THE UNIVERSITY OF UTAH

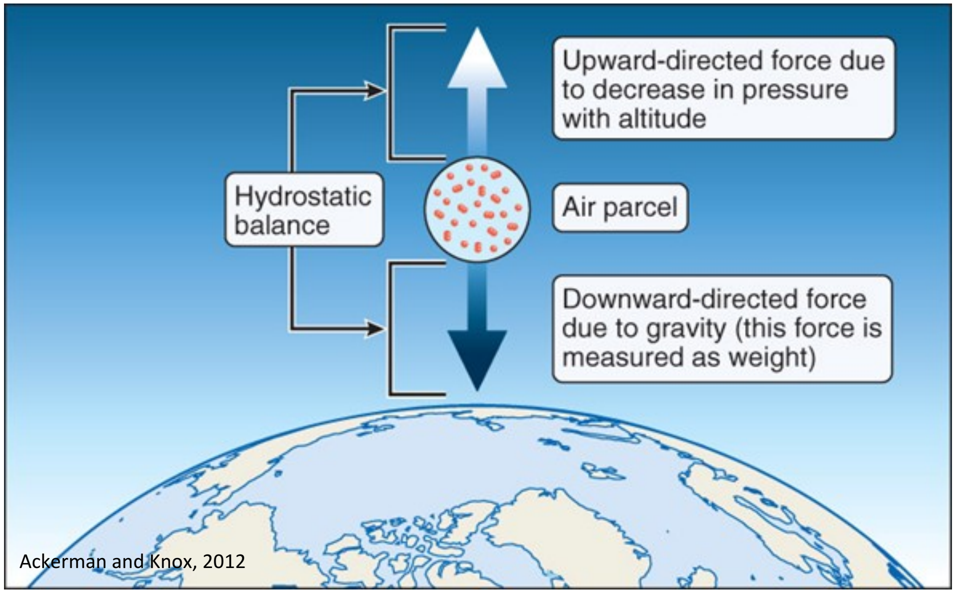
ATMOS 5130

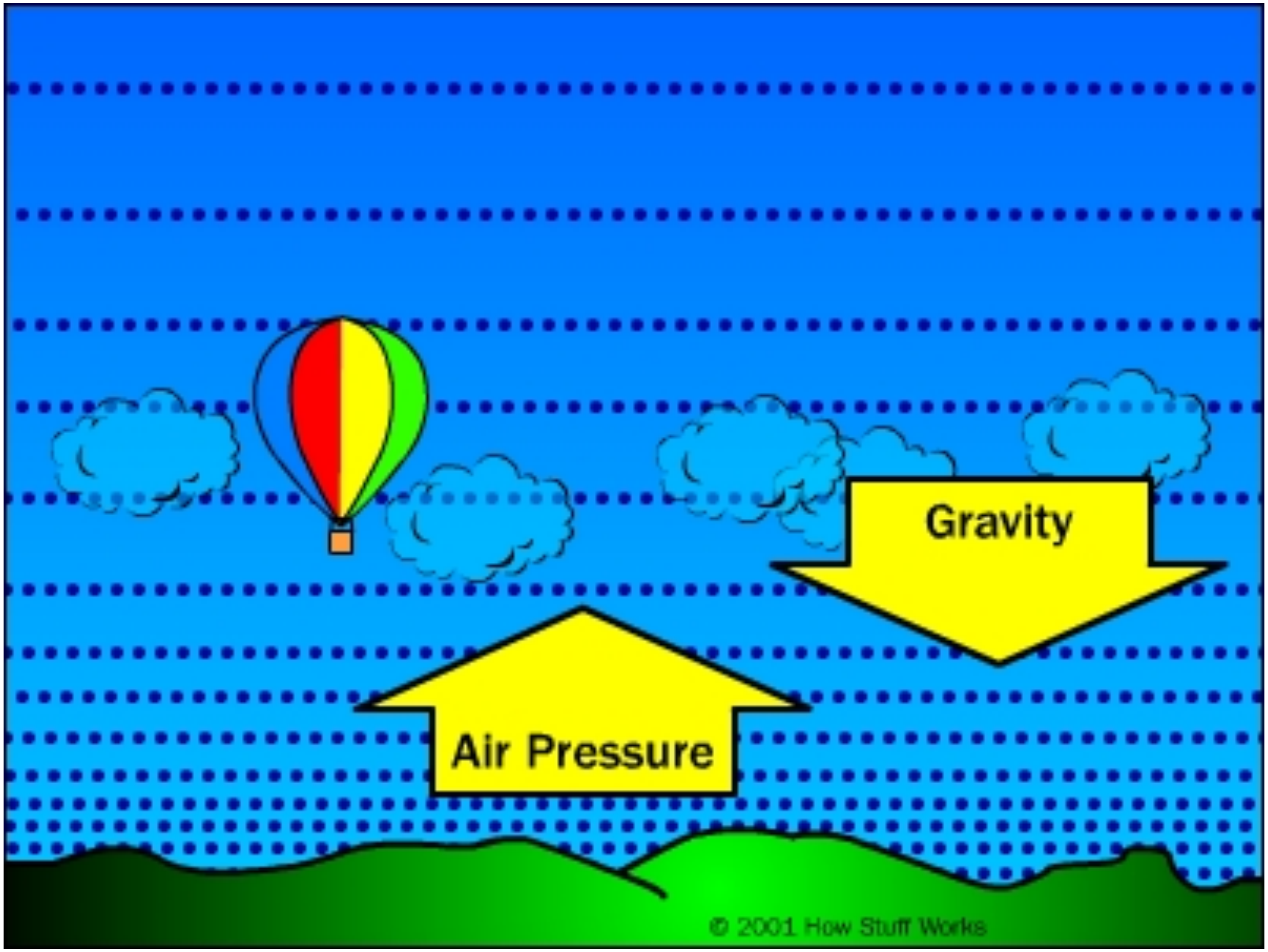
Lecture 5

- Atmospheric Pressure
  - Hydrostatic Balance
  - Geopotential Height
  - Hypsometric Equation

Introduction to Hydrostatic Balance

- Pressure at any point in the atmosphere equals the weight per unit area above that point.
- Pressure = Force per unit area
- Weight of an object = Force = F_g = mg
- g = acceleration due to gravity (9.81 m s^{-2}) at sea level




Gravity
Air Pressure
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Hydrostatic Equation

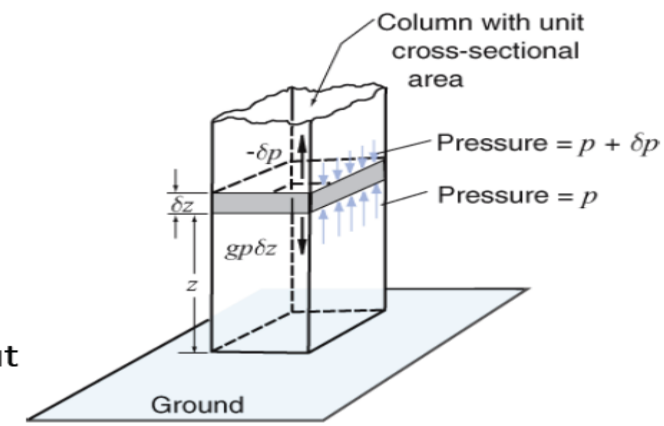
- Air pressures at any given height in the atmosphere is due to gravitational force of the air mass above that height,

$$p(z) = \int_z^\infty \rho(z) g dz$$

- Thus, change of surface pressure at a height is due to change of air mass above that height.
  - $dp(z)=-\rho g dz$ or $dp/dz=-\rho g$

The hydrostatic equation

- It is a good approximation for large-scale atmospheric and climate dynamic processes, but not a good approximation for fast mesoscale convective processes, such as supercell, tornadoes. Why?



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Hydrostatic Approximation for the Atmosphere

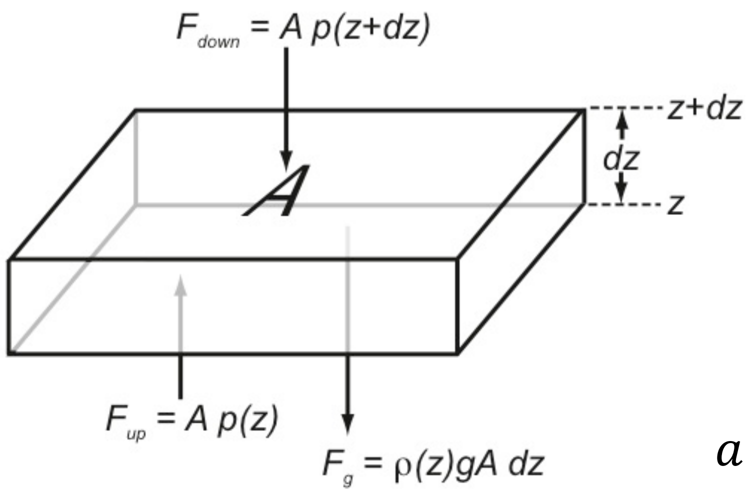

Fig. 4.1

$$\mathrm{F} = F_{up} - F_{down} - F_g$$

$$F = Ap(z) - Ap(z+dz) - \rho(Adz)g$$

<u>Divide by the Mass to get acceleration of atmosphere</u>

$$a = \frac{F}{M} = \frac{F}{\rho V} = \frac{F}{\rho A dz} = \frac{Ap(z) - Ap(z+dz) - \rho(Adz)g}{\rho A dz}$$

$$a = \frac{\cancel{A}[p(z) - p(z+dz) - \rho(dz)g]}{\rho \cancel{A} dz} = -{}^{1}/_{\rho}\left[\frac{p(z+dz)-p(z)}{dz}\right] - g$$

$$a + g = -{}^{1}/_{\rho}\frac{p(z+dz)-p(z)}{dz}$$

$$\boxed{-(a+\mathrm{g}) = {}^{1}/_{\rho}\frac{\partial p}{\partial z}}$$

Hydrostatic Approximation for the Atmosphere

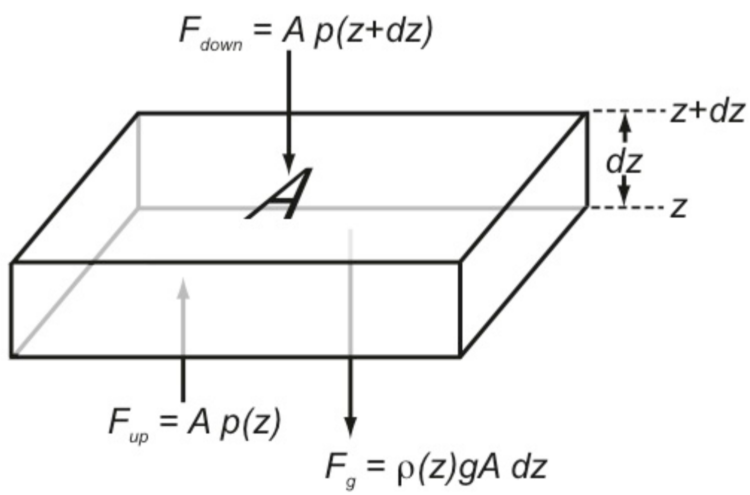

Fig. 4.1

$$\frac{\partial p}{\partial z} = -\rho(a+g)$$

In all but the most extreme meteorological conditions the vertical acceleration of of air parcels is much smaller than gravity.

$$\frac{\partial p}{\partial z} \approx -\rho g$$

Application to the Atmosphere

$$\frac{\partial p}{\partial z} = -\rho g = -\frac{pg}{R_d T_v}$$

$$\frac{1}{p}\frac{\partial p}{\partial z} = -\frac{g}{R_d T_v}$$

$$\boxed{\frac{\partial \ln p}{\partial z} = -\frac{g}{R_d T_v}}$$

The rate of change of the logarithm of pressure with height is inversely proportional to the absolute temperature and does not depend on pressure.

Pressure generally falls off exponentially with height!

Geopotential Height

- ✓ Earth's gravity has Altitude and Latitude dependences.
- ✓ Overall these are minor (few tenths of a percent under conditions of interest to meteorologist).
- ✓ Geopotential Height allows us a new reference (or coordinate) to accommodate these changes in gravity.
- ✓ Geopotential height is a vertical coordinate referenced to Earth's mean sea level — an adjustment to geometric height (elevation above mean sea level) using the variation of gravity with latitude and elevation.
- ✓ Thus it can be considered a "gravity-adjusted height".

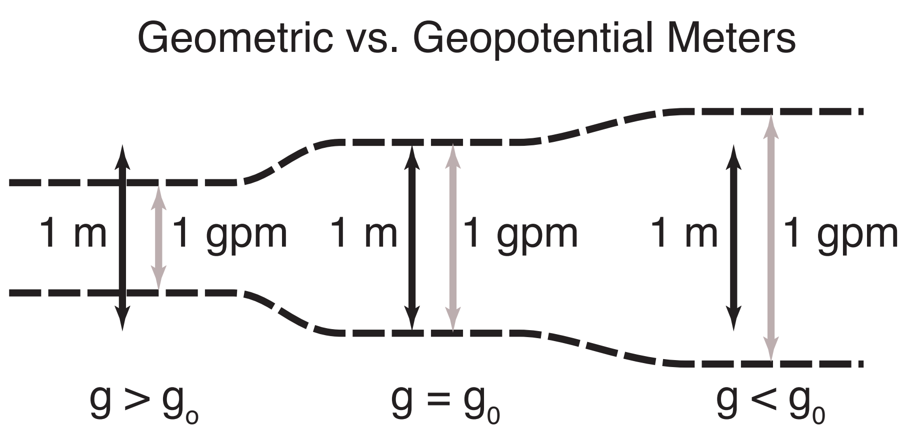

Fig. 4.2

Geopotential Height

At an elevation of z, the geopotential is defined

$$\phi(z) = \int_0^z g(\phi, h)\, dz$$

Where $g(\phi, h)$ is the acceleration due to gravity, ϕ is latitude and h is the geometric elevation (variation in latitude and elevation generally do not matter for meteorological applications).
Geopotential => Potential energy per unit mass

$$Z = \frac{\phi(z)}{g_0}$$

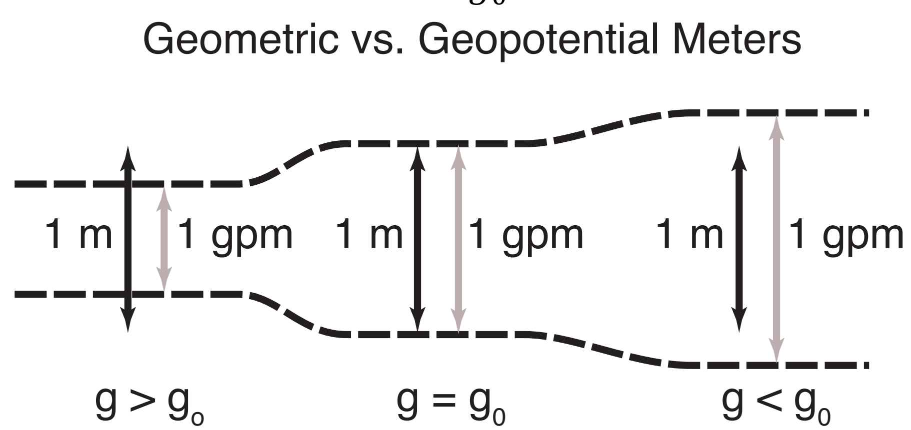

Fig. 4.2

Key Point

- When meteorologist reference height – it should be understood that they really mean geopotential height.

- This allows for the Hypsometric Equation

Hypsometric Equation

Given two pressure levels what is the thickness of the atmosphere (Δz)

$$\int_{z_1}^{z_2} dz = \frac{R_d}{g}\int_{p_1}^{p_2} T_v \, d\ln p$$

$$\Delta z = \frac{R_d \overline{T_v}}{g}\int_{p_1}^{p_2} d\ln p = \frac{R_d \overline{T_v}}{g} \ln\left[\frac{p_1}{p_2}\right]$$

The layer thickness (Δz) is directly proportional to mean virtual temperature ($\overline{T_v}$)

* Most of the time, T is used… very close if not too humid.

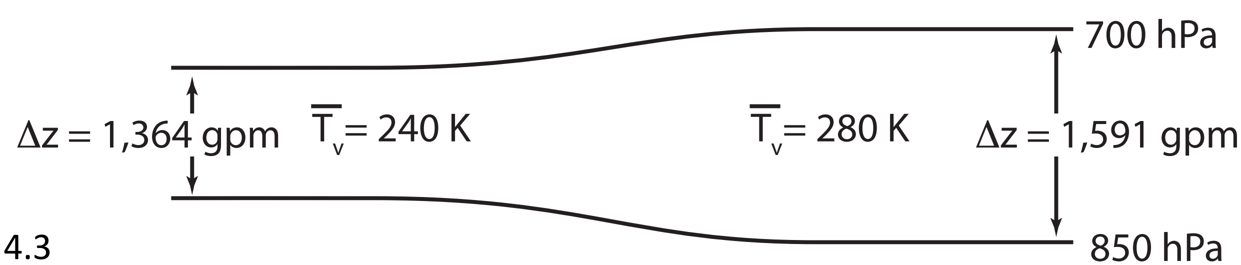

Fig. 4.3

In class problem

What is the 1000 to 500 millibar thickness of dry air that has an average temperature of 5° C?

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HINT : Since the air is dry Tv = T = 278 K

Z = (278 K)(1/9.8 ms^-2)(287 JK^-1kg^-1) LN (1000/500) = 5,643 geopotential meters

Units check: Kkgms^-2mK^-1kg^-1m^-1s^2 = m