THE UNIVERSITY OF UTAH

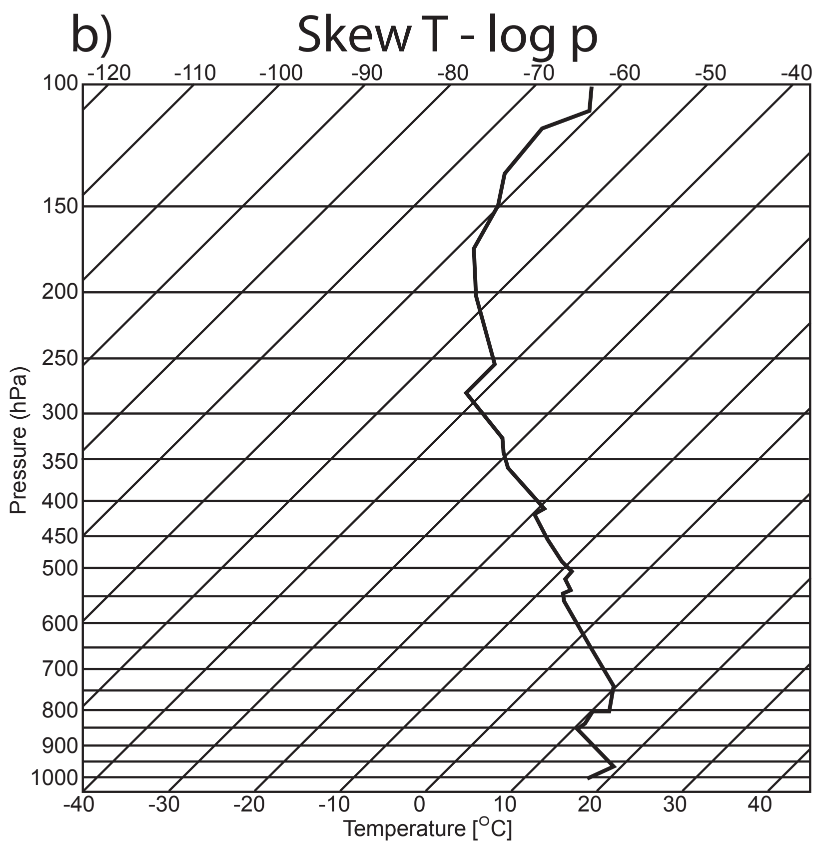
ATMOS 5130

Lecture 11

- Moist Processes – Part 2
  - Unsaturated – Skew T
  - Review Clausius – Clapeyron Equation
  - Saturation Mixing Ratio
  - Moist Adiabatic Lapse Rate
  - Lifting Condensation Level

Simplest form of Skew T

Review

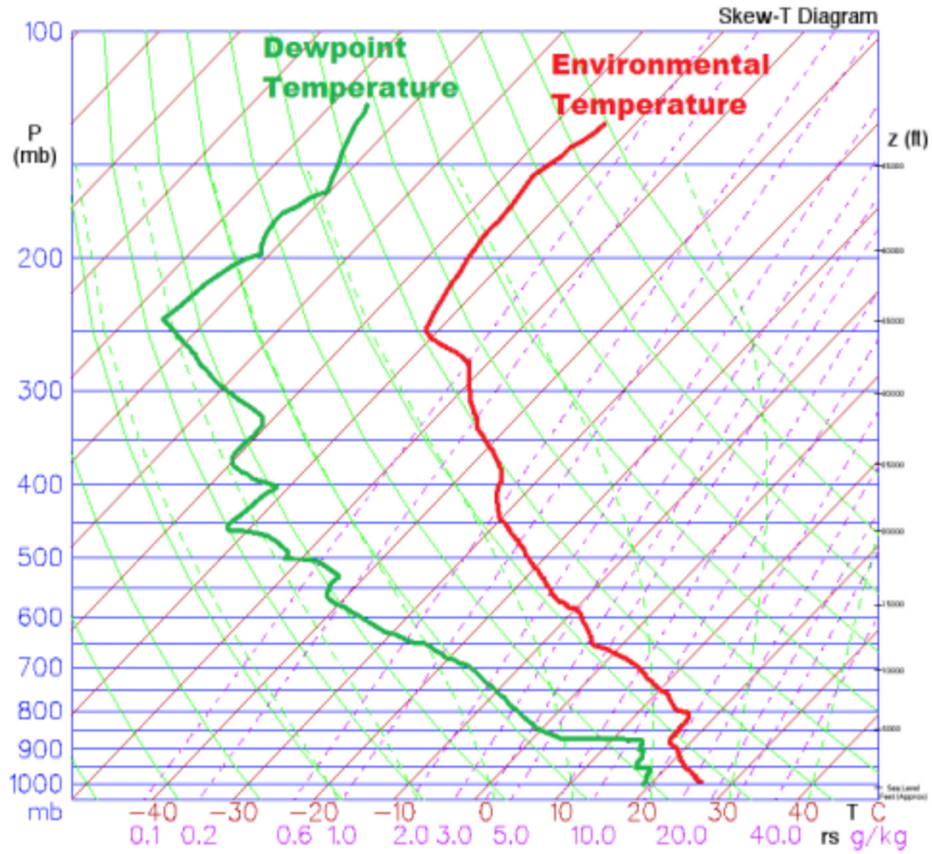


12Z 25 Aug 2005 Green Bay, WI

Standard Meteorological form of Skew T

Goal:

Understand meaning of all lines.



Review

Unsaturated Atmosphere

Review

Dry Adiabats

$$\Gamma_d = \frac{\partial T}{\partial z} = \frac{g}{c_p} = 9.8\ \text{K/km}$$

Potential Temperature

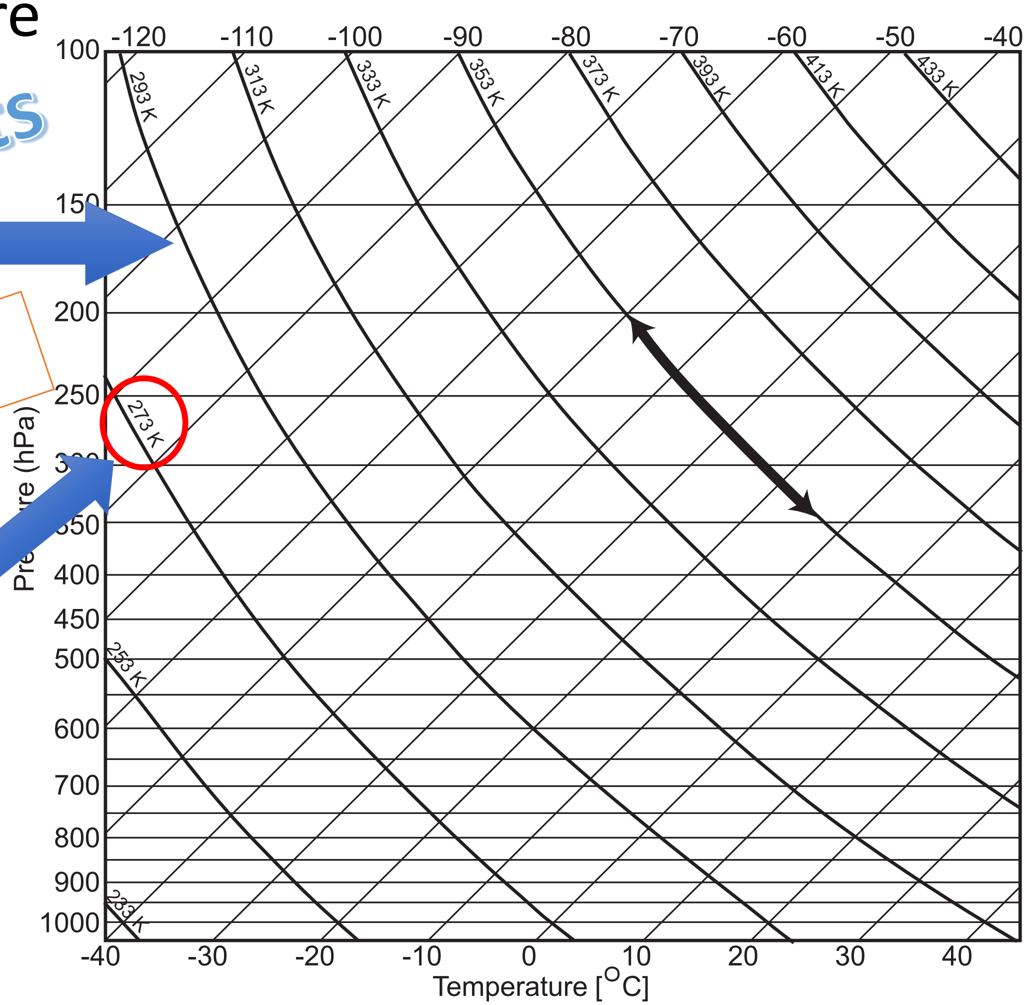


Fig. 5.3

The word adiabatic means that no outside heat is involved in the warming or cooling of the air parcels.

Clausius – Clapeyron equation for the atmosphere

$$e_s(T) = e_{s0} \exp\left[\frac{L}{R_v}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right]$$

Measured value of saturation vapor pressure at T_0

T_0 = 0 C = 273 K

e_{so} = 6.11 hPa

L = 2.50 x 10^6 J/kg

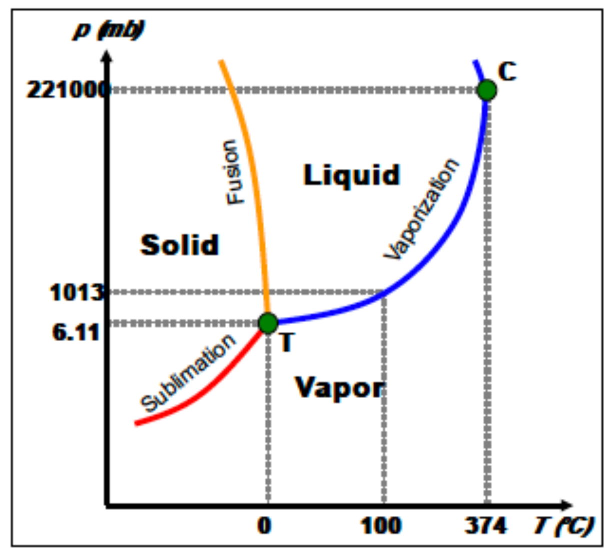
R_v = 461.5 J/(kg K)

$$e_s(T) \approx A \exp\left[\frac{-B}{T}\right]$$

A = 2.53 x 10^{11} Pa

B = 5420 K

Vapor Pressure with respect to water



Saturation Mixing Ratio

$$w_s(T,P) = \frac{\varepsilon e_s(T)}{p - e_s(T)}$$

$$Where:\ \varepsilon = \frac{R_d}{R_v}$$

$$w_s(T,P) \approx \frac{\varepsilon e_s(T)}{p}$$

Unique value of w_s fro any combination of T and p

Thus, lines may be drawn on a skew-T

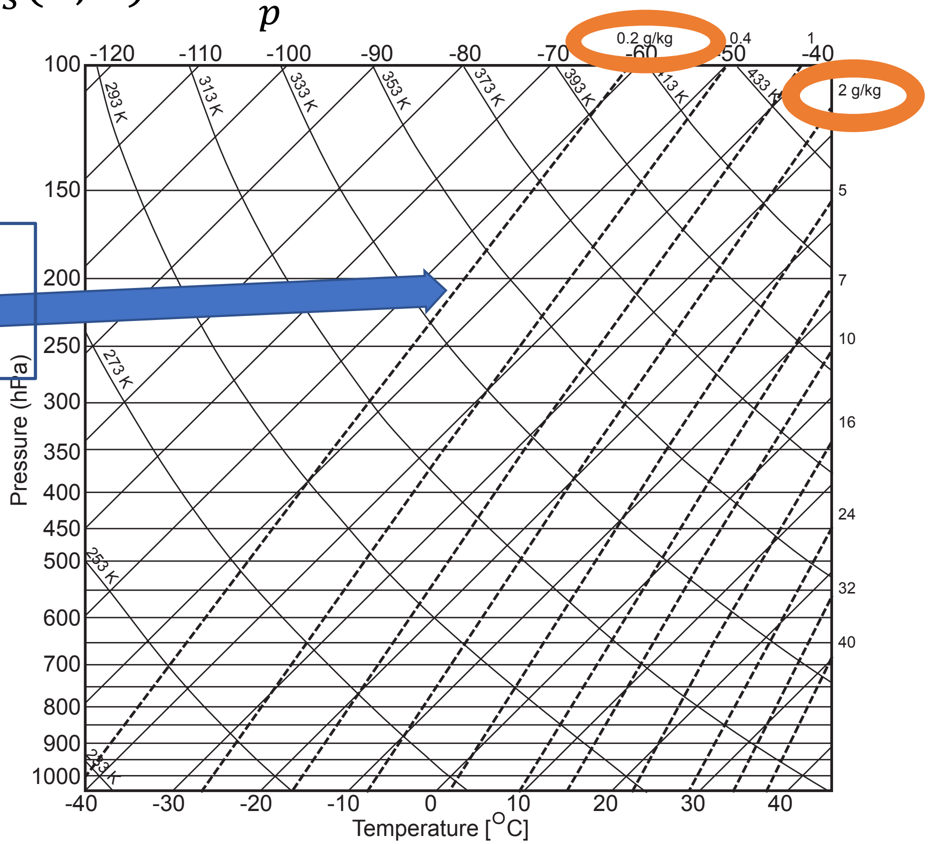
Saturation Mixing Ratio

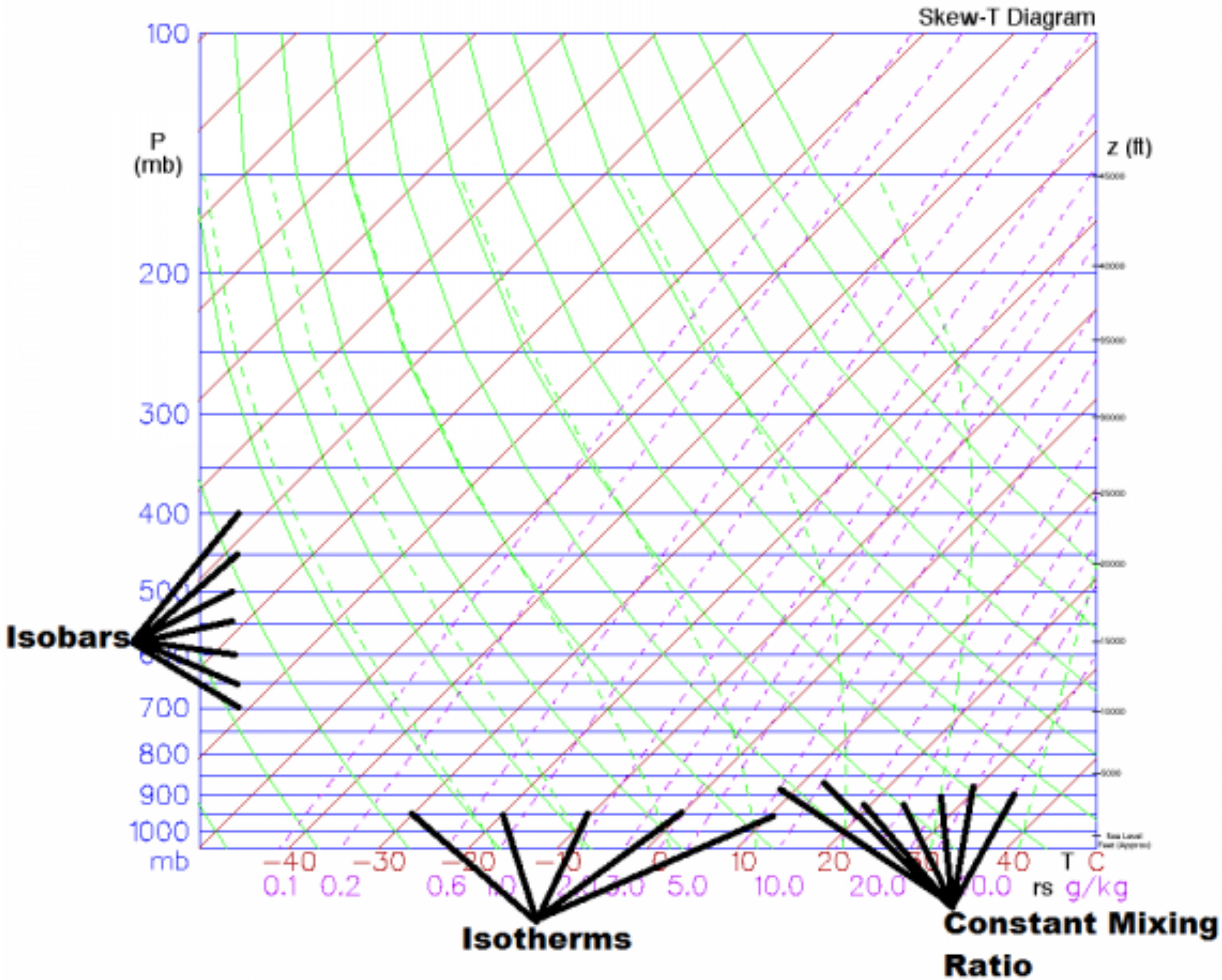
$$w_s(T,P) = \frac{\varepsilon e_s(T)}{p - e_s(T)}$$

$$w_s(T,P) \approx \frac{\varepsilon e_s(T)}{p}$$

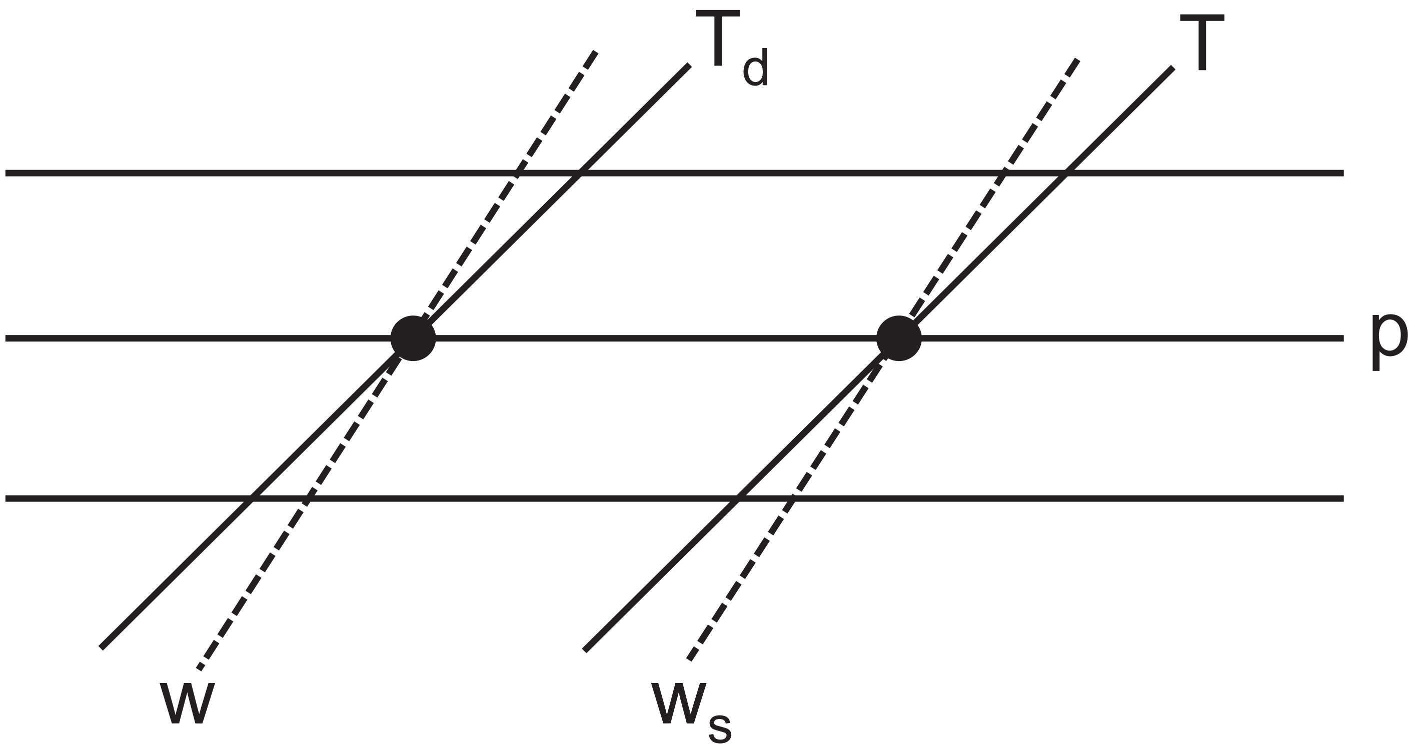
$$Where\text{: } \varepsilon = \frac{R_d}{R_v}$$

Dashed Line = Saturation Mixing Ratio




Skew-T Diagram
P
(mb)
z (ft)
100
200
300
400
500
700
800
900
1000
mb
-40
-30
-20
-10
0
10
20
40
T C
0.1
0.2
0.6
5.0
10.0
20.0
rs
g/kg
Isobars
Isotherms
Constant Mixing
Ratio

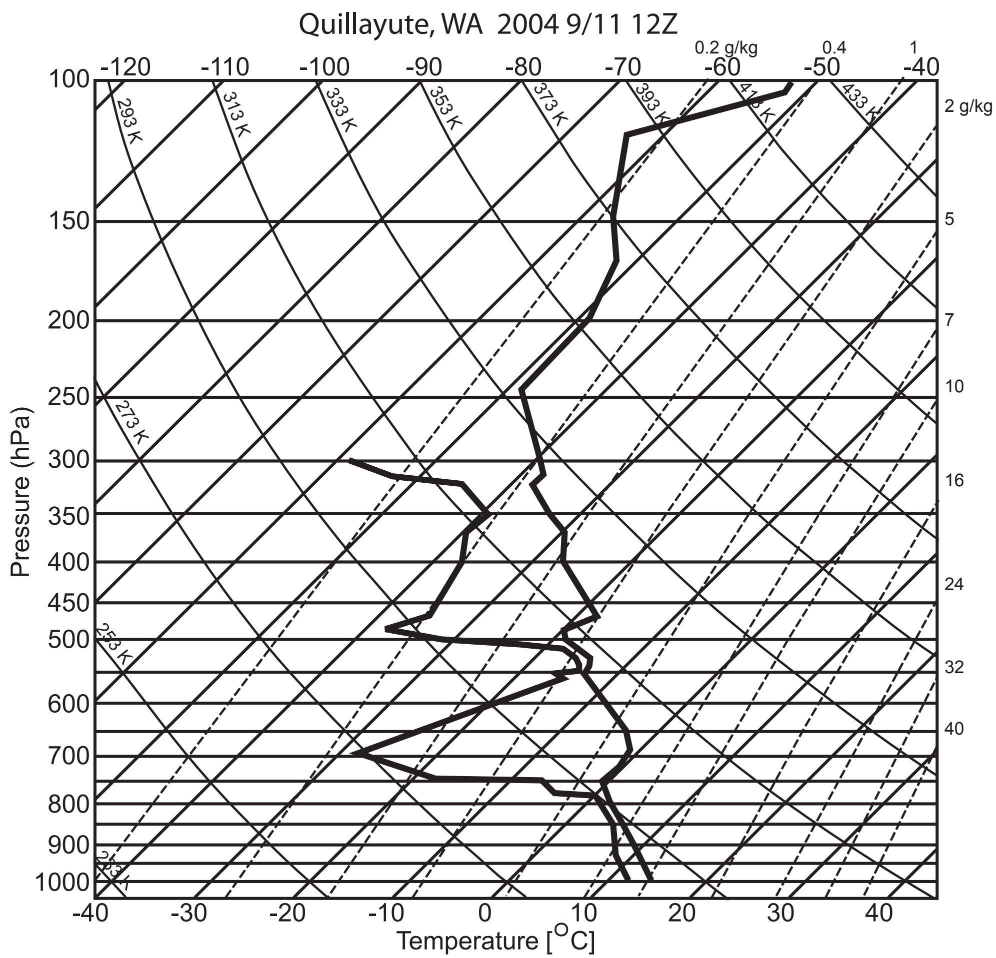
Dew Point and Mixing Ratio



Dew Point Depression and Relative Humidity

$$\Delta T_d = T - T_d$$

Where
T = air temperature
T_d = dew point temperature



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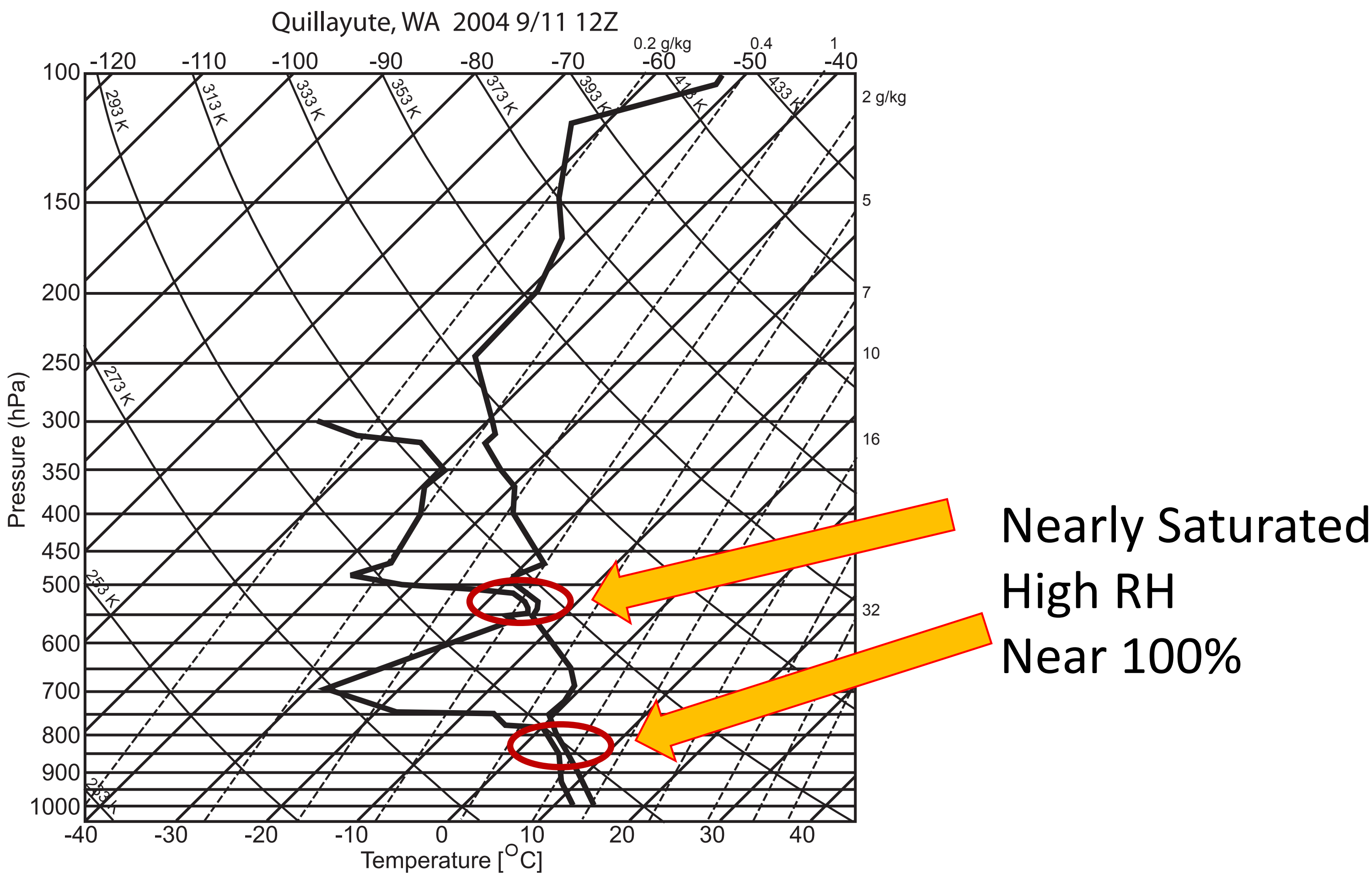

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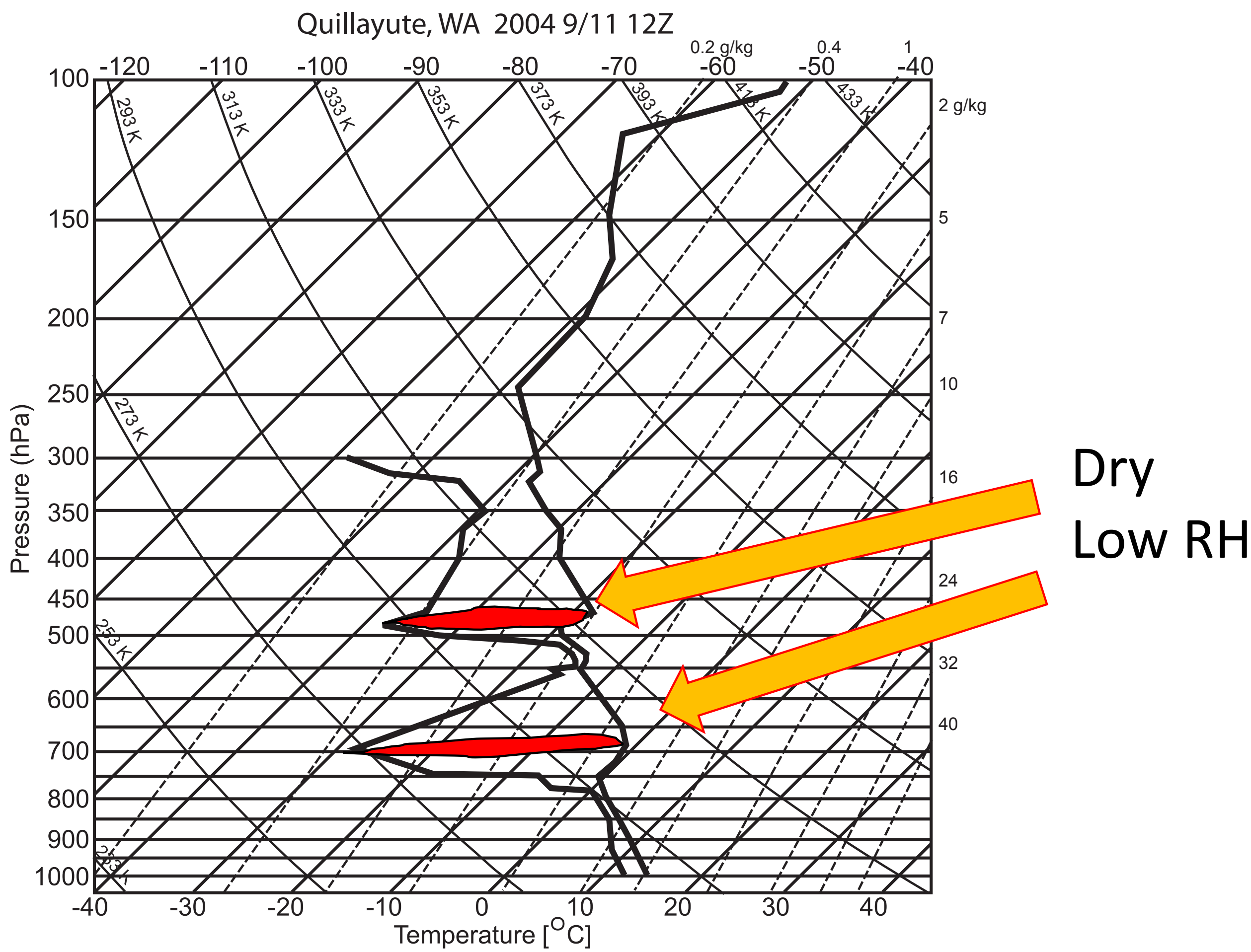
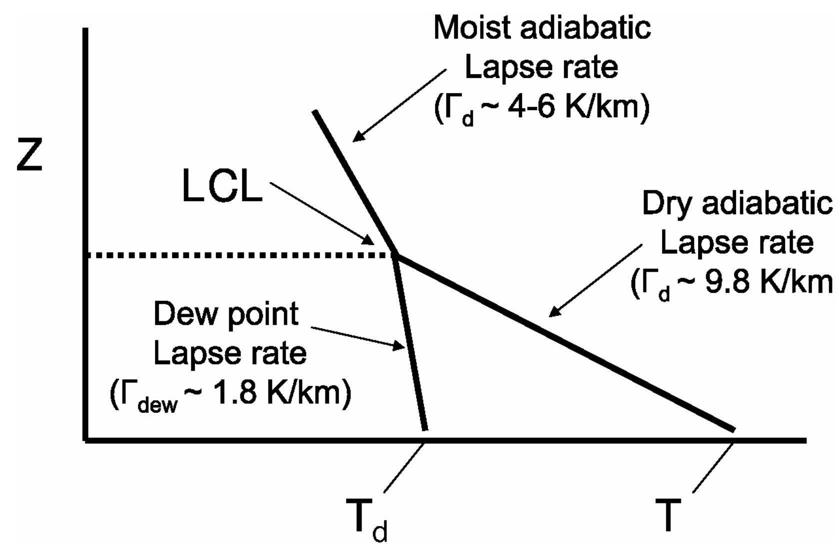


Lifting Condensation Level (LCL)

- Pressure at which saturation is achieved by a parcel during adiabatic ascent
- Level at which to expect a cloud base to form

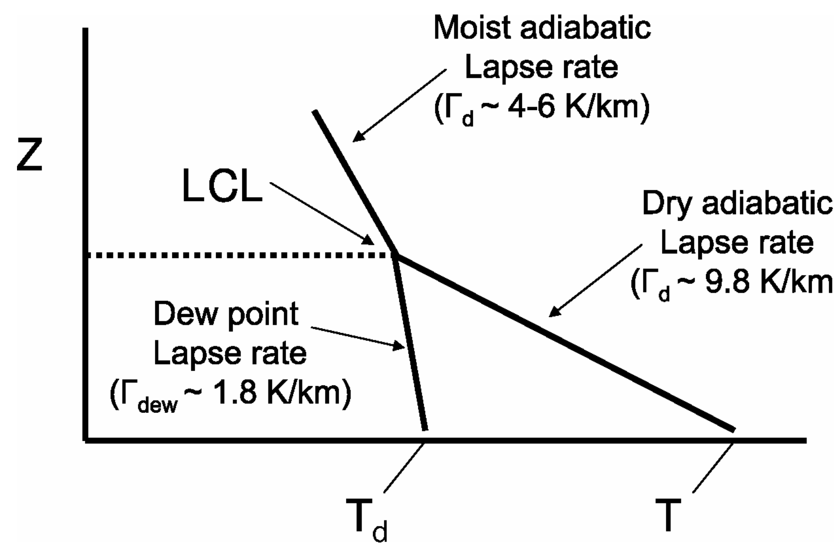
- Skew T – Intersection of the dry adiabat corresponding to the parcel temperature and the mixing ratio line corresponding to the parcel's dew point



Lifting Condensation Level (LCL) - Approximations

$$LCL \approx p \exp(-0.044 \Delta T_d)$$

Where dew point depression is given in degrees C



$$LCL(km) \approx \Delta T_d / 8$$

Where dew point depression is given in degrees K

Moist Adiabatic Lapse Rate

Also called saturation-adiabatic lapse rate or pseudoadiabatic
Recall adiabatic means no outside heat is involved in warming or cooling of air parcel

Why is the dry and moist adiabatic lapse rate different?

Water vapor in a rising parcel of air will condense when the air becomes cold enough.
The phase change from gas to liquid takes a little work from the water molecules.
As they are working, they release heat.
The heat decreases the cooling that occurs in the air parcel.
Therefore, a rising parcel of dry air cools faster than a moist parcel of air.
Conversely, a sinking parcel of dry air warms faster than a sinking parcel of moist air.

Clausius – Clapeyron equation for the atmosphere

$$\frac{de_s}{dT} = \frac{s_1 - s_2}{\alpha_2 - \alpha_1} = \frac{L}{T(\alpha_2 - \alpha_1)}$$

Assumption: $\alpha_2 >>> \alpha_1$

Specific Volume of liquid water

Specific Volume of water vapor

Review

$$\frac{de_s}{dT} = \frac{L}{T(\alpha_2)}$$

Substitute in the ideal gas law for water vapor $\frac{1}{\alpha_2} = \frac{e_s}{R_v T}$

$$\frac{de_s}{dT} = \frac{Le_s}{R_v T^2}$$

$$\frac{de_s}{e_s} = \frac{L}{R_v} * \frac{dT}{T^2}$$

$$\int_{e_{s0}}^{e_s} \frac{de_s}{e_s} = \frac{L}{R_v} \int_{T_0}^{T} \frac{dT}{T^2}$$

$$e_s(T) = e_{s0} \exp\left[\frac{L}{R_v}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right]$$

Special Cases of the First Law

Review

$$\delta q = c_v\, dT + p\, d\alpha$$
$$\delta q = c_p\, dT - \alpha\, dp$$

Special Significance in Meteorology!!

Dry Adiabatic Process: (δq = 0) negligible change of heat between the system and environment

$$c_v\, dT = -p\, d\alpha$$
$$c_p\, dT = \alpha\, dp$$

Moist Adiabatic Lapse Rate $\frac{dT}{dz} = \Gamma_s$

$$\delta q = c_p\, dT - \alpha\, dp$$

$$\delta q = -L_v\, dw_s$$

Now: $\delta q \neq 0$ as this represents the latent heat released to the air by condensation.

Heat generated by condensation of water (decreased vapor)

$$-L_v\, dw_s = c_p\, dT - \alpha\, dp$$

$$-L_v \left[\frac{\partial w_s}{\partial T} dT + \frac{\partial w_s}{\partial p} dp\right] = c_p\, dT - \alpha\, dp$$

$$\left[\alpha - L_v \frac{\partial w_s}{\partial p}\right] dp = \left[c_p + L_v \frac{\partial w_s}{\partial T}\right] dT$$

$$\frac{\left[\alpha - L_v \frac{\partial w_s}{\partial p}\right]}{\left[c_p + L_v \frac{\partial w_s}{\partial T}\right]} = \frac{dp}{dT}$$

Moist Adiabatic Lapse Rate

$$\frac{dT}{dz} = \Gamma_s$$

$$\frac{\left[\alpha - L_v \frac{\partial \boldsymbol{w_s}}{\partial \boldsymbol{p}}\right]}{\left[c_p + L_v \frac{\partial \boldsymbol{w_s}}{\partial \boldsymbol{T}}\right]} = \frac{dT}{dp}$$

RECALL

$$w_s(T,P) \approx \frac{\varepsilon e_s(T)}{p} \qquad Where:\ \varepsilon = \frac{R_d}{R_v}$$

So, $\frac{w_s}{\varepsilon} p \approx e_s$

And $\frac{de_s}{dT} = \frac{L_v e_s}{R_v T^2}$

$$\frac{\partial w_s}{\partial T} \approx \frac{\varepsilon}{p}\frac{de_s}{dT} = \frac{\varepsilon}{p}\frac{L_v e_s}{R_v T^2} = \frac{L_v w_s}{R_v T^2}$$

$$\frac{\partial w_s}{\partial p} \approx -\frac{\varepsilon}{p^2} e_s = -\frac{1}{p} w_s$$

$$\frac{\left[\alpha + L_v(\frac{1}{p} w_s)\right]}{\left[c_p + L_v \frac{L_v w_s}{R_v T^2}\right]} = \frac{\left[\alpha + (\frac{L_v}{p} w_s)\right]}{\left[c_p + \frac{L_v^2 w_s}{R_v T^2}\right]} = \frac{\left[\alpha + (\frac{L_v \alpha}{R_d T} w_s)\right]}{\left[c_p + \frac{L_v^2 w_s}{R_v T^2}\right]} = \frac{\alpha}{c_p} \frac{\left[1 + (\frac{L_v}{R_d T} w_s)\right]}{\left[1 + \frac{L_v^2 w_s}{c_p R_v T^2}\right]} = \frac{dT}{dp}$$

Moist Adiabatic Lapse Rate

$$\frac{\alpha}{c_p}\frac{\left[1+(\frac{L_v}{R_dT}w_s)\right]}{\left[1+\frac{L_v^2 w_s}{c_pR_vT^2}\right]} = \frac{dT}{dp}$$

Invoking the hydrostatic equation

$$gdz \approx -\alpha dp$$

$$-\frac{\alpha}{c_p}\frac{\left[1+(\frac{L_v}{R_dT}w_s)\right]}{\left[1+\frac{L_v^2 w_s}{c_pR_vT^2}\right]}\frac{g}{\alpha} = \frac{dT}{dp}\frac{dp}{dz}$$

$$\Gamma_s \equiv -\frac{dT}{dz} = \left[\frac{1+\frac{Lw_s}{R_dT}}{1+\frac{L^2w_s}{R_vc_pT^2}}\right]\frac{g}{c_p} = \left[\frac{1+\frac{Lw_s}{R_dT}}{1+\frac{L^2w_s}{R_vc_pT^2}}\right]\Gamma_d$$

Moist Adiabatic Lapse Rate

$$\frac{\alpha}{c_p}\frac{\left[1+\left(\frac{L_v}{R_dT}w_s\right)\right]}{\left[1+\frac{L_v^2w_s}{c_pR_vT^2}\right]}=\frac{dT}{dp}$$

$$\alpha=\frac{R_dT}{p}$$

$$\frac{d\ln T}{d\ln p}\approx\left[\frac{1+\frac{Lw_s}{R_dT}}{1+\frac{L^2w_s}{R_vc_pT^2}}\right]\frac{R_d}{c_p}$$

If w_s is approximately zero (as is the case for very cold parcels)

Then this reduces to the differential form of Poisson’s equation.

SO NO distinction between dry and moist adiabatic ascent.

Potential Temperature
(<u>Poisson's Equation</u>)

$$c_p \, dT = \alpha \, dp = \frac{R_d T}{p} dp$$

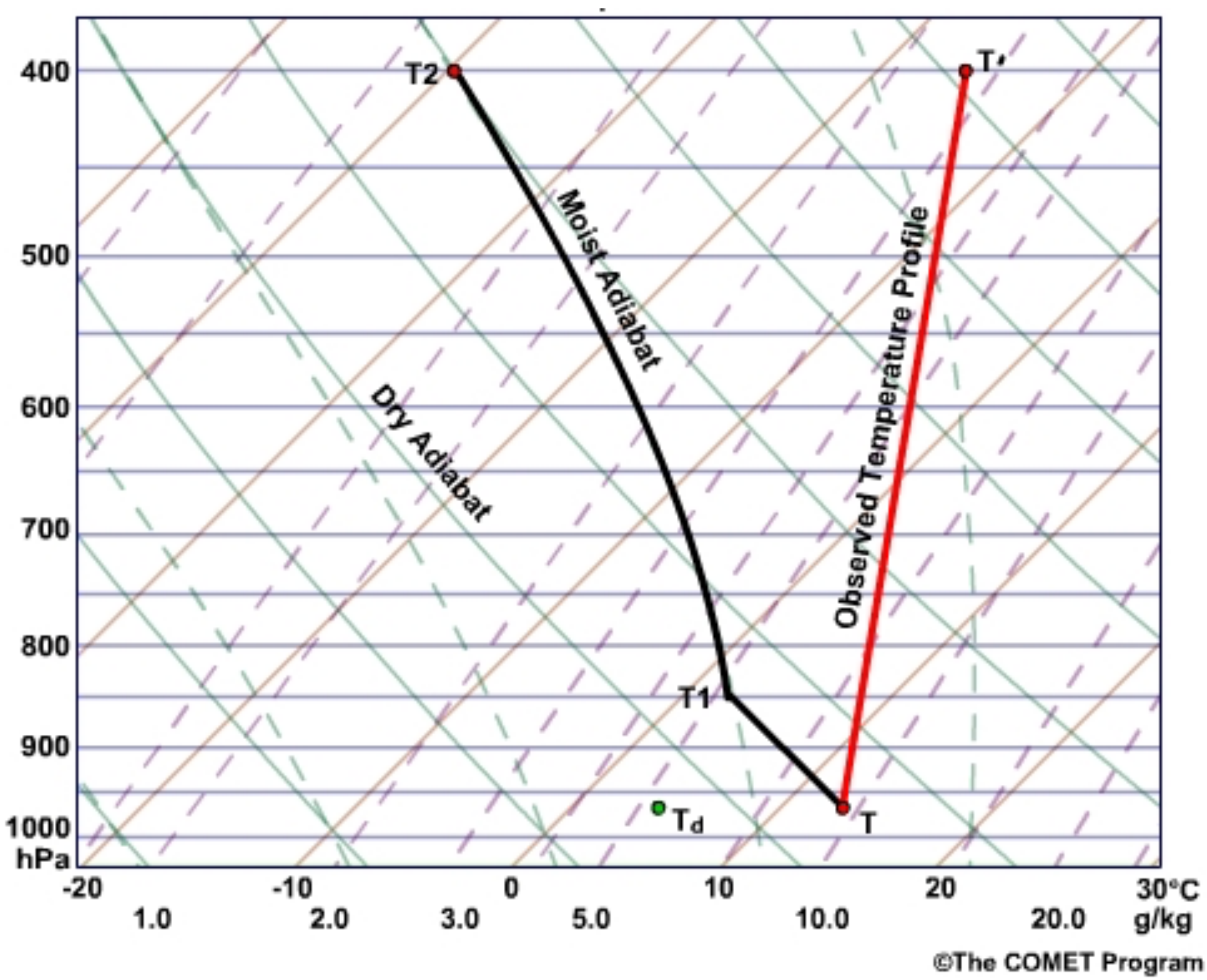
$$\boxed{\frac{1}{T} dT = \frac{R_d}{c_p} \frac{1}{p} dp}$$

Review

$$\int_{T_0}^{T} \frac{1}{T} dT = \frac{R_d}{c_p} \int_{p_0}^{p} \frac{1}{p} dp$$

$$\boxed{\frac{T}{T_0} = \left(\frac{p}{p_0}\right)^{\kappa}}$$

$$\kappa = \frac{R_d}{c_p} \approx 0.286$$



400
500
600
700
800
900
1000
hPa
T2
T'
Moist Adiabat
Dry Adiabat
Observed Temperature Profile
T1
T
Td
-20
-10
0
10
20
30°C
1.0
2.0
3.0
5.0
10.0
20.0
g/kg
©The COMET Program

In Class problem:

Problem 7.10:

Using the skew-T diagram at the back of the book determine the saturation vapor pressure at a temperature of – 20 C

In Class problem:

Problem 7.10:

Using the skew-T diagram determine the saturation vapor pressure at a temperature of – 20 C

Assumed sea level (1013 mb).

~ 0.78 g/kg