THE UNIVERSITY OF UTAH

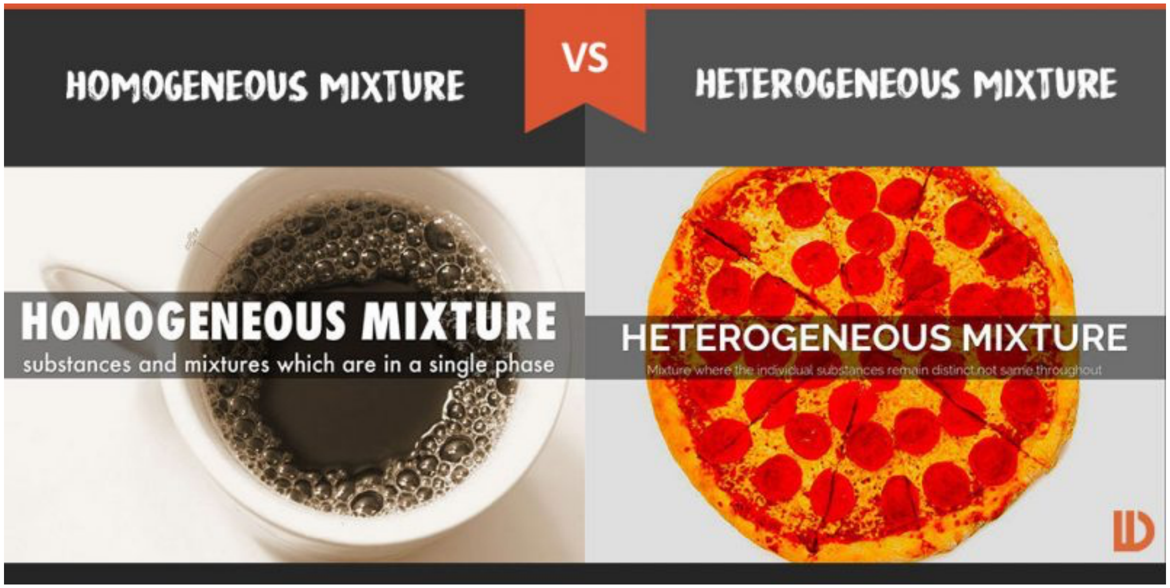
ATMOS 5140

Lecture 5 – Chapter 4

- Reflection and Refraction in Homogeneous Medium
    - Index of Refraction
    - Angle of Reflection and Angle of Refraction
    - Reflectivity

Homogeneous Medium

- Medium is smooth and uniform on scales comparable to the wavelength of the radiation



Homogeneous Medium

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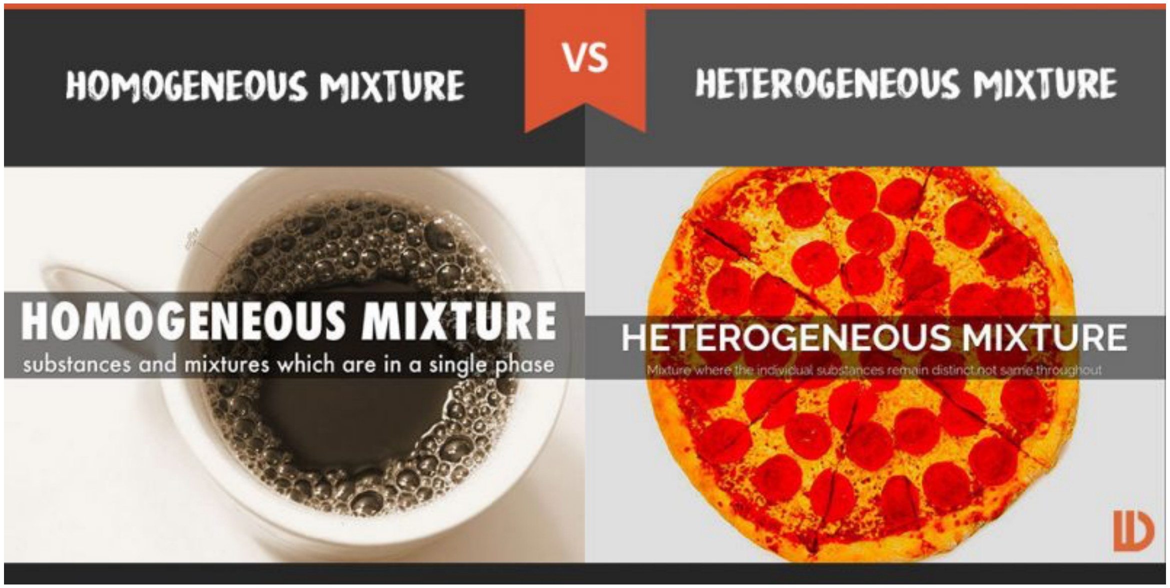


Yet both homogeneous to a 100 m radiowave!

Maxwell’s Equations for plane waves

Review

$$\vec{\mathbf{k}} = \vec{\mathbf{k}}' + i\vec{\mathbf{k}}''$$

If $\vec{k}''$ = 0, then the amplitude of the wave is constant
Then the medium is non absorbing!

$$\vec{\mathbf{E}}_c = \vec{\mathbf{E}}_0 \exp(-\vec{\mathbf{k}}'' \cdot \vec{\mathbf{x}}) \exp[i(\vec{\mathbf{k}}' \cdot \vec{\mathbf{x}} - \omega t)],$$

$$\vec{\mathbf{H}}_c = \vec{\mathbf{H}}_0 \exp(-\vec{\mathbf{k}}'' \cdot \vec{\mathbf{x}}) \exp[i(\vec{\mathbf{k}}' \cdot \vec{\mathbf{x}} - \omega t)]$$

Maxwell's Equations for plane waves

Review

In a nonvacuum, we can write

$$|\vec{\mathbf{k}}'| + i|\vec{\mathbf{k}}''| = \omega\sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}}\sqrt{\varepsilon_0\mu_0} = \frac{\omega N}{c},$$

where the complex *index of refraction* N is given by

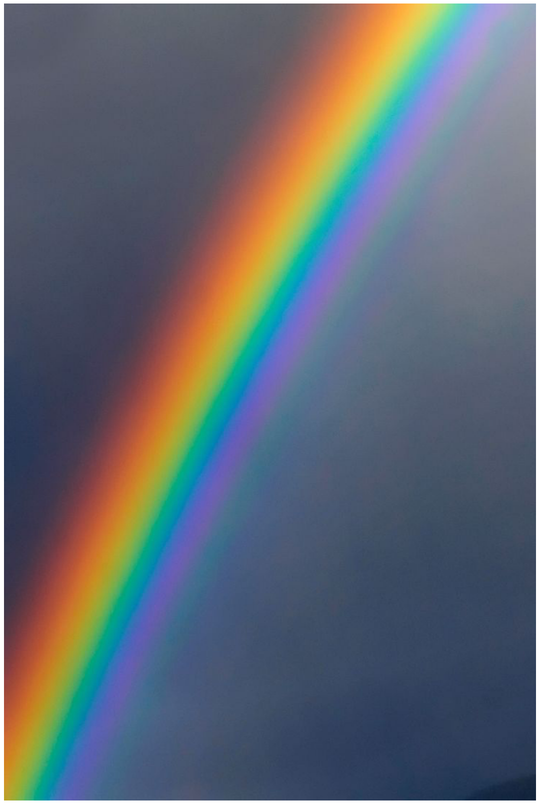
$$\boxed{N \equiv \sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}} = \frac{c}{c'}\,,}$$

Refractive Index

- The refractive index of a material is critical in determining the scattering and absorption of light, with the imaginary part of the refractive index having the greatest effect on absorption.

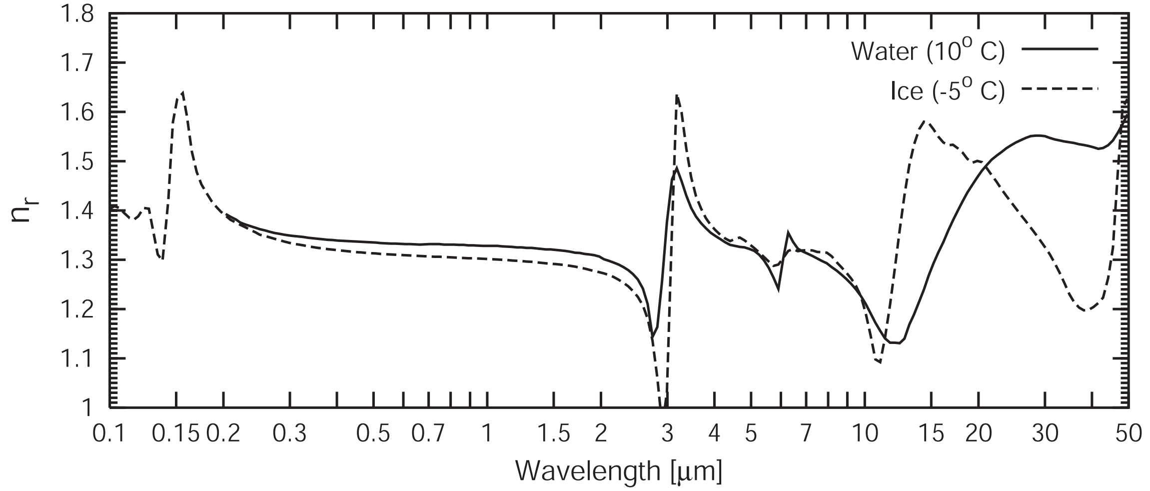
- The refractive index is NOT a constant for any substance but depends strongly on wavelength, and to a lesser degree, temperature &pressure

Rainbow

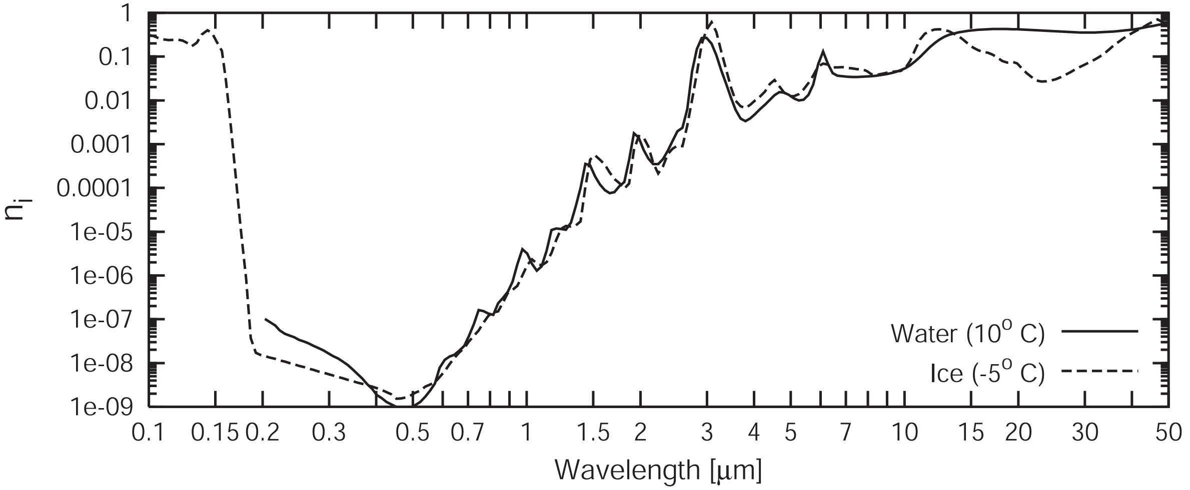


- Results from variation in the real part of the refractive index with wavelength of rain drops

(a) Index of Refraction of Water and Ice (Real Part)



(b) Index of Refraction of Water and Ice (Imag. Part)



Key Points

$n_r \approx 1.333$ in visible bands

$n_i \approx 1.0003$ in the visible bands
Close to zero absorption

Imaginary part of N => Absorption

Review

Consider scalar amplitude of the wave

$$E = |\vec{\mathbf{E}}_0 \exp(-\vec{\mathbf{k}}'' \cdot \vec{\mathbf{x}})|$$

Now go back to our definition of flux

$$F = F_0[\exp(-\vec{\mathbf{k}}'' \cdot \vec{\mathbf{x}})]^2 = F_0 \exp(-2\vec{\mathbf{k}}'' \cdot \vec{\mathbf{x}}).$$

Recall that our imaginary part of wave vector is responsible for absorption

$$|\vec{\mathbf{k}}''| = \frac{\omega}{c}\mathrm{Im}\{N\} = \frac{\omega n_i}{c} = \frac{2\pi\nu n_i}{c}$$

$$F = F_0 e^{-\beta_\mathrm{a} x}$$

Absorption

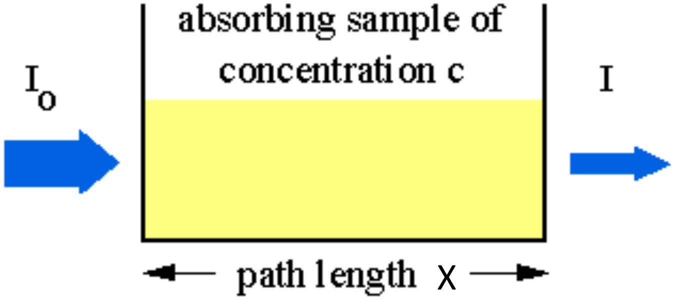
Review

$$\beta_a = 4\pi n_i/\lambda$$

$\frac{1}{\beta_a}$= distance required for the wave’s energy to be attenuated to e^{-1} (about 37%)

Beers - Lambert Law

$$I(x) = I_0 \, e^{-\beta_a x}$$



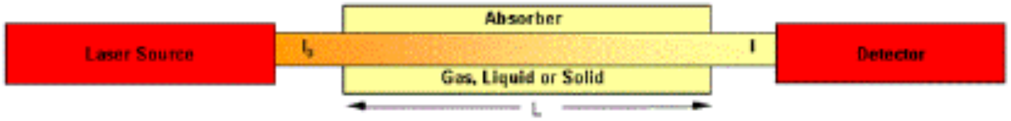
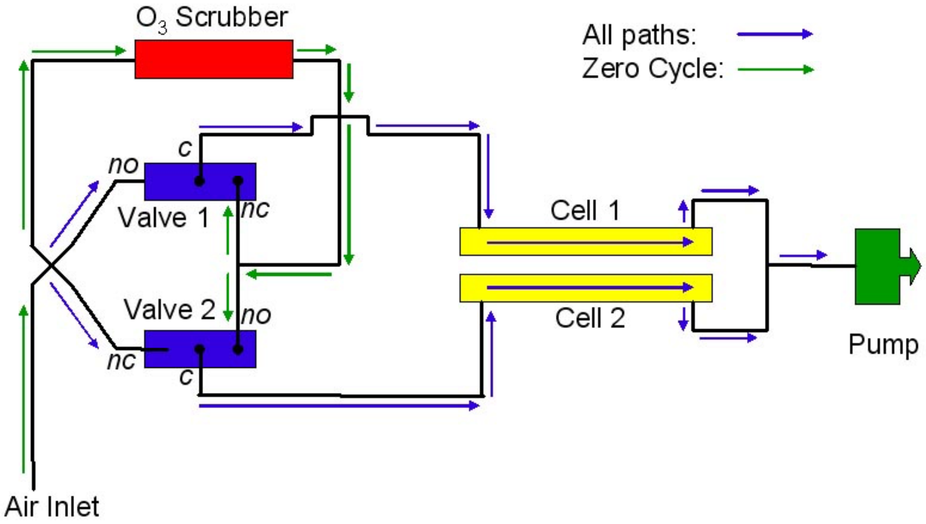


Figure 1: Laser absorption based on Beer-Lambert law.

Beers - Lambert Law

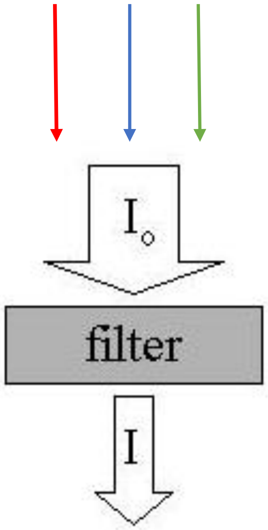
$$I(x) = I_0\, e^{-\beta_a x}$$

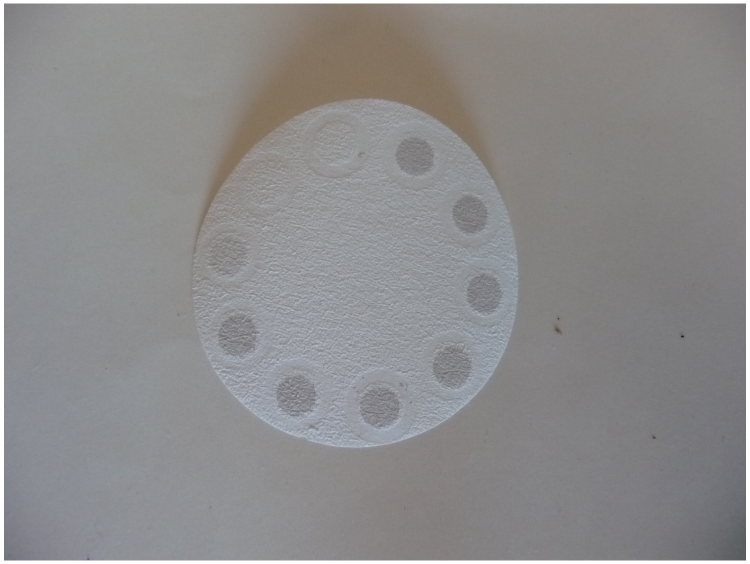
- UV Absorption at 254 nm



Transmission

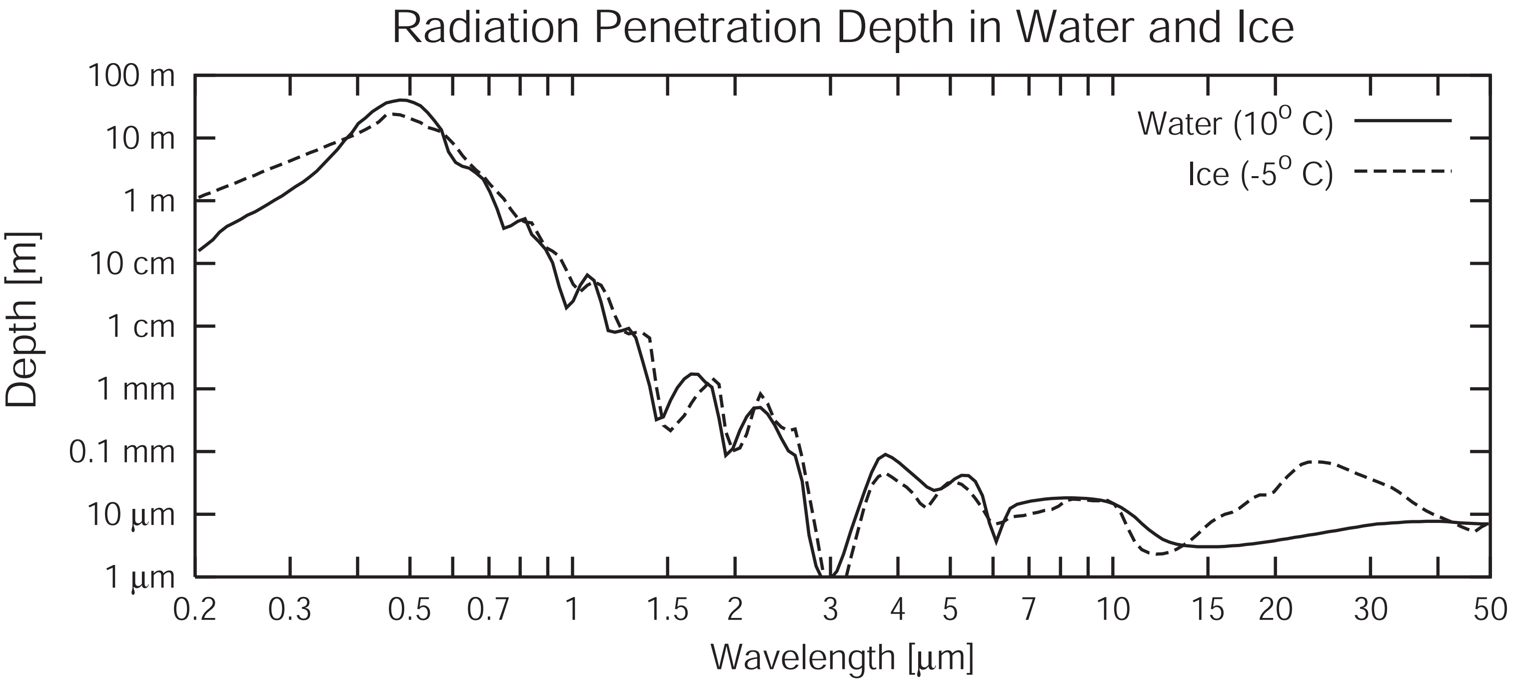
$$t(x) \equiv \frac{I(x)}{I_0} = e^{-\beta_a x}$$



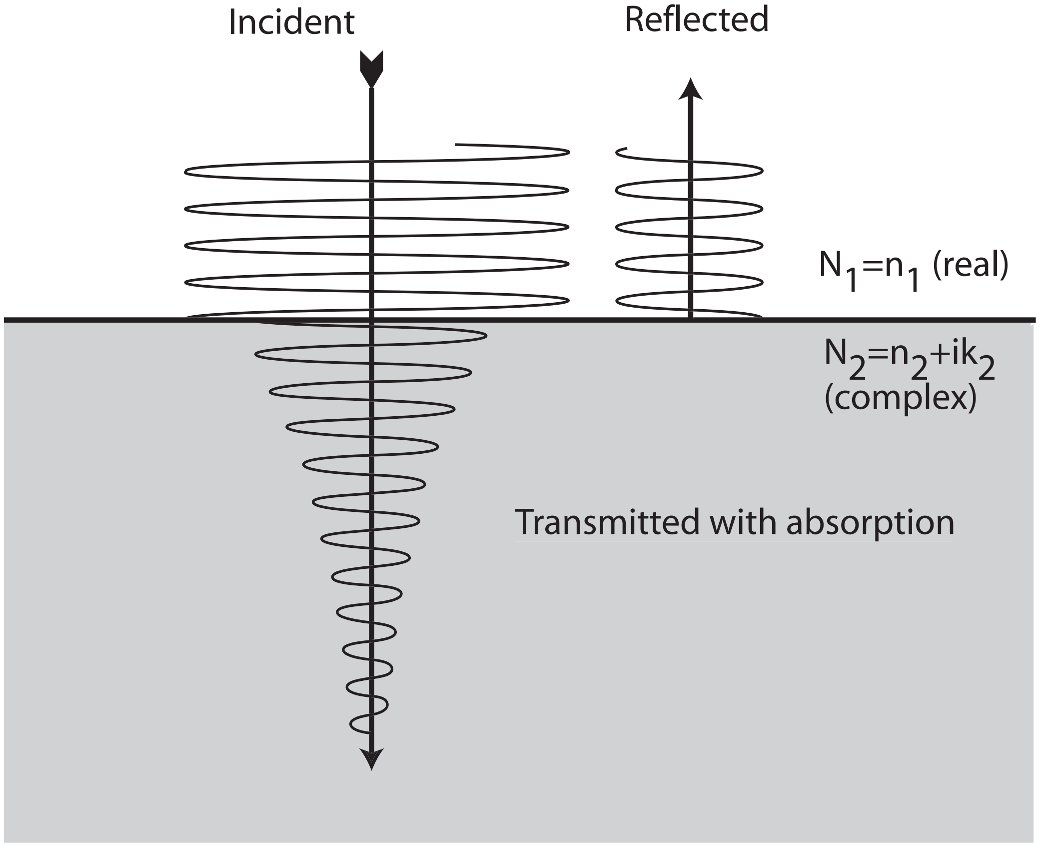


Penetration depth

$$D = \frac{1}{\beta_a} = \frac{\lambda}{4\pi n_i}$$



Reflection vs. Transmission

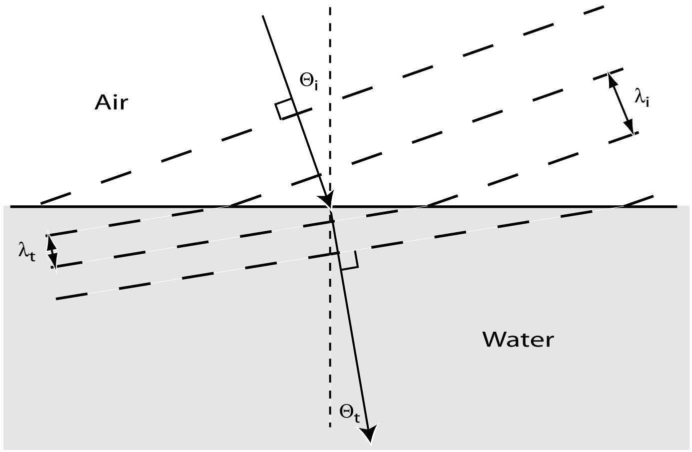


Refraction and Reflection

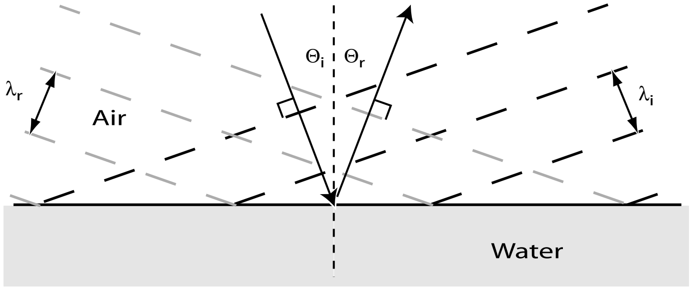
When an EM wave encounters a boundary between two homogeneous media having different indices of refraction, some of the energy is **reflected,** while the remainder passes through the boundary and may be altered from the original direction, and thus experience **refraction.**

Refraction and Reflection

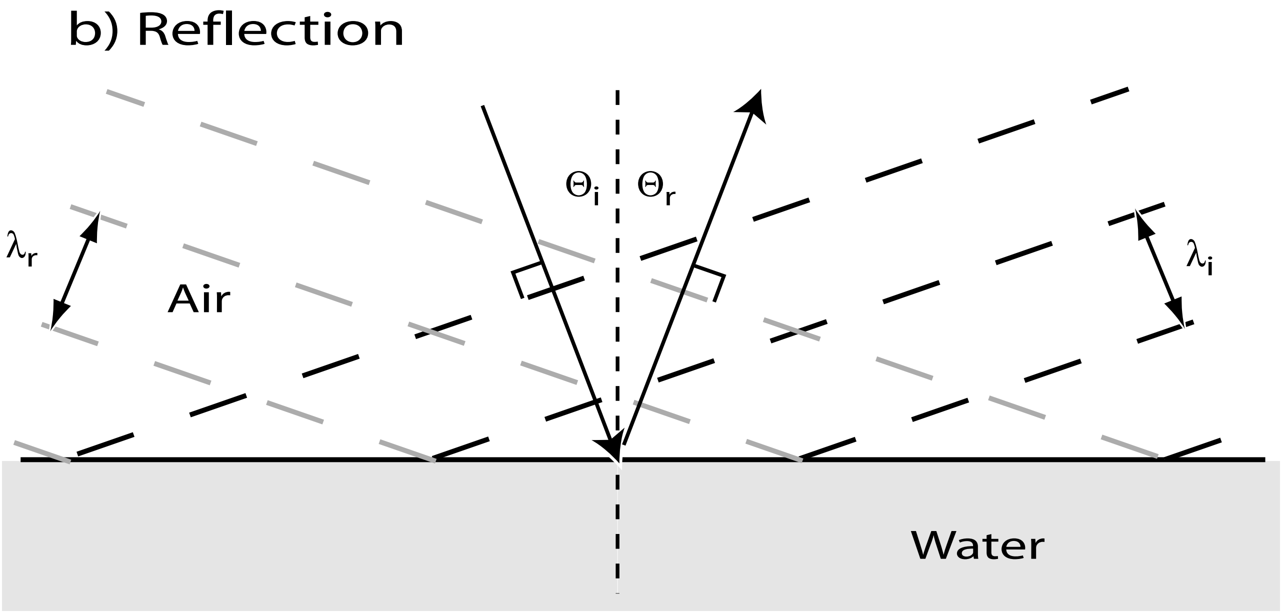
a) Refraction



b) Reflection



Reflection



When

$$\Theta_i = \Theta_r$$

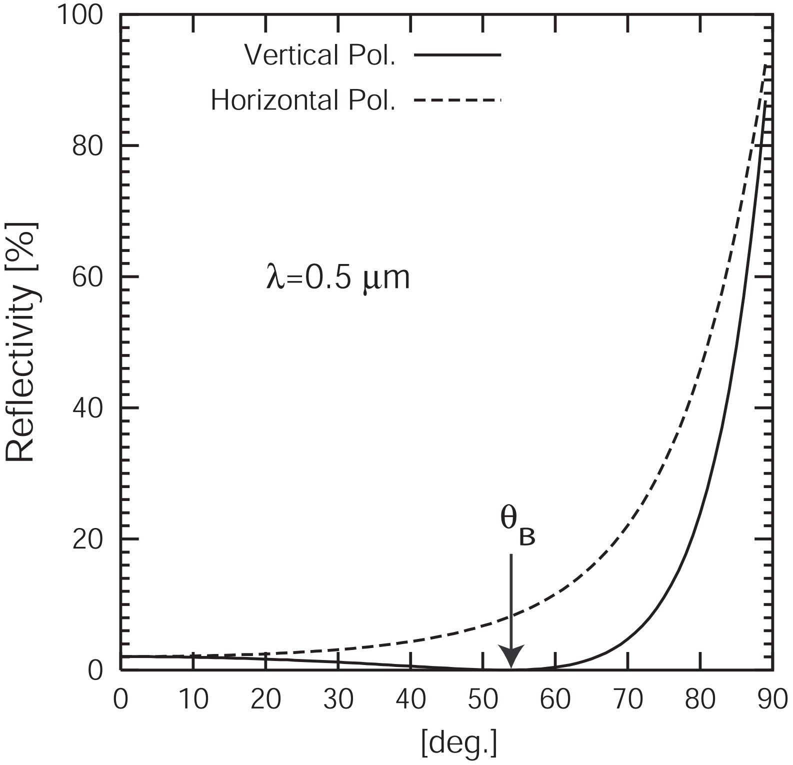
Specular reflection

Smooth surface in comparison to wavelength of light

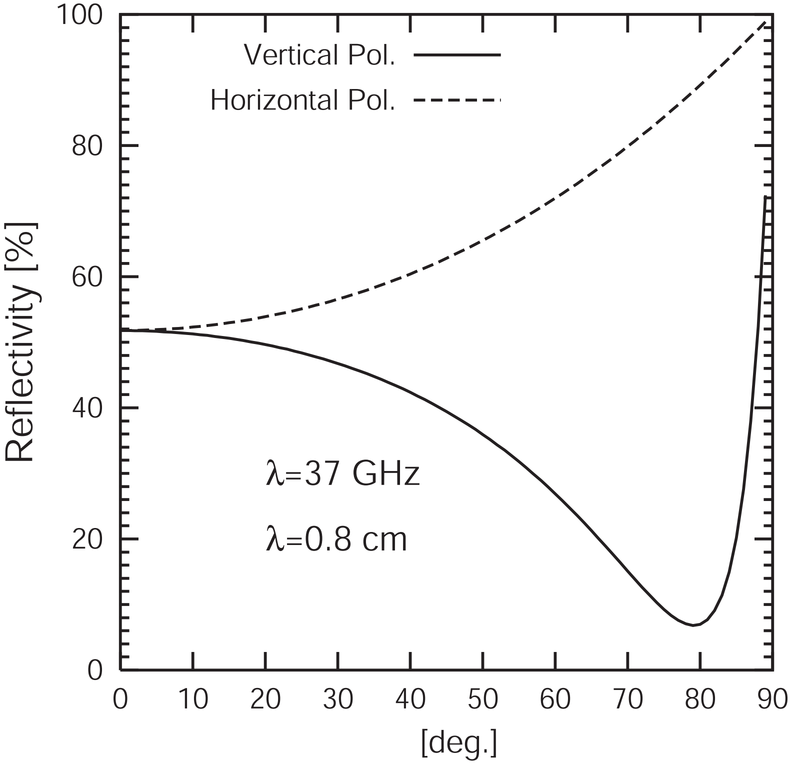
Reflectivity

- What fraction of the beam is reflected
- Polarization of the incident radiation matters!
- Frensnel Relations

Reflectivity

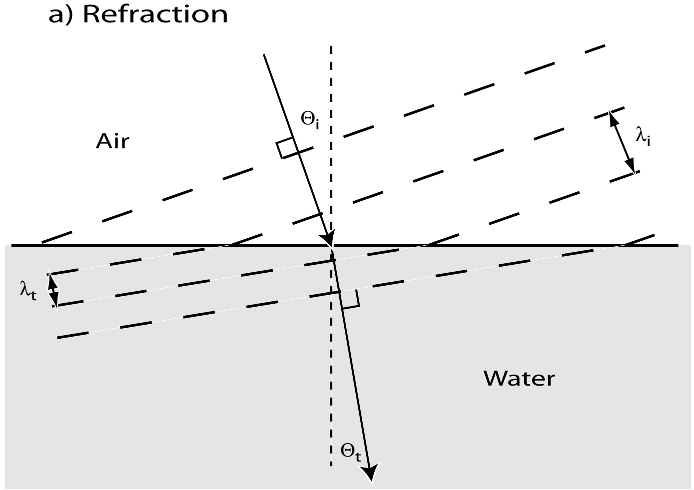


(a) Reflectivity of Water (Visible)



(b) Reflectivity of Water (Microwave)

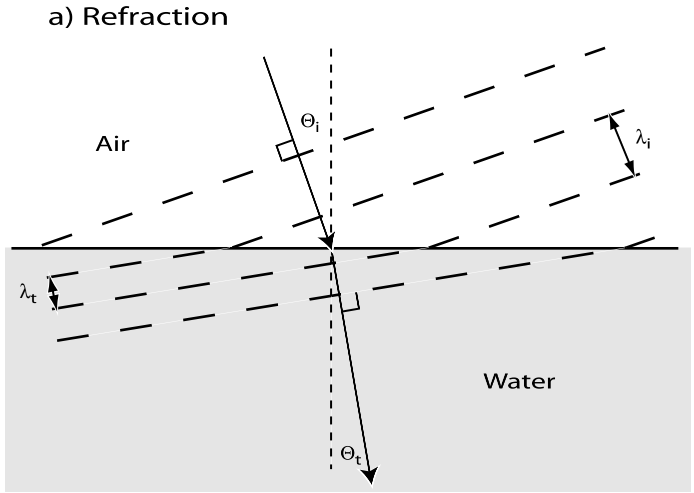
Angle of Refraction



Snell's Law

$$\frac{\sin \Theta_t}{N_1} = \frac{\sin \Theta_i}{N_2}$$

Angle of Refraction



Snell's Law

$$\frac{\sin \Theta_t}{N_1} = \frac{\sin \Theta_i}{N_2}$$

Critical Angle – Point of total reflection

$$\Theta_0 = \arcsin\left(\frac{N_1}{N_2}\right)$$