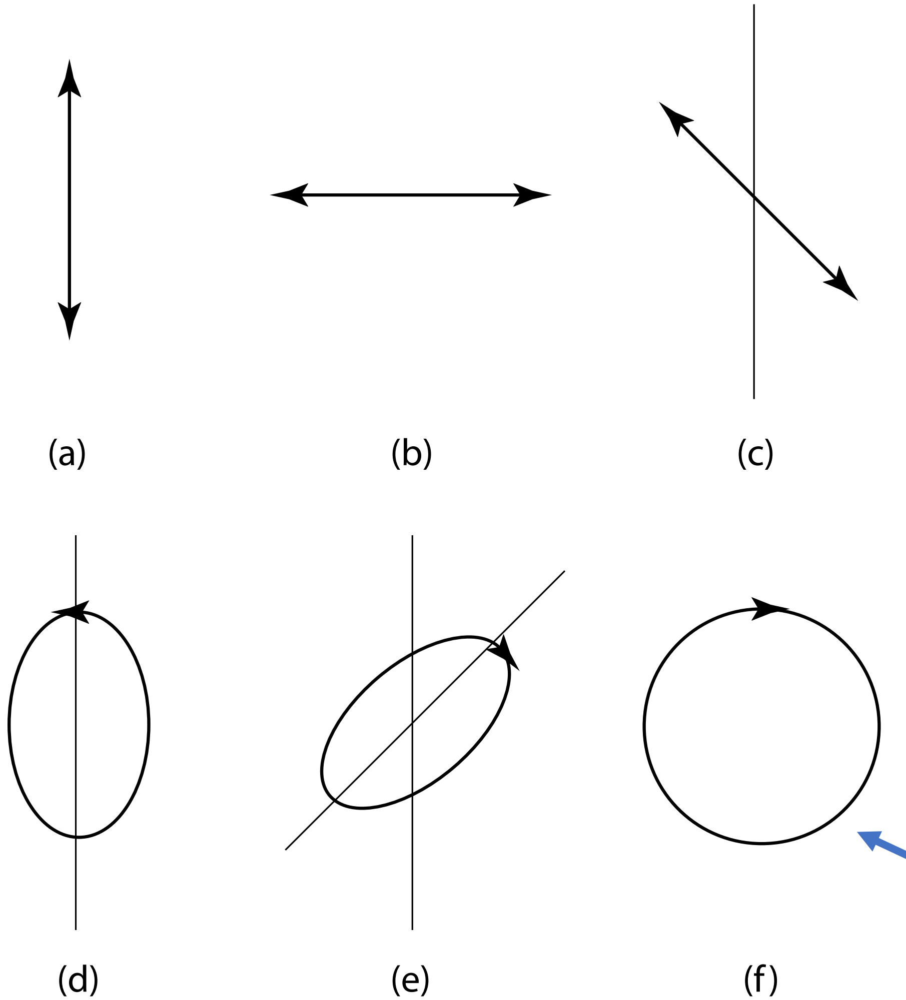


parameter applying to transverse waves that specifies the geometrical orientation of the oscillations

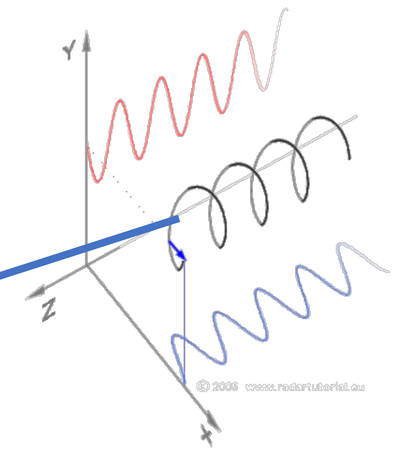
the "polarization" of electromagnetic waves refers to the direction of the electric field.

Polarization



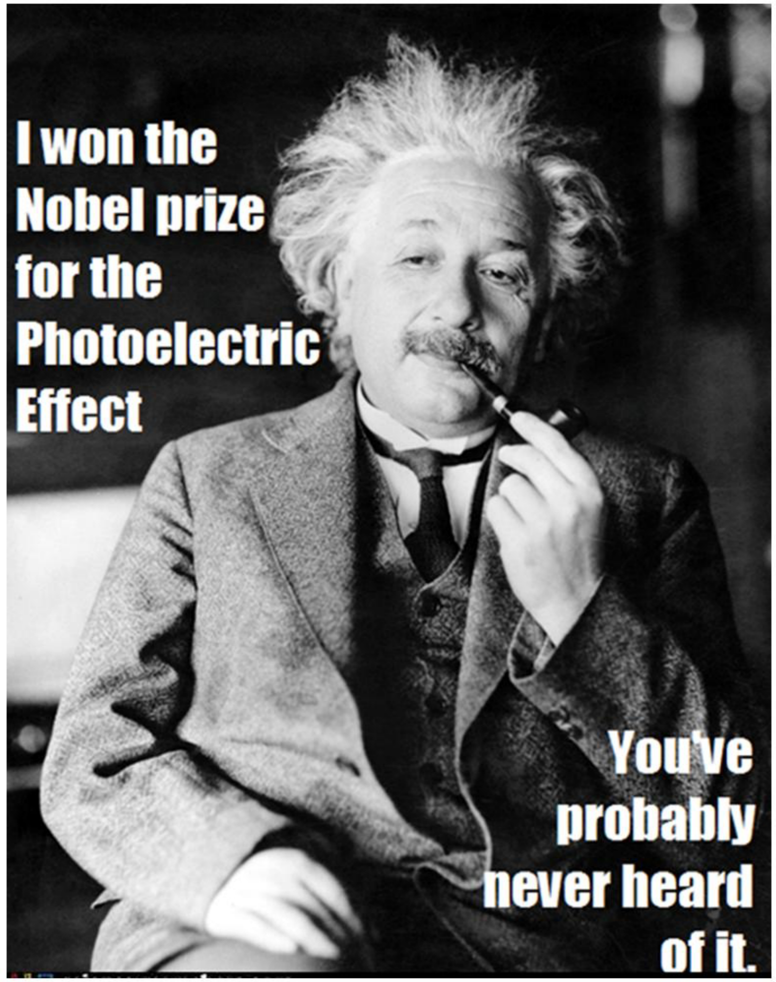


Linear polarization



Circular polarization

Elliptical polarization (Hybrid of Linear and Circular)


I won the Nobel prize for the Photoelectric Effect
You've probably never heard of it.

Quantum Properties of Radiation

Einstein's Photoelectric Equation:

When a photon of energy $h\nu$ falls on a metal surface, the energy of the photon is absorbed by the electron and is used in two ways:

i) A part of energy is used to overcome the surface barrier and come out of the metal surface. This part of the energy is called '**work function**' ($\Phi = h\nu_0$).

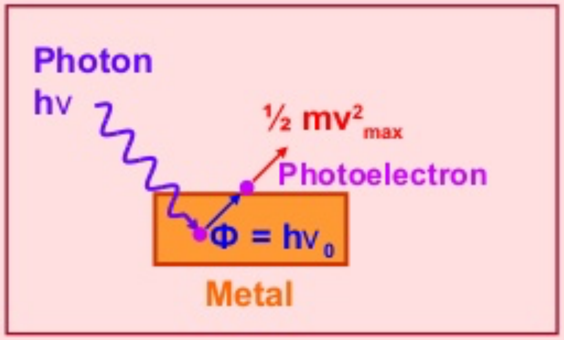
ii) The remaining part of the energy is used in giving a velocity 'v' to the emitted photoelectron. This is equal to the maximum kinetic energy of the photoelectrons ($\frac{1}{2} mv^2_{max}$) where 'm' is mass of the photoelectron.

According to law of conservation of energy,

$$h\nu = \Phi + \frac{1}{2} mv^2_{max}$$

$$= h\nu_0 + \frac{1}{2} mv^2_{max}$$

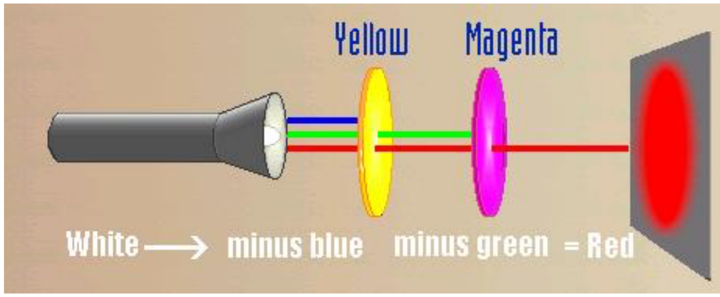
$$\frac{1}{2} mv^2_{max} = h(\nu - \nu_0)$$

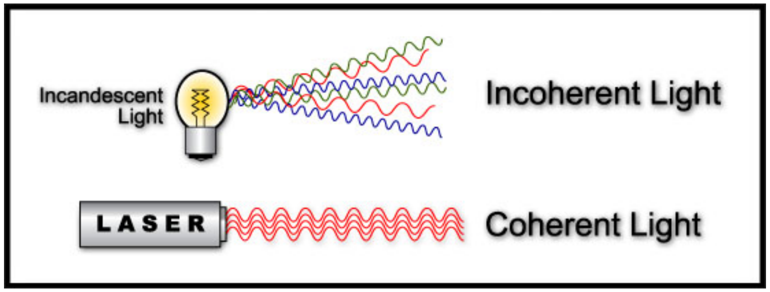


Wave vs. Particle

- Wave Nature of Radiation matters
  - Computing Scattering and reflection properties of atmospheric particles
    - Air molecules, aerosol, cloud droplets, rain drops
    - And Surfaces
- Quantized Nature of radiation
  - Absorption and Emission of radiation by INDIVIDUAL Atoms and molecules
    - Photochemical reactions

- Broadband Radiation
  - Wide range of frequencies
- Monochromatic Radiation Radiation
  - Single frequency (one color)
  - More commonly use: quasi monochromatic (range of frequencies)
- Coherent
  - Perfect synchronization
  - Artificial Source – radar, lidar, microwave
- Incoherent Radiation
  - Not phase locked
  - No synchronization
  - Natural radiation in the lower atmosphere





Monochromatic Flux

$$F_\lambda = \lim_{\Delta\lambda \to 0} \frac{F(\lambda, \lambda + \Delta\lambda)}{\Delta\lambda},$$

W m^{-2} um^{-1}

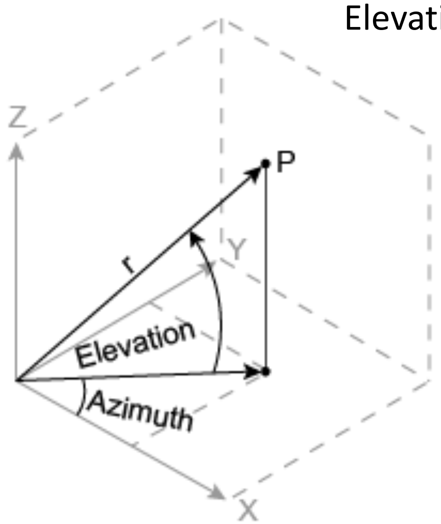
Broadband Flux

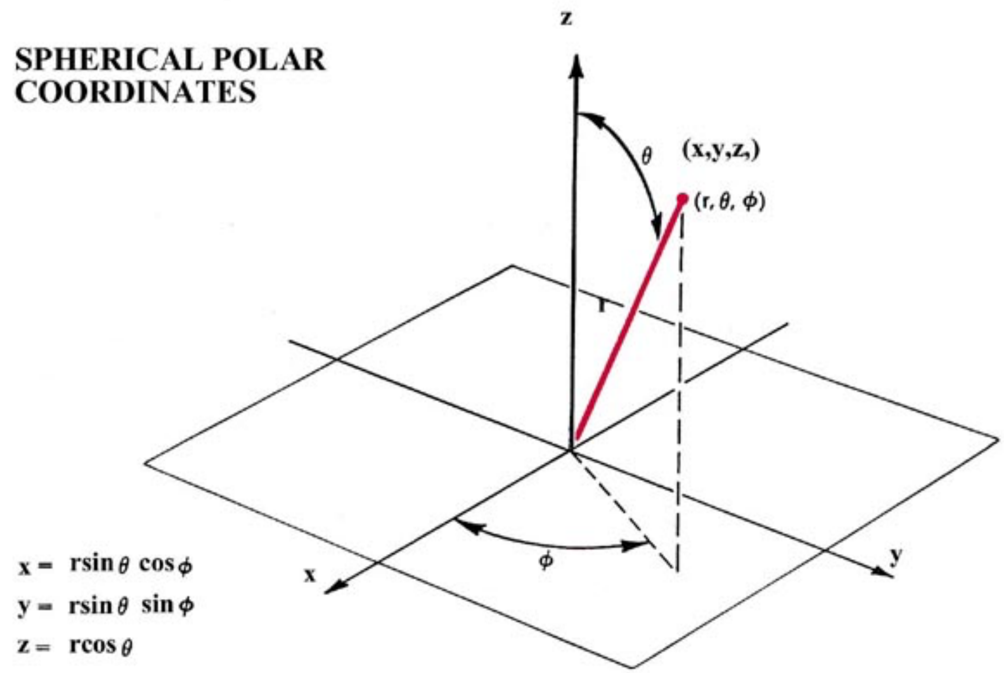
$$F(\lambda_1, \lambda_2) = \int_{\lambda_1}^{\lambda_2} F_\lambda \, d\lambda \, .$$

W m^{-2}

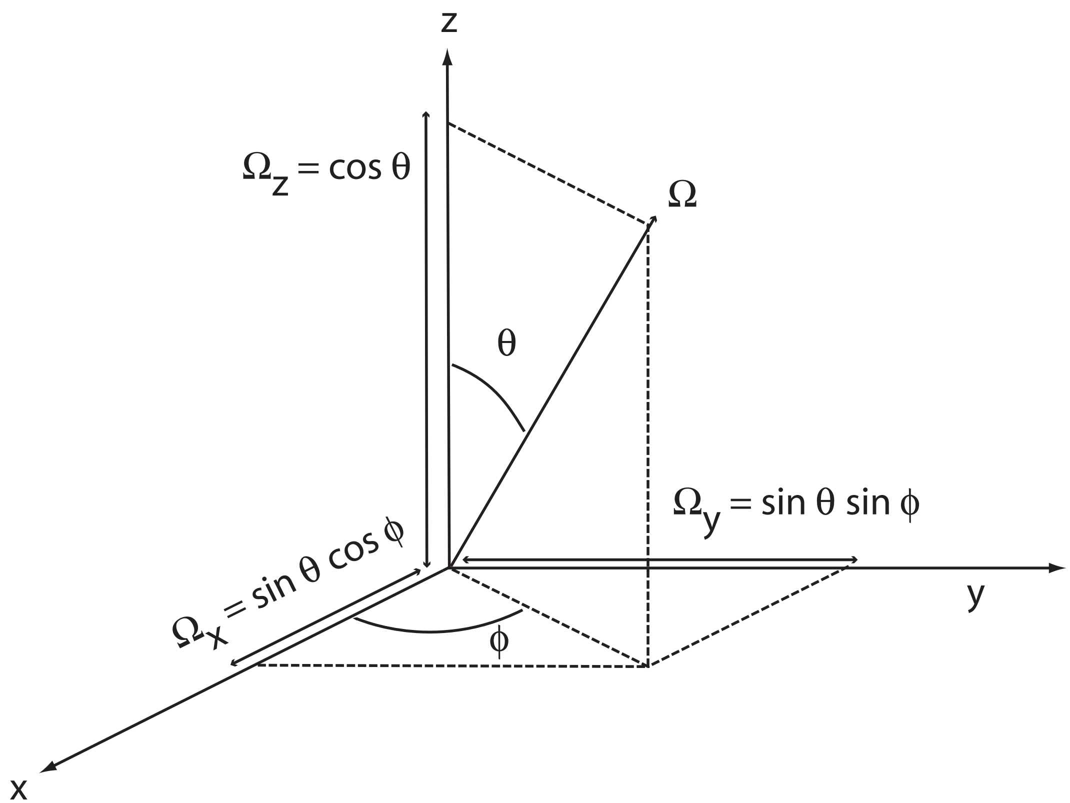
Spherical Coordinates

Elevation or Zenith

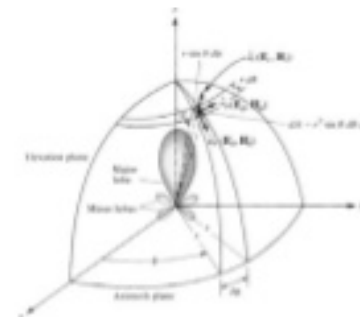




Spherical Coordinates



▪Steradian



one steradian subtends an area of

$$A = r^2$$

4π steradians in entire sphere

$$dA = r^2 \sin\theta \, d\theta \, d\phi$$

$$d\Omega = \frac{dA}{r^2} = \sin\theta \, d\theta \, d\phi$$

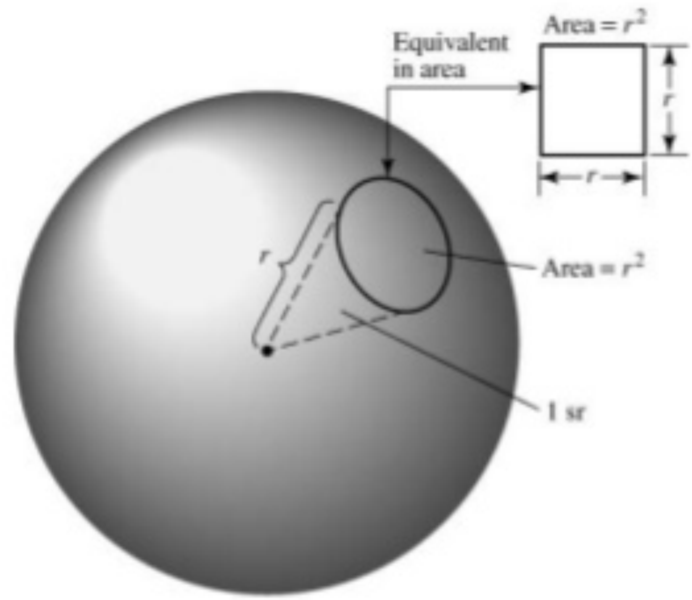
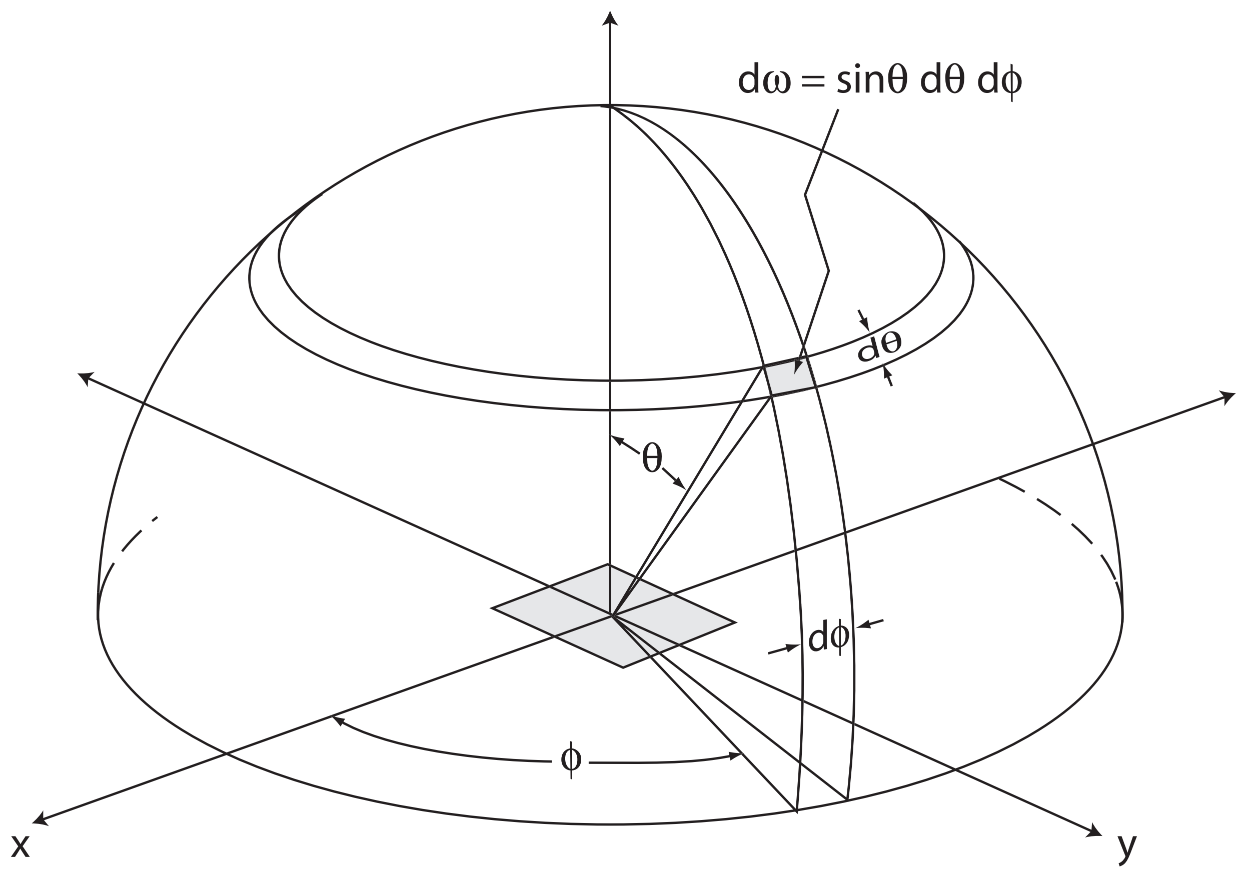


Fig. 2.10(b) Geometrical arrangements for defining a steradian.

Steradian

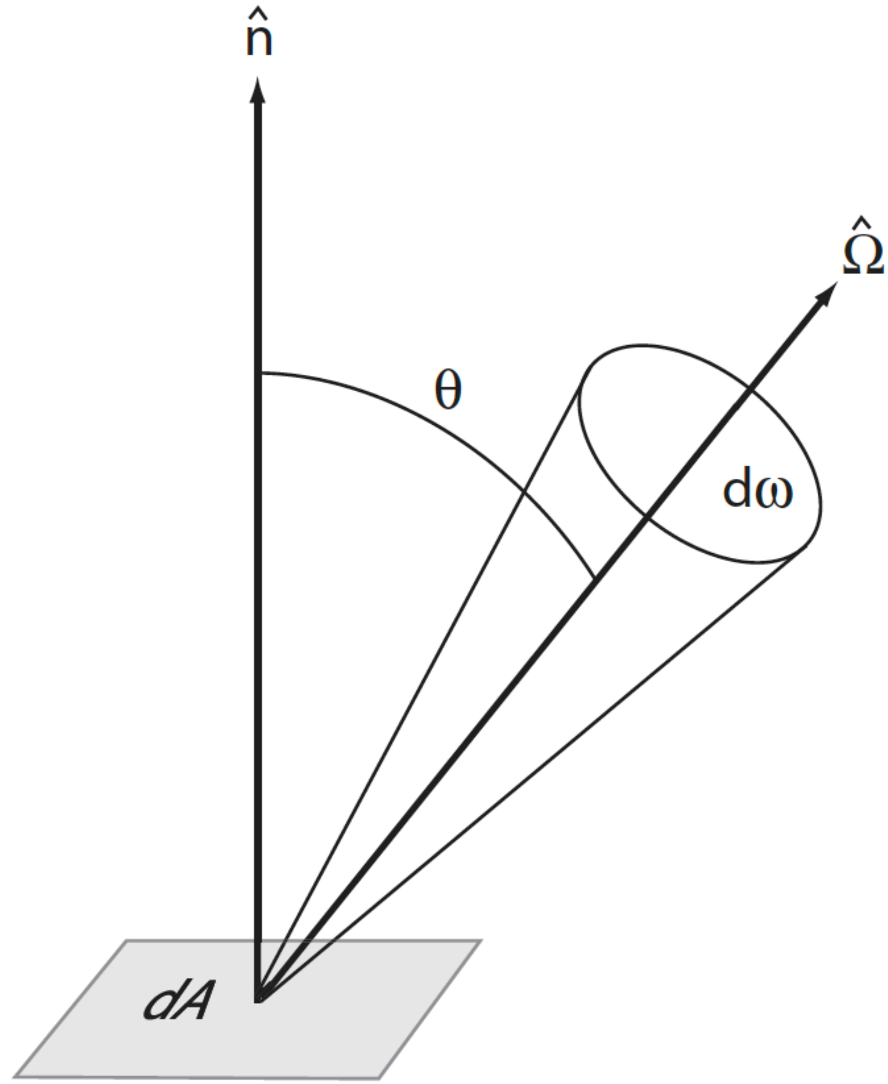
$$\int_{4\pi} d\omega = \int_0^{2\pi}\int_0^{\pi} \sin\theta \, d\theta \, d\phi = 2\pi \int_0^{\pi} \sin\theta \, d\theta = 4\pi.$$



Intensity

$$I(\hat{\mathbf{\Omega}}) = \frac{\delta F}{\delta \omega} \, .$$

$$\mathrm{dA} \propto \cos\theta = \hat{n} \cdot \hat{\Omega}$$



Relationship between Flux and Intensity

Upward Flux – Integrate over upper hemisphere

$$F^{\uparrow} = \int_0^{2\pi}\int_0^{\pi/2} I^{\uparrow}(\theta,\phi)\cos\theta\sin\theta\;d\theta d\phi\;,$$

Downward Flux – Integrate over lower hemisphere

$$F^{\downarrow} = -\int_0^{2\pi}\int_{\pi/2}^{\pi} I^{\downarrow}(\theta,\phi)\cos\theta\sin\theta\;d\theta d\phi\;.$$

Relationship between Flux and Intensity

Key fact: For the special case that the intensity is *isotropic* — that is, I is a constant for all directions in the hemisphere, then the above integrals can be evaluated to yield

$$\boxed{F = \pi I \, .} \tag{2.60}$$

Relationship between Flux and Intensity

Key fact: The *net flux* is defined as the difference between upward- and downward-directed fluxes:

$$\boxed{F^{\text{net}} \equiv F^{\uparrow} - F^{\downarrow}\ ,} \tag{2.61}$$

which can be expanded as

$$F^{\text{net}} = \int_0^{2\pi}\int_0^{\pi} I(\theta,\phi)\cos\theta\sin\theta\ d\theta d\phi = \int_{4\pi} I(\hat{\mathbf{\Omega}})\hat{\mathbf{n}}\cdot\hat{\mathbf{\Omega}}\ d\omega. \tag{2.62}$$