THE UNIVERSITY OF UTAH

ATMOS 5130

Lecture 6

- Atmospheric Pressure
  - Pressure Profiles for Idealized Atmosphere

Goal

- Understand how temperature, pressure and altitude are related in the atmosphere.

Recall from last lecture

Hypsometric Equation

$$\Delta z = \frac{R_d \overline{T_v}}{g} \ln\left[\frac{p_1}{p_2}\right]$$

Allow for calculation of pressure at any height

Pressure Profiles for Idealized Atmosphere

- <u>Constant Density Atmosphere</u>
  - Unrealistic
  - Better representation of the ocean
  - Atmosphere had the same mass as now, but had a constant density it would be only 8.3 km deep

$$\frac{\partial p}{\partial z} \approx -\rho g$$ Hydrostatic Equation

$$\int_{p_0}^{p(z)} dp = -\rho g \int_0^z dz$$

$$\boxed{p(z) = p_0 - \rho g z}$$

Pressure decreases linearly with height

Pressure Profiles for Idealized Atmosphere

- <u>Constant Density Atmosphere</u>
  - Pressure is decreasing with height
  - Density is constant
  - SO, Temperature must decease with height

$$p(z) = p_0 - \rho g z$$

$$p = \rho R_d T$$ IDEAL GAS LAW

$$\frac{\partial p}{\partial z} = \rho R_d \frac{\partial T}{\partial z}$$ Differentiate with respect to elevation

Substitute in **Hydrostatic Equation** → $\frac{\partial p}{\partial z} = -\rho g$

$$\boxed{\frac{\partial T}{\partial z} = -\frac{g}{R_d} = \text{-34.1 C/km}}$$

Constant Density Atmosphere

$$p(z) = p_0 - \rho g z$$

Autoconvective Lapse Rate

$$\boxed{\frac{\partial T}{\partial z} = -\frac{g}{R_d} = \text{-34.1 C/km} = \Gamma_{auto}}$$

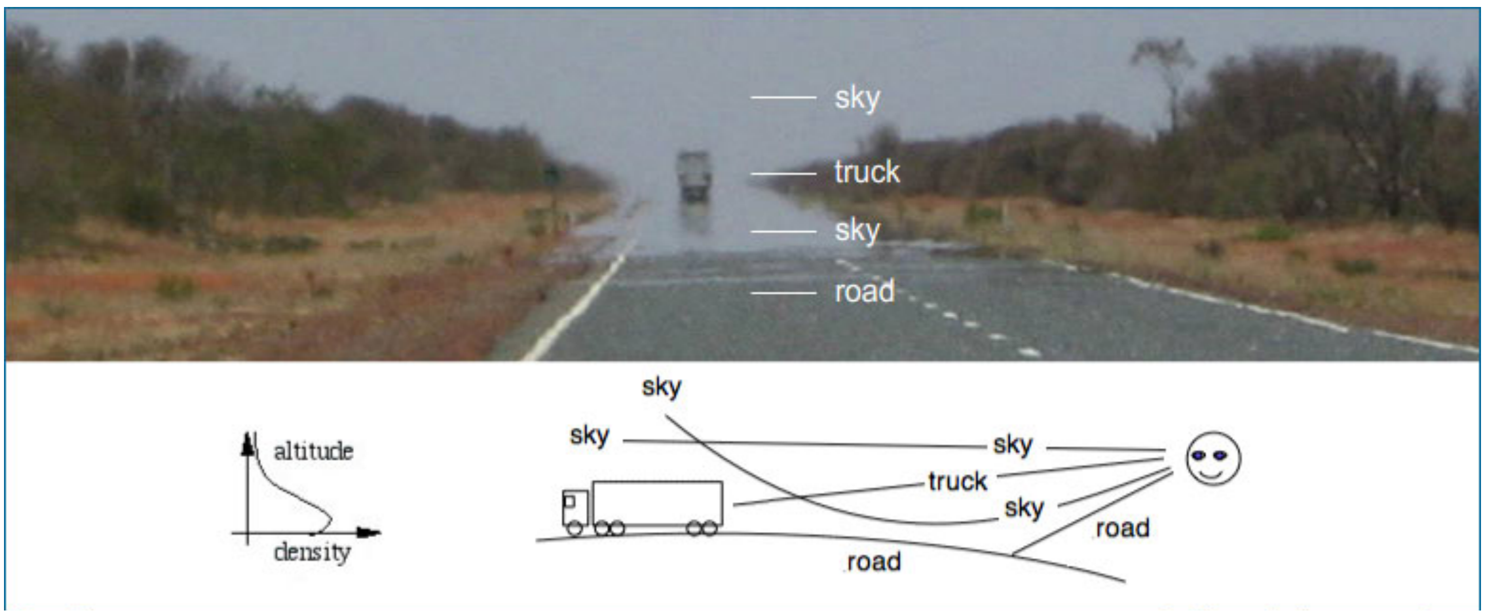
$When$: $\Gamma > \Gamma_{auto}$

Density above greater than below! Density Inversion => Creates a mirage

Above a very hot road, a layer of warm air whose density is lower than air above.

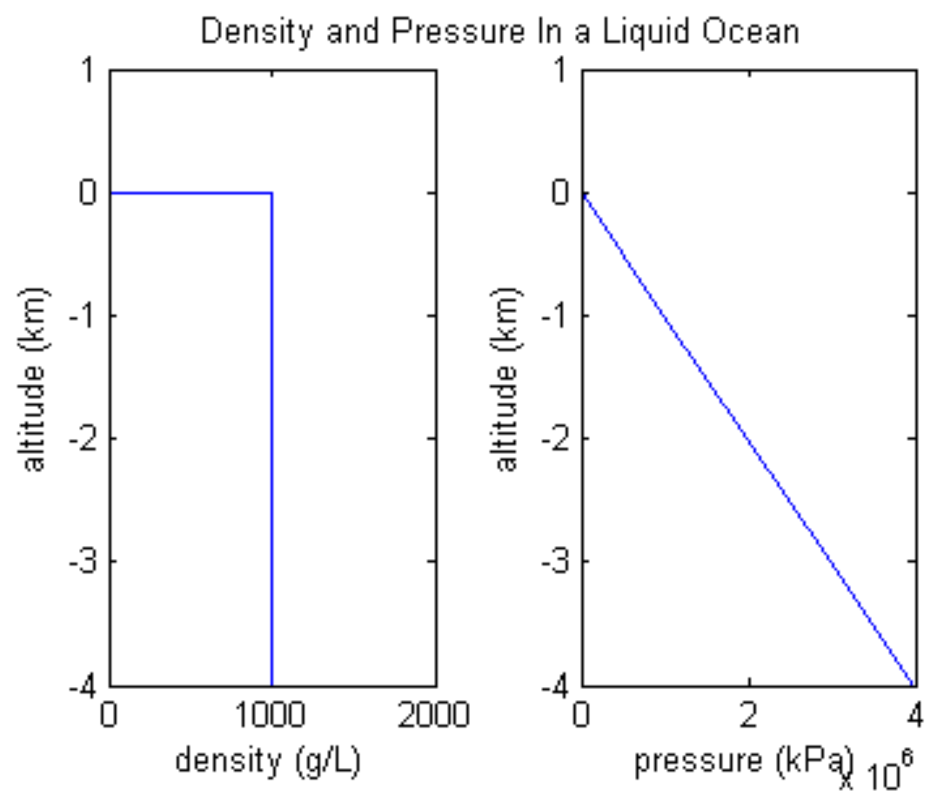
The denser air slows light very slightly more than the less dense layer.

So, one sees sky where the road should be, we often interpret this as a reflection caused by water on the road



Pressure Profiles for Idealized Atmosphere

Constant Density



Pressure Profiles for Idealized Atmosphere

- **Isothermal Atmosphere**

$$\frac{dp}{dz} = -\rho g$$ Hydrostatic Equation

Again substitute in the ideal gas law,

$$\frac{dp}{dz} = -\rho g = -\frac{pg}{R_d T}$$

Solve

$$\frac{1}{p}\,dp = -\frac{g}{R_d T}\,dz$$

$$\int_{p_0}^{p} \frac{1}{p}\,dp = -\frac{g}{R_d T}\int_0^z dz$$

$$\ln\left(\frac{p}{p_0}\right) = -\frac{gz}{R_d T}$$

$$\boxed{p(z) = p_0 \exp\left(-\frac{z}{H}\right)}$$

Where:

$$\boxed{H = \frac{R_d T_0}{g}}$$

Scale Height

Constant Density Atmosphere

$$p(z) = p_0 - \rho g z$$

Scale Height (H): solve for z, when p(z) = 0,

$$H = \frac{p_0}{\rho g}$$

substitute the Ideal Gas Law

$$H = \frac{R_d T_0}{g}$$

In other words: scale height is the increase in altitude for which the atmospheric pressure decreases by a factor of e^{-1} or about 37% of its original value.

Approximate scale heights for selected Solar System bodies

Venus: 15.9 km
Earth: 8.3 km
Mars: 11.1 km
Jupiter: 27 km
Saturn: 59.5 km
Titan: 40 km
Uranus: 27.7 km
Neptune: 19.1–20.3 km
Pluto: ~60 km

Elevation	Density
0	ρ_0
H	$(1/e)\rho_0 = 0.368\rho_0$
2H	$(1/e^2)\rho_0 = 0.135\rho_0$
3H	$(1/e^3)\rho_0 = 0.050\rho_0$
4H	$(1/e^4)\rho_0 = 0.018\rho_0$

Pressure Profiles for Idealized Atmosphere

- Constant Lapse Rate atmosphere

$$T = T_0 - \Gamma z$$

  - Normally Γ >0, temperature decreases with altitude
  - When Γ >0, temperature increases with altitude
    - INVERSION

Pressure Profiles for Idealized Atmosphere

- Constant Lapse Rate atmosphere

$$T = T_0 - \Gamma z$$

Take the hydrostatic equation and combine with Ideal Gas Law

$$\frac{dp}{dz} = -\rho g = -\frac{pg}{R_d(T_0 - \Gamma z)}$$

$$\frac{1}{p}dp = -\frac{g}{R_d}\left(\frac{dz}{T_0 - \Gamma z}\right)$$

$$\int_{p_0}^{p}\frac{1}{p}dp = -\frac{g}{R_d}\int_0^z \frac{dz}{T_0 - \Gamma z}$$

$$\ln\left(\frac{p}{p_0}\right) = \frac{g}{R_d\Gamma}\ln\left(-\frac{T_0 - \Gamma z}{T_0}\right)$$

$$\boxed{p(z) = p_0\left(\frac{T_0 - \Gamma z}{T_0}\right)^{\frac{g}{R_d\Gamma}}}$$

Pressure Profiles for Idealized Atmosphere

- Constant Lapse Rate atmosphere

$$T = T_0 - \Gamma z$$

Take the hydrostatic equation and combine with Ideal Gas Law

$$\frac{dp}{dz} = -\rho g = -\frac{pg}{R_d(T_0 - \Gamma z)}$$

$$\frac{1}{p}dp = -\frac{g}{R_d}\left(\frac{dz}{T_0 - \Gamma z}\right)$$

$$\int_{p_0}^{p}\frac{1}{p}dp = -\frac{g}{R_d}\int_0^z \frac{dz}{T_0 - \Gamma z}$$

$$\ln\left(\frac{p}{p_0}\right) = \frac{g}{R_d\Gamma}\ln\left(-\frac{T_0 - \Gamma z}{T_0}\right)$$

$T(z)$

$$\boxed{p(z) = p_0\left(\frac{T_0 - \Gamma z}{T_0}\right)^{\frac{g}{R_d\Gamma}}}$$

Pressure Profiles for Idealized Atmosphere

- Constant Lapse rate atmosphere

$$p(z) = p_0 \left(\frac{T(z)}{T_0}\right)^{\frac{g}{R_d \Gamma}}$$

Valid for $\Gamma < 0$
Exponent of the ratio is negative, so pressure still decreases with height

$\frac{g}{R_d \Gamma}$ is ratio of autoconvective to actual lapse rate

So, what happens if $\boldsymbol{\Gamma = \Gamma_{auto}}$?

Pressure Profiles for Idealized Atmosphere

- Constant Lapse rate atmosphere

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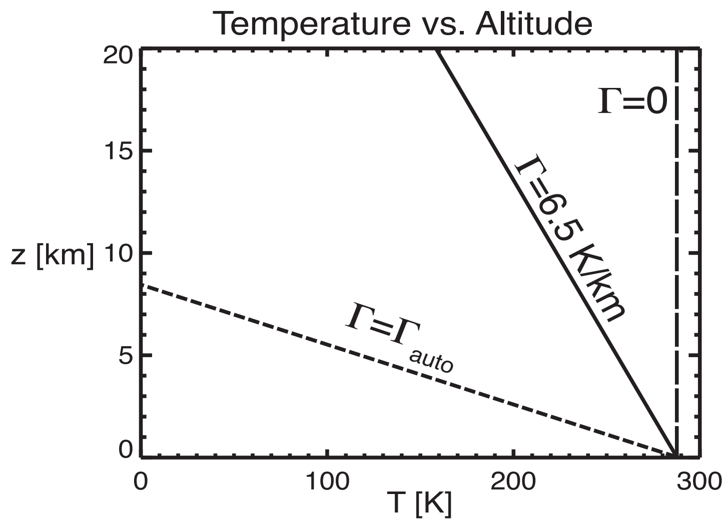
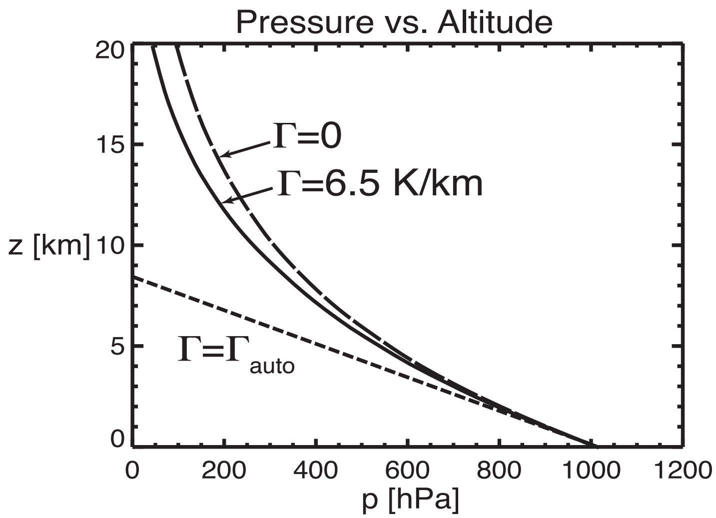
So, what happens if $\boldsymbol{\Gamma = \Gamma_{auto}}$?

$p(z) = p_0 \left(\frac{T(z)}{T_0}\right)$ - Pressure is linear function of z



Γ = 0 ; Isothermal
Γ = 6.5 K/km; Constant Lapse Rate
$\Gamma = \Gamma_{auto}$; Autoconvective; Constant Density = g/R_d

Fig. 4.4



Γ = 0 ; Isothermal
Γ = 6.5 K/km; Constant Lapse Rate
$\Gamma = \Gamma_{auto}$; Autoconvective Constant Density

Isothermal

$$p(z) = p_0 \exp\left(-\frac{z}{H}\right)$$

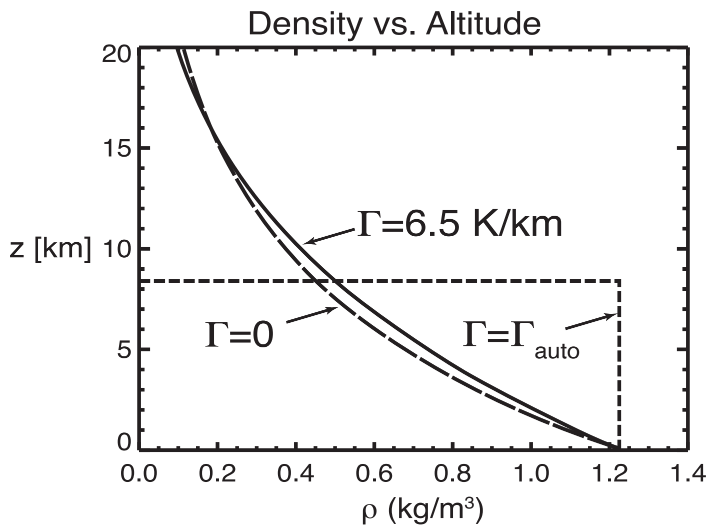
Constant Lapse Rate

$$p(z) = p_0 \left(\frac{T(z)}{T_0}\right)^{\frac{g}{R_d \Gamma}}$$

Autoconvective

$$p(z) = p_0 - \rho g z$$

Fig. 4.4



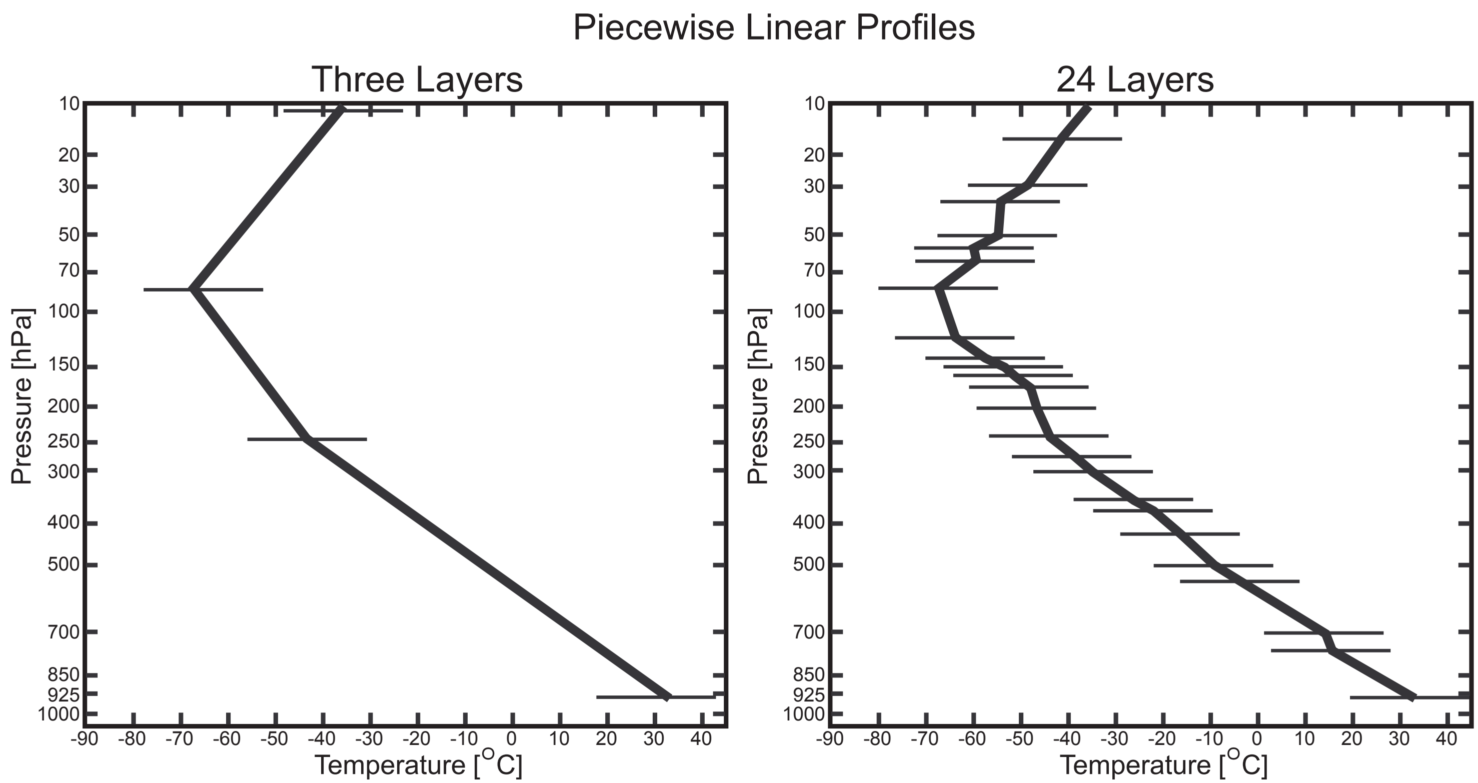
Γ = 0 ; Isothermal
Γ = 6.5 K/km; Constant Lapse Rate
$\Gamma = \Gamma_{auto}$; Autoconvective Constant Density

Fig. 4.4

Piecewise Linear Temperature Profile

- Ultimate Approximation is model based on series of stacked layers
- Each layer exhibits a different constant lapse rate
- Generally speaking, if a layer is thin enough, then all the above methods give you about the same result.

$$\boxed{p_{i+1} = p_i \left(\frac{T_{i+1}}{T_i}\right)^{\frac{g}{R_d \Gamma_i}}}$$



Piecewise Linear Profiles
Three Layers
24 Layers
Pressure [hPa]
10
20
30
50
70
100
150
200
250
300
400
500
700
850
925
1000
-90 -80 -70 -60 -50 -40 -30 -20 -10 0 10 20 30 40
Temperature [°C]

Fig. 4.5