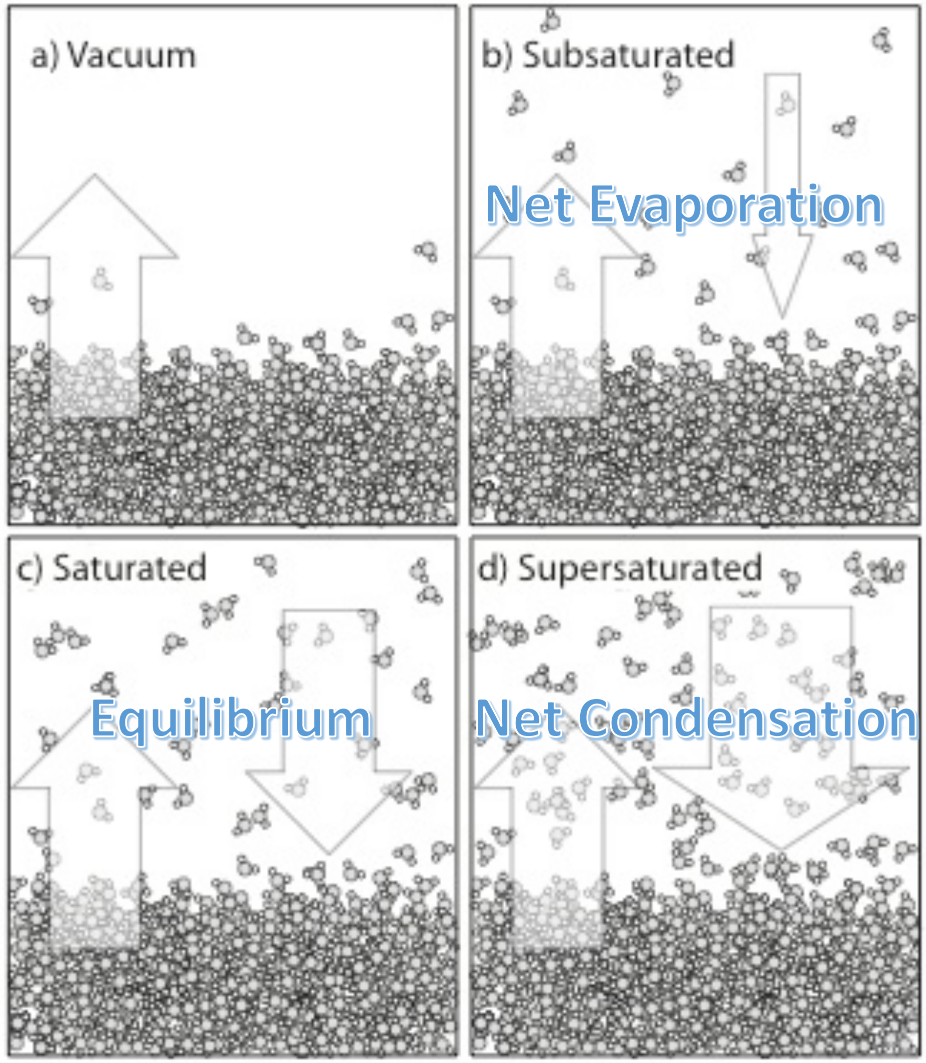
THE UNIVERSITY OF UTAH

ATMOS 5130

Lecture 10

- Moist Processes – Part 1
  - Relative Humidity
  - Dew Point
  - Clausius Clapeyron Equation
    - Latent Heat
    - Gibbs Energy



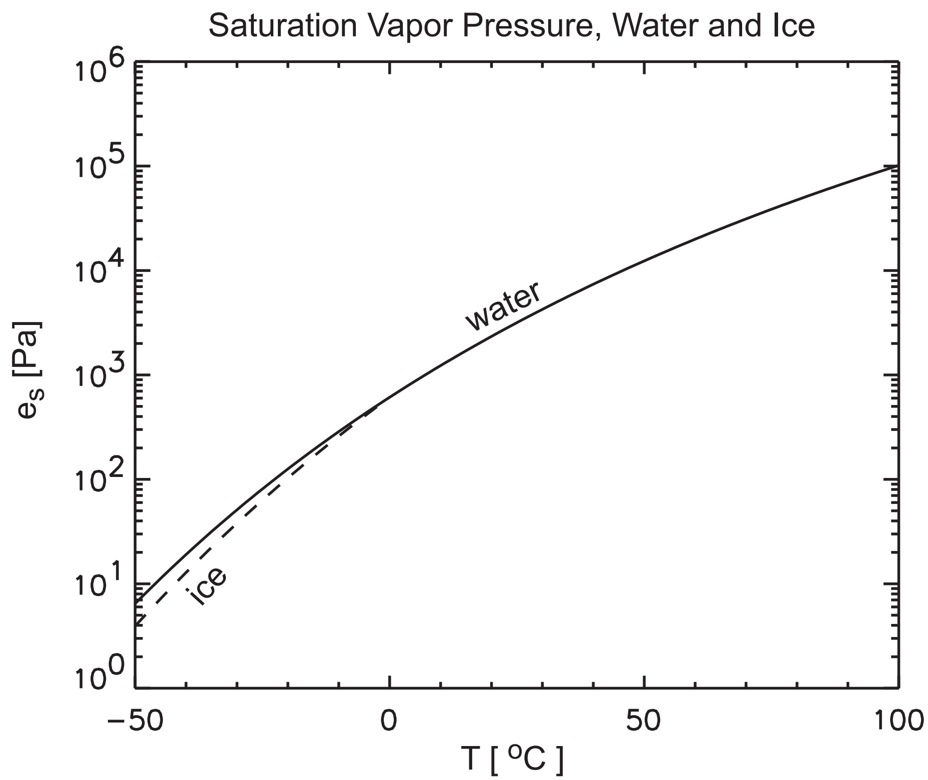
Vapor Pressure (e)

$e < e_s$

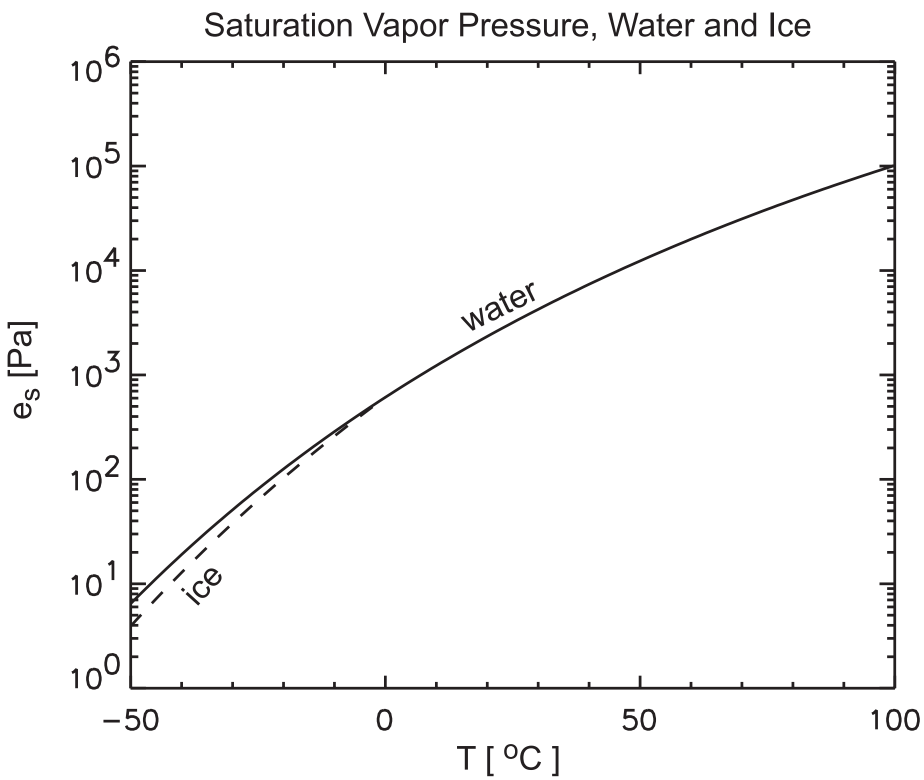
$e = e_s$

Vapor Pressure

$e > e_s$



Why does it take longer to cook at higher altitudes?



Why does it take longer to cook at higher altitudes?

Lower boiling point results in less heat.

General Rule:
For foods that cook in 20 minutes or less at sea level, add 1 minute of cooking time for each 1,000 feet (310 meters) of elevation.

For items taking more than 20 minutes to cook, add 2 minutes for each 1,00 feet of elevation.

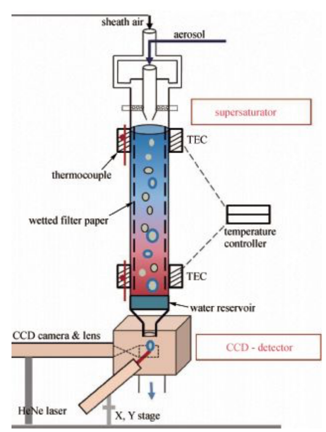
Relative humidity

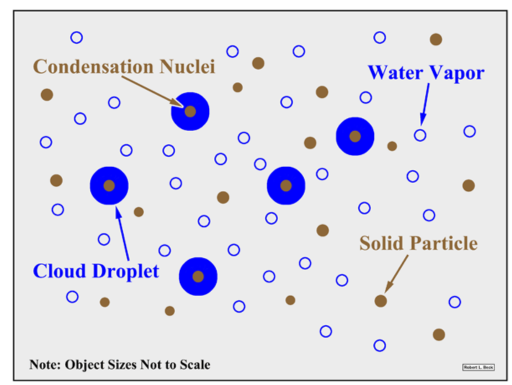
$$RH = \frac{e}{e_s(T)}$$

Supersaturation $\quad e > e_s\ (T)$

$$S \equiv RH\ - 100\%$$

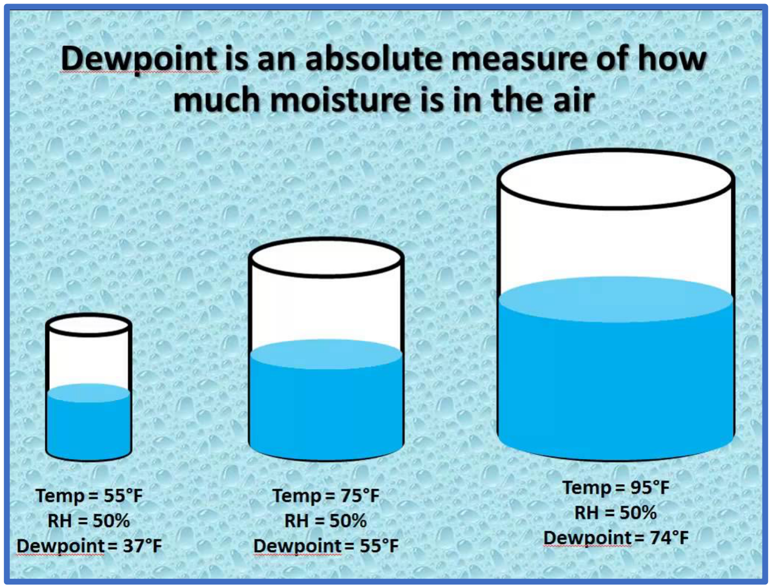
$S \approx 0.5\ - 2\%$ in convective updrafts





Dew Point

$$e_s(T_d) = e$$



Advection Fog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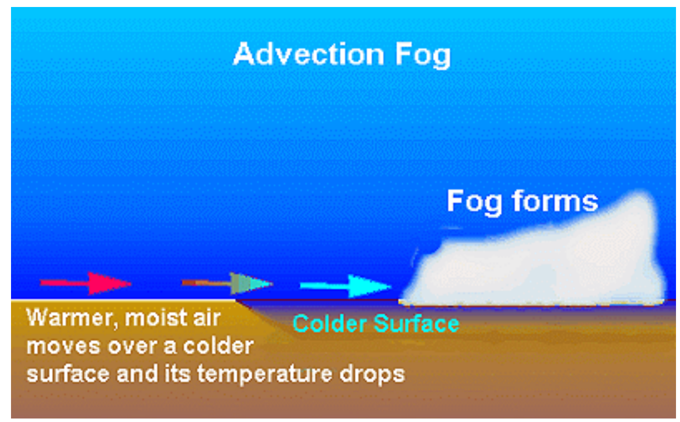
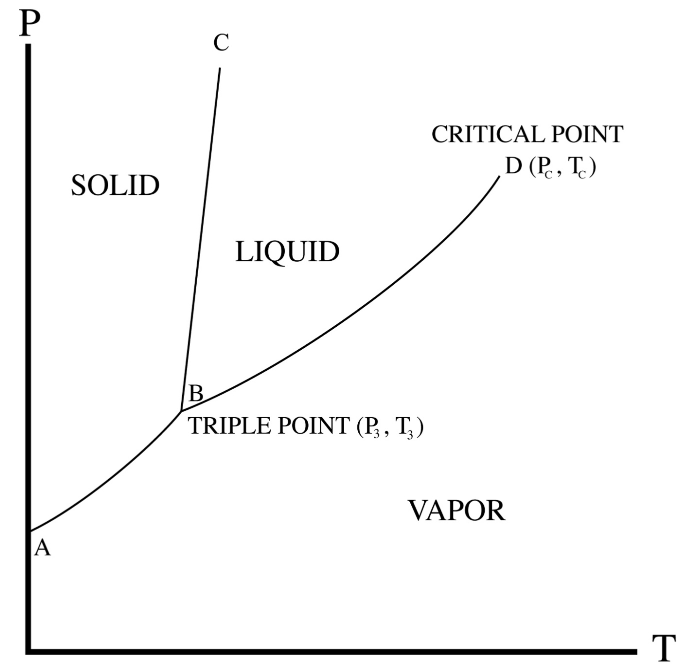
Warm humid air moves over cold water, chilling the air to its dewpoint.



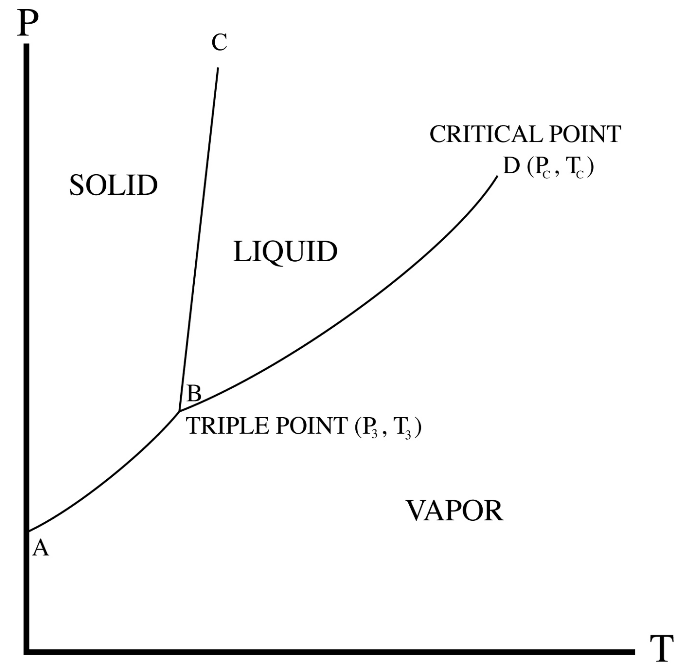
The Clausius-Clapeyron Equation

- Used to find the relationship between pressure and temperature along phase boundaries.

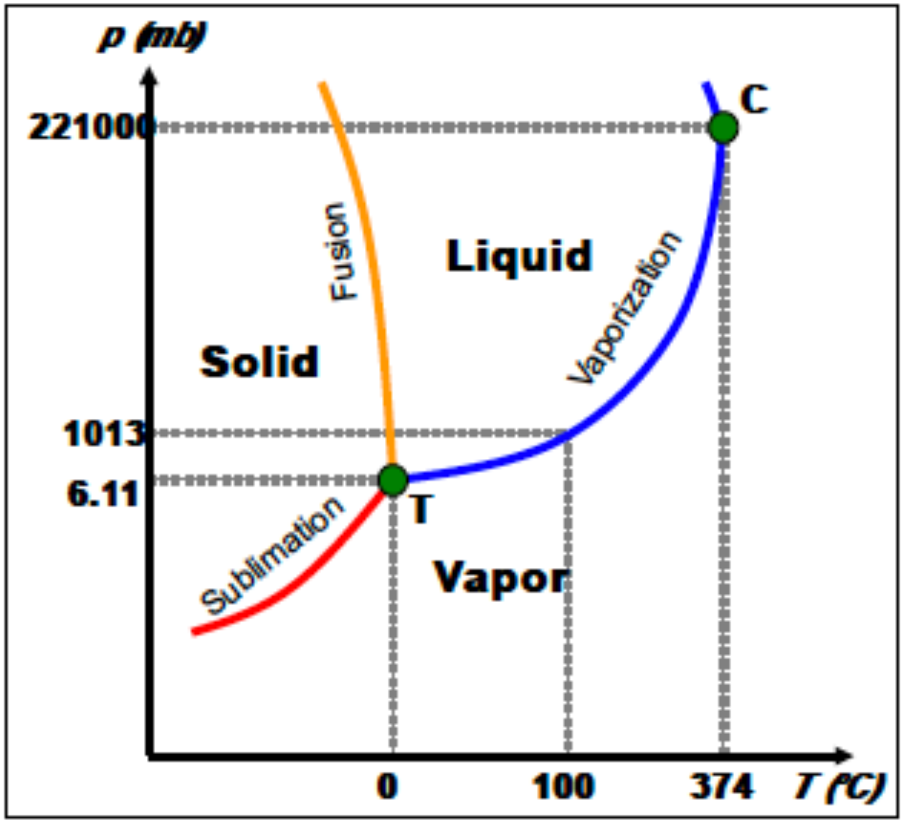


The Clausius-Clapeyron Equation

- Equilibrium vapor pressure – the pressure of the vapor phase in equilibrium with a condensed (solid or liquid) phase **(Line from A-D)**
- Boiling temperature – the temperature at which the equilibrium vapor pressure of a liquid equals the external atmospheric pressure **(Line from B-D)**
- Critical temperature – the temperature above which there is no liquid-vapor phase transition: **Point D,** referred to as the *critical point*.
    - For every homogeneous substance, there is one *critical point*, with P_C and T_C unique to the substance.
- Melting temperature – the temperature at which, for a given pressure, the solid and liquid phases coexist in equilibrium **(Line from B to C)**
- Triple point – a fixed point in P - T space corresponding to equilibrium between three phases: **Point B (P_3,T_3)**

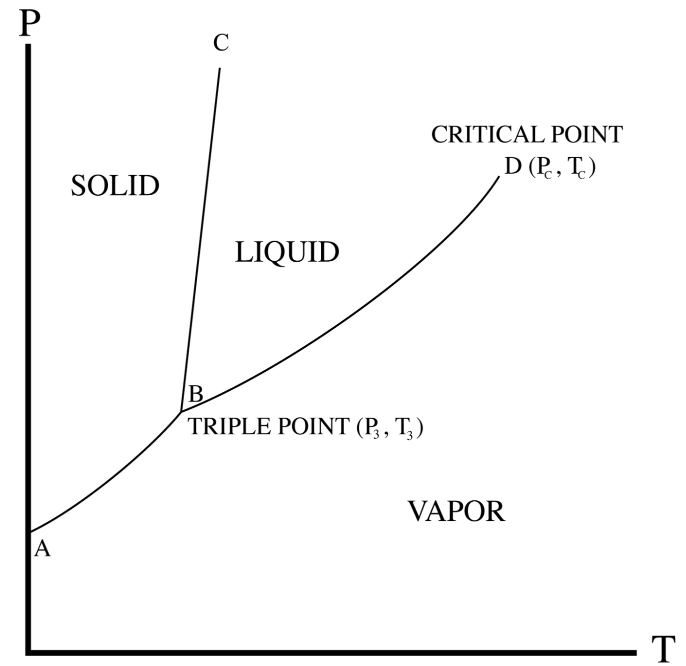


Clausius Clapeyron Diagram for H_2O



The Clausius-Clapeyron Equation

- Used to find the relationship between pressure and temperature along phase boundaries.
- Need to derive:
  - Latent Heat
  - Gibbs Energy



Latent Heat

- Total energy required to convert a unit mass of a substance from one phase to another while keeping the pressure and temperature constant is called the *specific enthalpy* of the phase change.

- More commonly called <u>Latent Heat</u>

Enthalpy – Sensible Heat

Review

Specific Enthalpy – intensive units

$$h \equiv u + p\alpha$$

Internal energy

Mechanical work

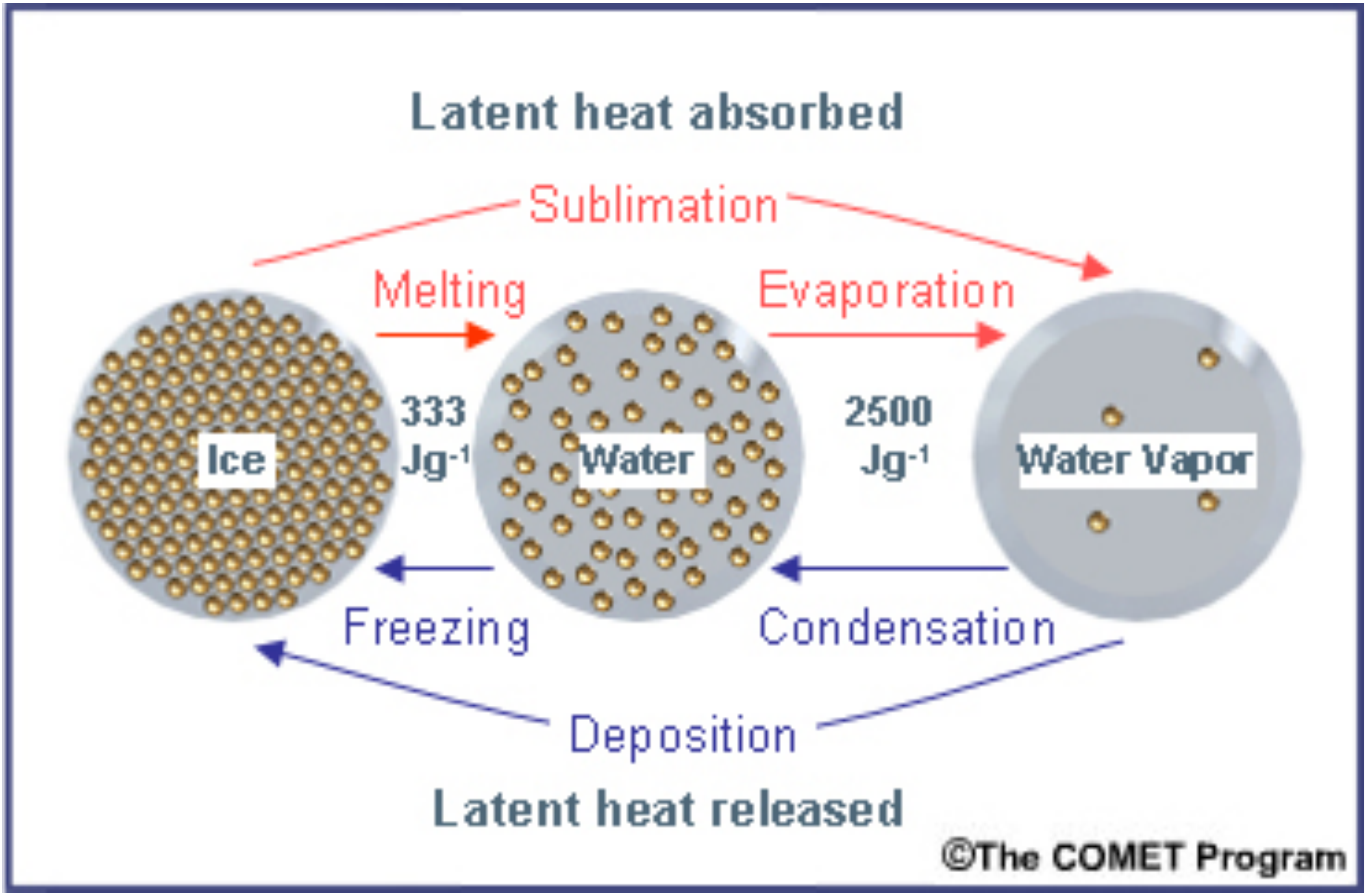
$$dh \equiv du + d(p\alpha) = du + \alpha dp + p d\alpha$$

$$\boxed{\delta q = du + p\, d\alpha}$$

Substitute in 1st law of Thermo

$dh - \alpha dp = du + p d\alpha$

$$\delta q = dh - \alpha\, dp$$



Latent heat absorbed
Sublimation
Melting
Evaporation
Ice
333 Jg-1
Water
2500 Jg-1
Water Vapor
Freezing
Condensation
Deposition
Latent heat released
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Latent Heat

Specific Enthalpy at constant pressure and temperature

Specific Enthalpy – intensive units

$$h \equiv u + p\alpha$$

Internal energy

Mechanical work

Constant Pressure

$$L = dh \equiv du + d(p\alpha) = \int \delta q = \int_{u_1}^{u_2} du + \int_{\alpha_1}^{\alpha_2} p d\alpha$$

$$p = e_s$$

$$L = (u_2 - u_1) + e_s(\alpha_2 - \alpha_1)$$

Specific Entropy

NEW STATE VARIABLE

Review

$$\oint \frac{1}{T}\delta q = \oint \frac{1}{T} c_v dT + \oint \frac{1}{T} p \, d\alpha$$

$$\oint \frac{1}{T}\delta q = c_v \oint \frac{dT}{T} + R \oint \frac{1}{\alpha} \, d\alpha$$

$$\boxed{\oint \frac{1}{T}\delta q = 0 \ \equiv ds}$$

Latent Heat

Specific Enthalpy at constant pressure and temperature

Specific Enthalpy – intensive units

$$h \equiv u + p\alpha$$

Internal energy

Mechanical work

Constant Temperature

$$L = T\int \frac{\delta q}{T} = \int_{s_1}^{s_2} ds = T(s_2 - s_1)$$

$$L = T(s_2 \; - s_1)$$

Gibbs Energy

$$L = (u_2 - u_1) + e_s(\alpha_2 \ - \alpha_1)$$

$$L = T(s_2 \ - s_1)$$

Combine

$$(u_2 - u_1) + e_s(\alpha_2 \ - \alpha_1) = T(s_2 \ - s_1)$$

Rearrange

$$u_1 + e_s(\alpha_1) - T(s_1) = u_2 + e_s(\alpha_2) - T(s_2)$$

Gibbs Energy = $\mathrm{G} = u + e_s(\alpha) - T(s)$

$G_1 = G_2$

Remember Gibbs energy is not constant for temperature and pressure

We assumed isothermal, isobaric change in phase to get here.

Gibbs Energy

Latent Heat Equation
in 2 forms

Gibbs Energy = $G = u + e_s(\alpha) - T(s)$

$$dG = du + e_s(d\alpha) + \alpha de_s - T(ds) - sdT$$

Recall: $du + e_s d\alpha = \delta q = Tds$

$$dG = du + e_s(d\alpha) + \alpha de_s - (du + e_s d\alpha) - sdT$$

$$dG = \alpha de_s - sdT$$

$G_1 = G_2$

$$\alpha_1 de_s - s_1 dT = \alpha_2 de_s - s_2 dT$$

- Remember Gibbs energy is not constant for temperature and pressure.
- Gibbs energy is the same for both the liquid and vapor phases when the two are at equilibrium (same pressure and temperature)

Clausius – Clapeyron equation

$\alpha_1 de_s - \mathrm{s}_1 \mathrm{dT} = \alpha_2 de_s - \mathrm{s}_2 \mathrm{dT}$

$\alpha_1 \frac{de_s}{dT} - \mathrm{s}_1 = \alpha_2 \frac{de_s}{dT} - \mathrm{s}_2$

$$\frac{de_s}{dT} = \frac{\mathrm{s}_1 - \mathrm{s}_2}{\alpha_2 - \alpha_1}$$

Recall: $L = T(s_2 - s_1)$

$$\frac{de_s}{dT} = \frac{\mathrm{s}_1 - \mathrm{s}_2}{\alpha_2 - \alpha_1} = \frac{L}{T(\alpha_2 - \alpha_1)}$$

Clausius – Clapeyron equation for the atmosphere

$$\frac{de_s}{dT} = \frac{s_1 - s_2}{\alpha_2 - \alpha_1} = \frac{L}{T(\alpha_2 - \alpha_1)}$$

Assumption: $\alpha_2 >>> \alpha_1$

Specific Volume of liquid water

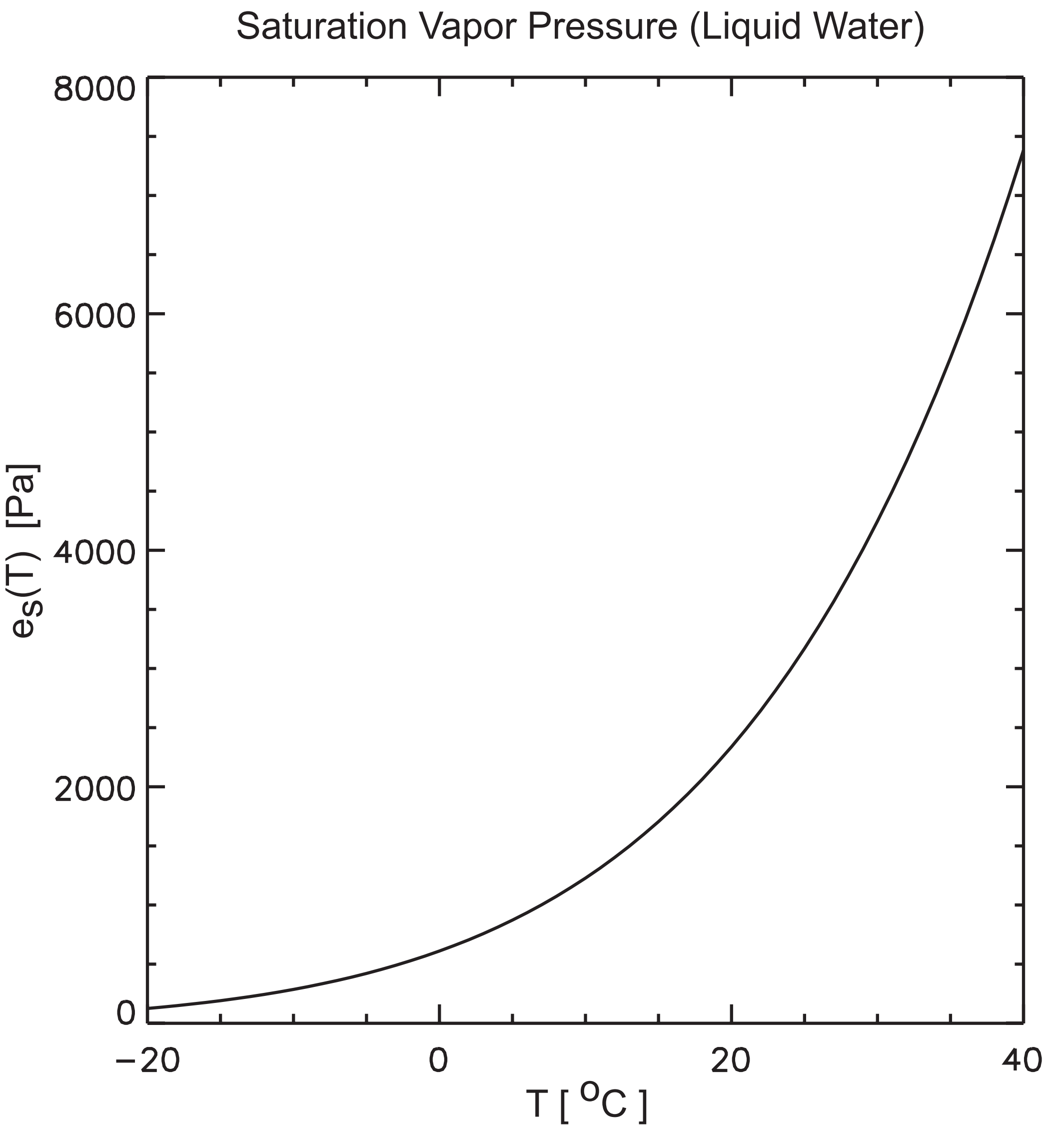
Specific Volume of water vapor

$$\frac{de_s}{dT} = \frac{L}{T(\alpha_2)}$$

Substitute in the ideal gas law for water vapor $\frac{1}{\alpha_2} = \frac{e_s}{R_v T}$

$$\frac{de_s}{dT} = \frac{Le_s}{R_v T^2} \Rightarrow \frac{de_s}{e_s} = \frac{L}{R_v} * \frac{dT}{T^2}$$

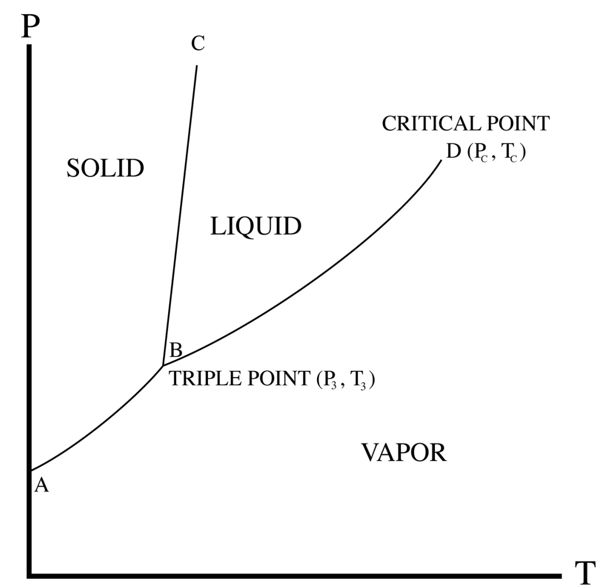
$$\int_{e_{s0}}^{e_s} \frac{de_s}{e_s} = \frac{L}{R_v} \int_{T_0}^{T} \frac{dT}{T^2} \Rightarrow \boxed{e_s(T) = e_{s0} \exp\left[\frac{L}{R_v}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right]}$$



Saturation Vapor Pressure (Liquid Water)
8000
6000
4000
2000
0
−20
0
20
40
T [oC]
es(T) [Pa]

Vapor Pressure with respect to ice

$$\boxed{e_i(T) = e_{i0} \exp\left[\frac{L_s}{R_v}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right]}$$



Supercooled water!

At $T < 0$ C ➡ $e_i(T) < e_s(T)$

So, at subfreezing temperature, an environment that is **saturated** with respect to water will be **supersaturated** with respect to ice.

Conversely, an environment that is **saturated** with respect to ice will be **subsaturated** with respect to supercooled liquid water