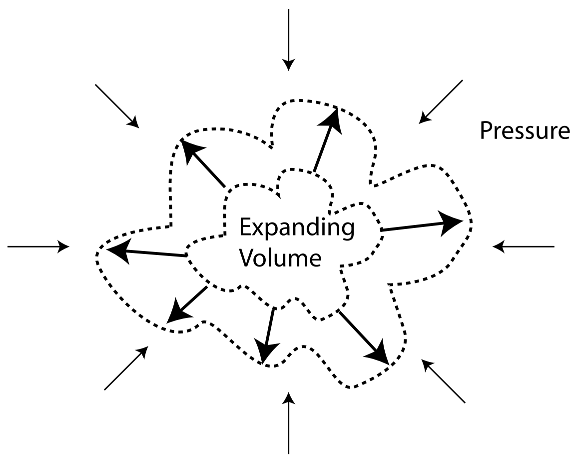
THE UNIVERSITY OF UTAH

ATMOS 5130

Lecture 7

- The First Law and Its Consequences
  - Pressure-Volume Work
  - Internal Energy
  - Heat Capacity
  - Special Cases of the First Law

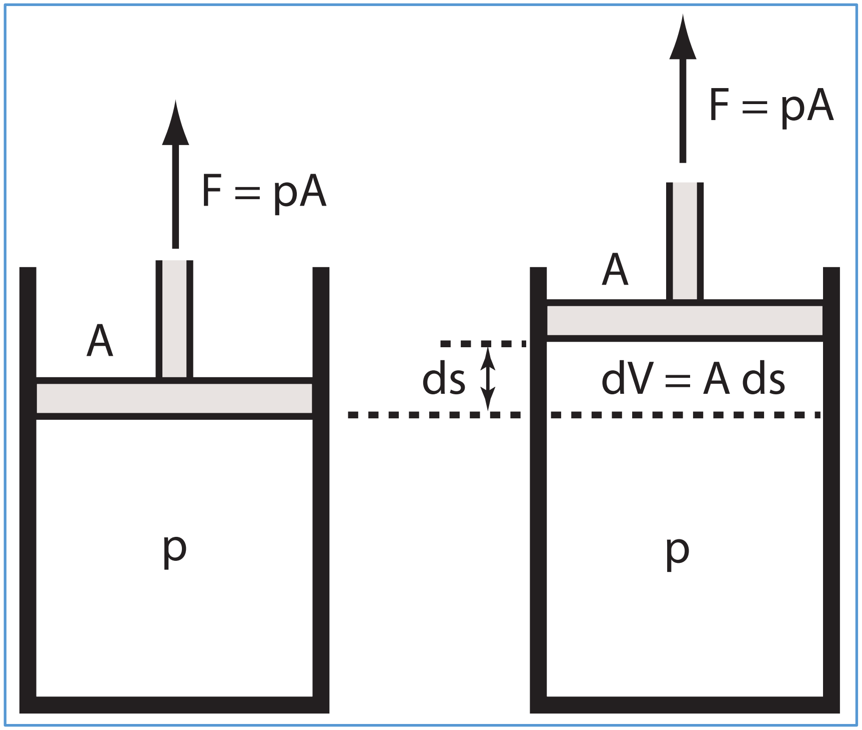
Pressure-Volume Work



$$\delta W = \overrightarrow{f} \cdot d\vec{x}$$

$$\delta W = F\, ds$$

Pressure-Volume Work



$$\delta W = F\,ds$$

$$\delta W = pA\,ds$$

$$\delta W = p\,dV$$

Pressure-Volume Work

$$\delta W = p\, dV$$

Intensive Units – work or energy per unit mass

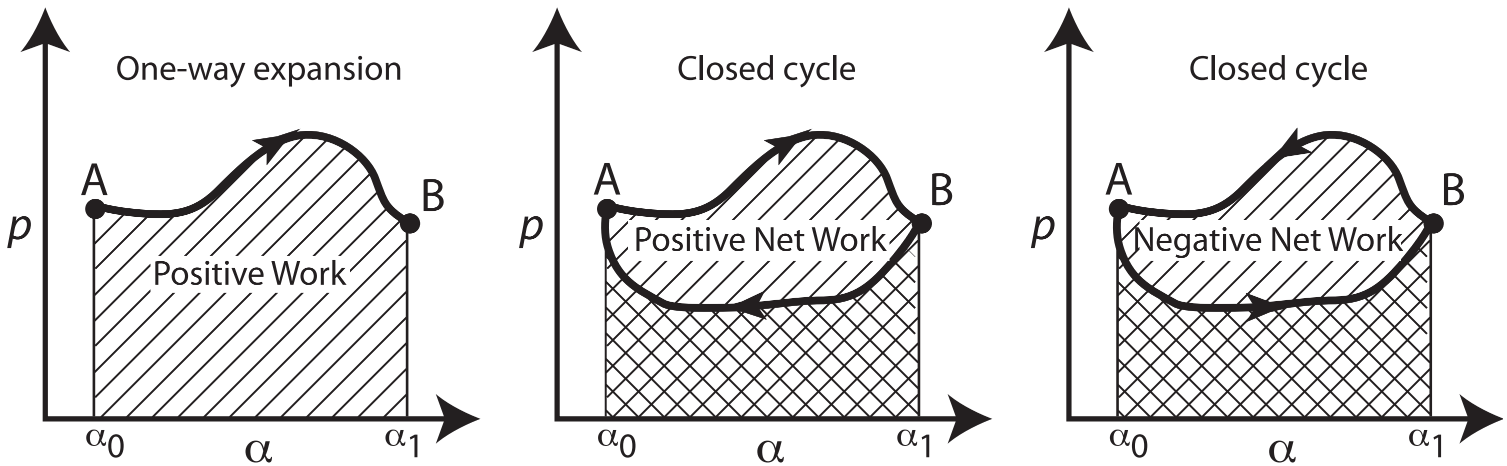
$$w \equiv \frac{W}{M}$$

$$\delta w \equiv p \frac{dV}{M}$$

$$\boxed{\delta w \equiv p\, d\alpha}$$

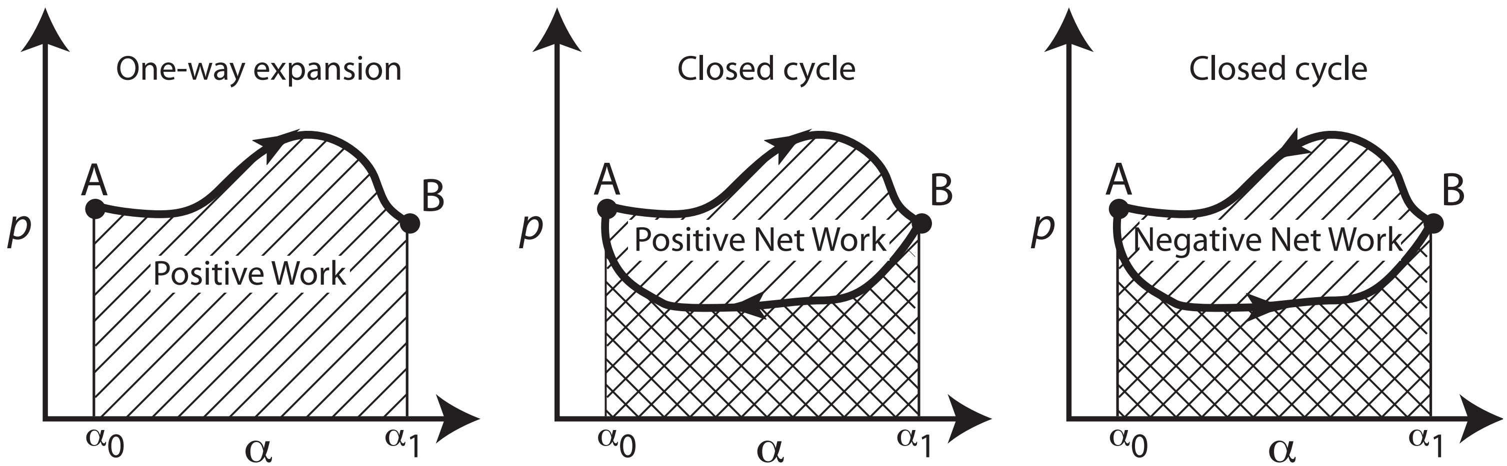
Where α is the specific volume

$$\boxed{w = \int \delta w \equiv \int_{\alpha_0}^{\alpha_1} p\, d\alpha}$$



The Total work done by (or on) a parcel during a transition from one state to the next depends on how it gets there!

$$w = \oint p d\alpha \equiv \int_{\alpha_0}^{\alpha_1} p_1(\alpha) d\alpha + \int_{\alpha_1}^{\alpha_0} p_2(\alpha) d\alpha$$



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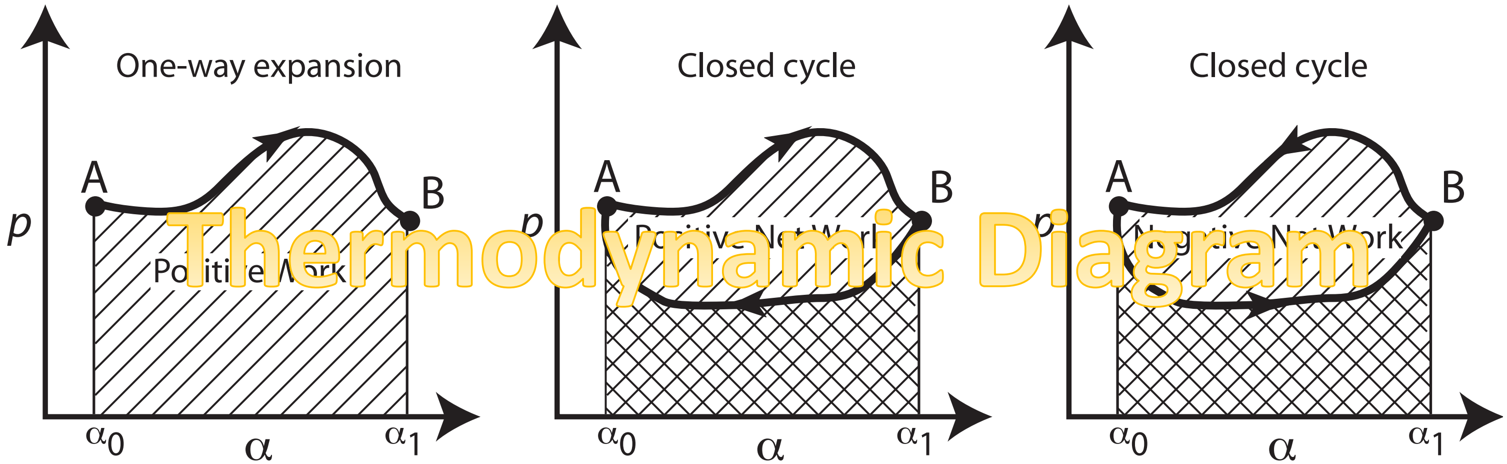
$w = \oint p d\alpha$ = CLOSED CYCLE

Thermodynamic Diagram



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Thermodynamic Diagram

- ANY diagram where the total work involved in a process is proportional to the area enclosed by the curve representing that process
- Skew-T diagram

First Law of Thermodynamics

- Special Case of the Law of Conservation of Energy
- Energy stored by the parcel is its **internal energ**y

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$$\delta w = -du + \delta q$$

Where δw = increment of **work** (per unit mass

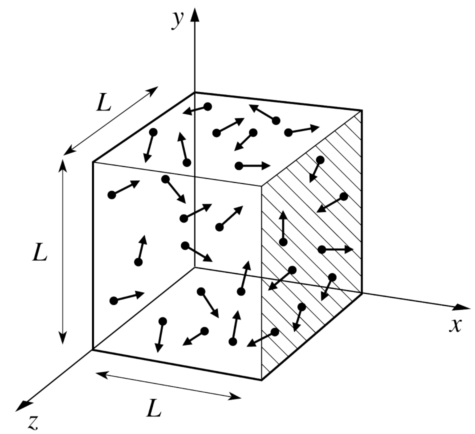
du = change in the **internal energy** (per unit mass)

δq = increment of **heat energy** (per unit mass)

Recall: $\delta w \equiv p\, d\alpha$

$$\delta q = -du + p\, d\alpha$$

Joule's Law



Recall from Lecture 3

$$\delta q = -du + p\,d\alpha$$

- Ideal gas (*kinetic theory of gases*) – molecules do not exert attractive or repulsive forces

- From equation above the internal energy (u) is proportional to the temperature of the air parcel !

Joule’s Law

Recall from Lecture 3

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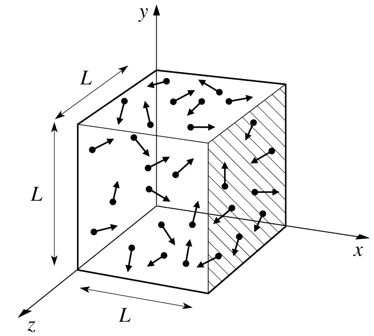

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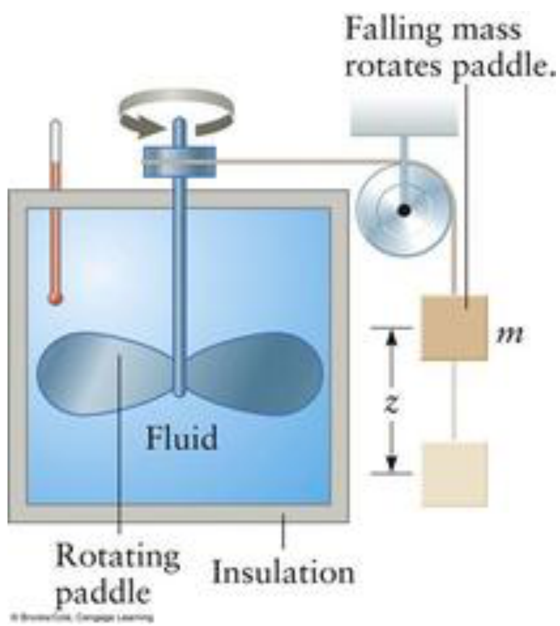


Heat Capacity

Add heat (δq)
Then temperature will increase (from T to T+dT)

$$\frac{\delta q}{dT} = \textit{Heat capacity of the material (or specific heat)}$$

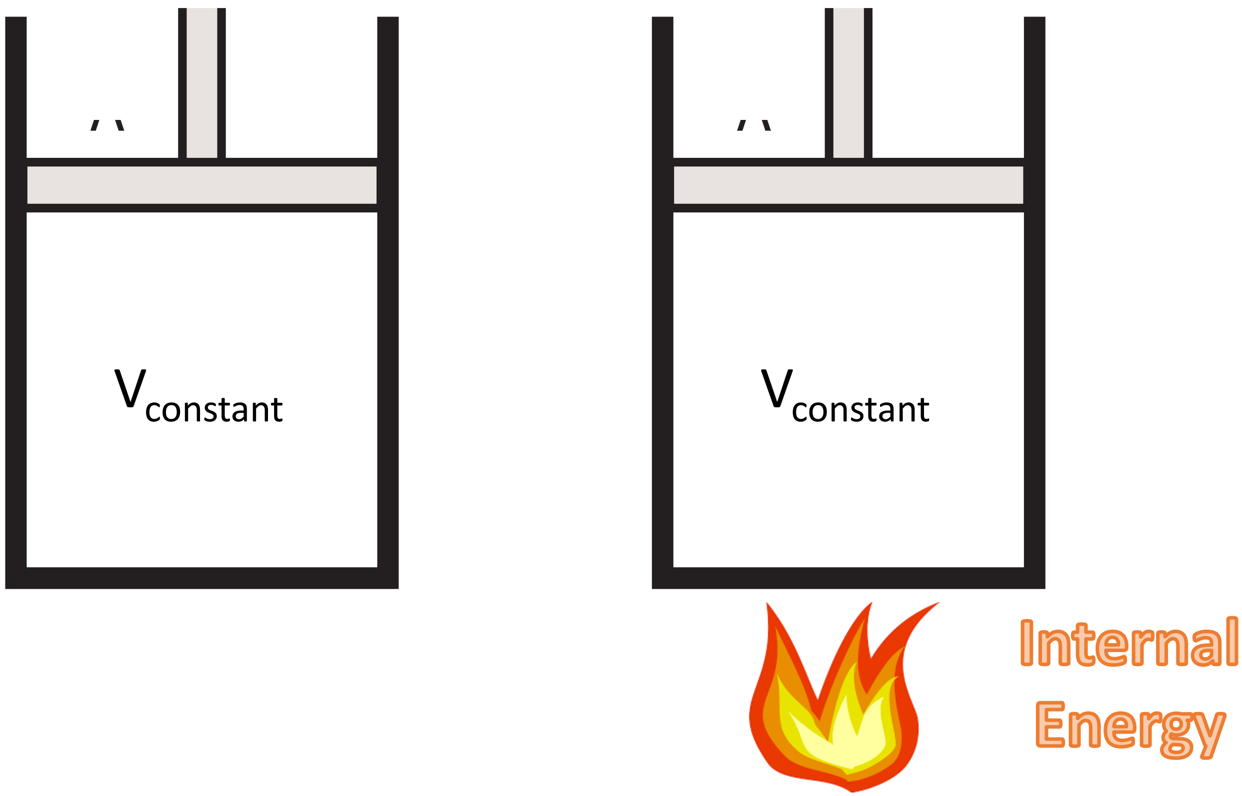
Heat Capacity at Constant Volume = Isochoric case

$$c_v \equiv \left(\frac{\delta q}{dT}\right)_{\alpha=constant}$$

Heat Capacity at Constant Pressure = Isobaric case

$$c_p \equiv \left(\frac{\delta q}{dT}\right)_{p=constant}$$

Heat Capacity at Constant Volume = Isobaric case



Heat Capacity at Constant Volume = Isochoric case

$$c_v \equiv \left(\frac{\delta q}{dT}\right)_{\alpha=constant}$$

$$Recall, Joule's\ Law:\quad \delta q = -du + p\ d\alpha$$

Since volume is constant, $d\alpha = 0$

(So, heat equals internal energy – no work done on system)

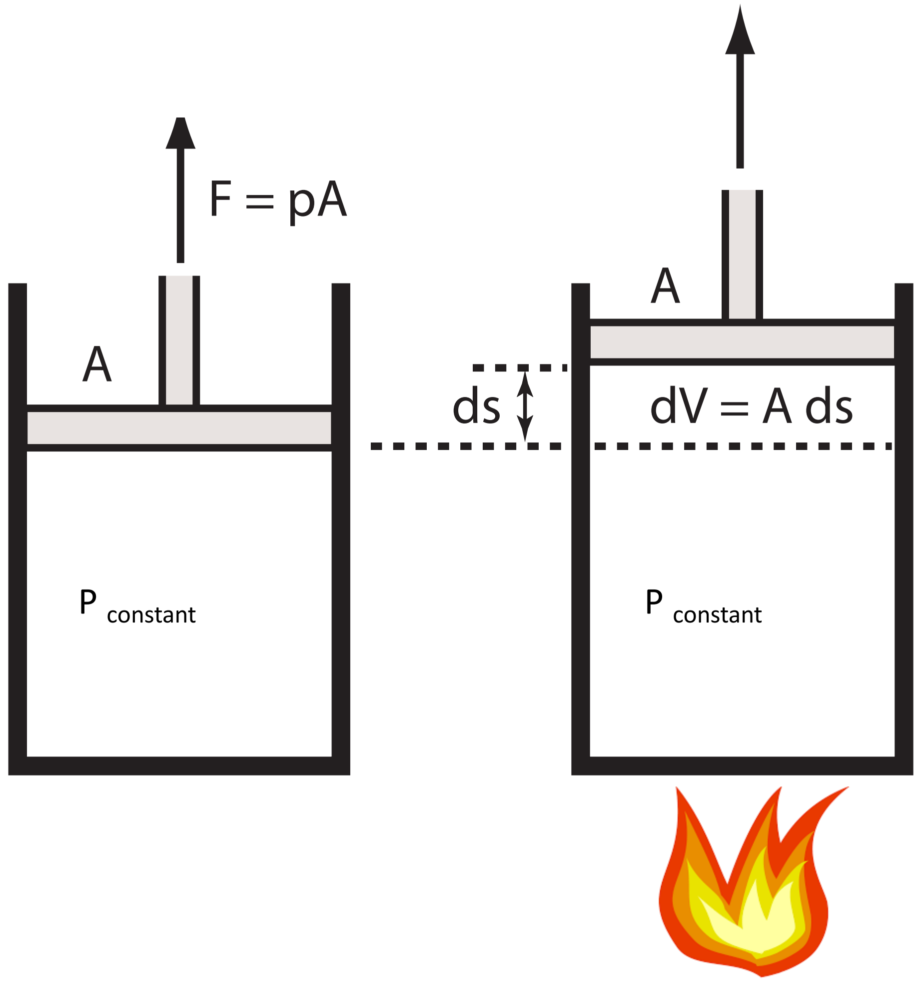
$$c_v = \frac{du}{dT}$$

$$\boxed{du = c_v\ dT}$$

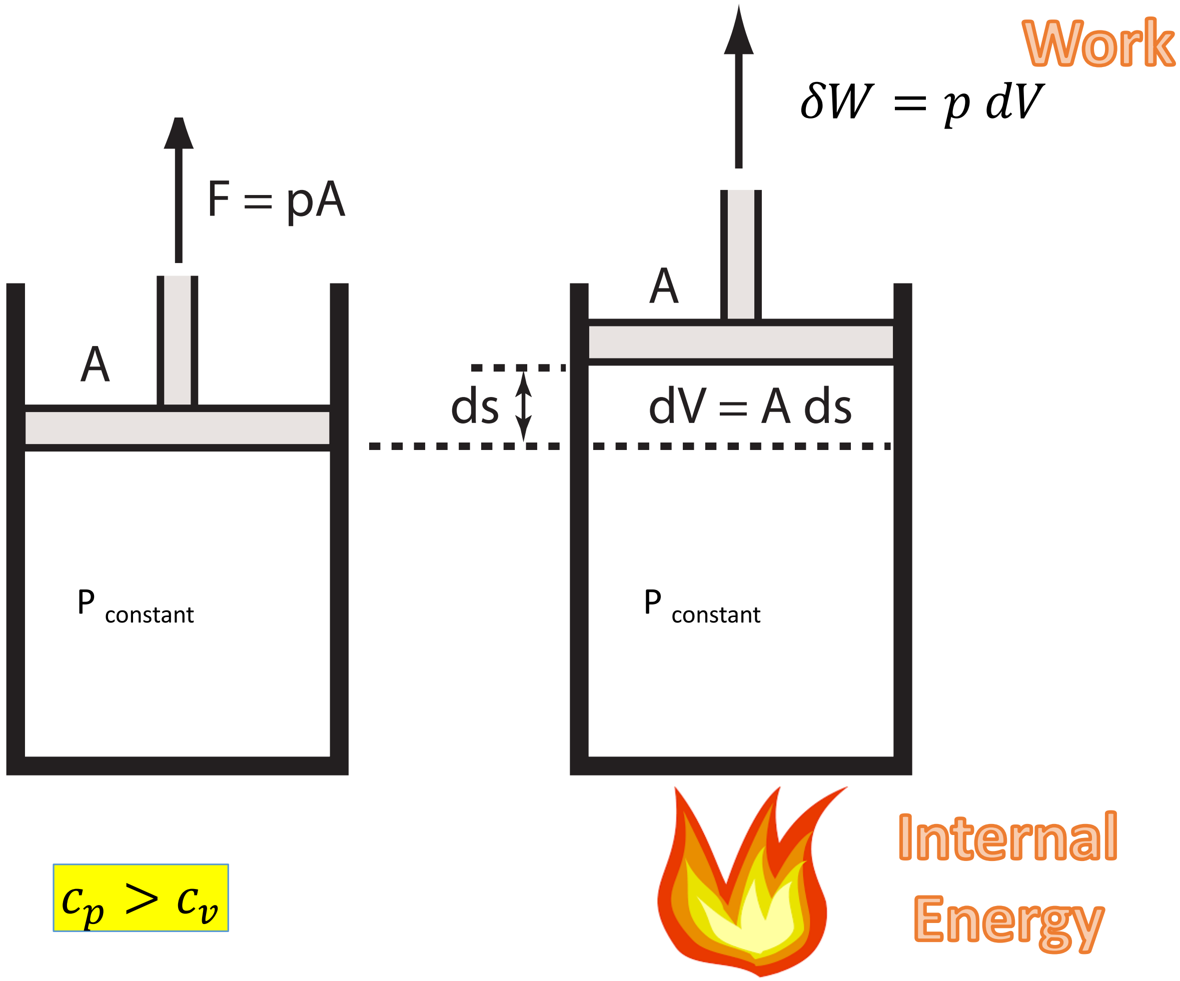
$$\boxed{\delta q = c_v\ dT + p\ d\alpha}$$

$$\boxed{c_v = 718\ J\ kg^{-1}K^{-1}}$$

Heat Capacity at Constant Pressure = Isobaric case



Heat Capacity at Constant Pressure = Isobaric case



$c_p > c_v$

Heat Capacity at Constant Pressure = Isobaric case

$$c_p \equiv \left(\frac{\delta q}{dT}\right)_{p=constant}$$

$$c_p = \left(\frac{du + pd\alpha}{dT}\right)_{p=constant}$$

Product rule of differentiation

$d(p\alpha) = p\ d\alpha + \ \alpha\ dp$

Substitute the ideal gas law (R is constant)

$d(RT) = \ R\ dT = p\ d\alpha + \ \alpha\ dp$

$p\ d\alpha = \ R\ dT - \ \alpha\ dp$

$p\ d\alpha = \ R\ dT, where\ p\ is\ constant$

$$c_p = \left(\frac{du + R\ dT}{dT}\right)_{p=constant}$$

$$c_p = \frac{du + R\ dT}{dT} = \ \frac{du}{dT} + R$$

$$\boxed{c_p = c_v + R}$$

Review of Heat Capacity

$$c_p = c_v + R$$

$$du = c_v \, dT$$

$$\delta q = c_v \, dT + p \, d\alpha$$

$$\delta q = (c_p - R) \, dT + p \, d\alpha$$

Recall $R \, dT = p \, d\alpha + \alpha \, dp$

$$\delta q = c_p \, dT - \alpha \, dp$$

$$c_v = \frac{5}{2} R$$

Atmosphere ➡ Diatomic Gas

$$c_v = 718 \, J \, kg^{-1} K^{-1}$$

$$c_p = 1005 \, J \, kg^{-1} K^{-1} = 718 \, J \, kg^{-1} K^{-1} + 287 J \, kg^{-1} K^{-1}$$

https://www.khanacademy.org/test-prep/mcat/physical-processes/kinetic-molecular-theory-of-gas/v/heat-capacity-at-constant-volume-and-pressure

Special Cases of the First Law

$$\delta q = c_v \, dT + p \, d\alpha$$

$$\delta q = c_p \, dT - \alpha \, dp$$

Isobaric Process: (dp = 0)

$$\delta q = c_p \, dT$$

Isothermal Process

Isochoric Process

Adiabatic Process

Special Cases of the First Law

$$\delta q = c_v\, dT + p\, d\alpha$$
$$\delta q = c_p\, dT - \alpha\, dp$$

Isobaric Process: (dp = 0)

$$\delta q = c_p\, dT = \left(\frac{c_p}{c_v}\right) du$$

Recall
$$du = c_v\, dT$$

Isothermal Process

Isochoric Process

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Special Cases of the First Law

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Isobaric Process: (dp = 0)

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Isothermal Process: (dT=0)

$$\delta q = -\alpha \, dp = p \, d\alpha = \delta w$$

Isochoric Process

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Special Cases of the First Law

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Adiabatic Process: (δq = 0) negligible change of heat between the system and environment

$$c_v\, dT = -p\, d\alpha$$
$$c_p\, dT = \alpha\, dp$$

Special Cases of the First Law

$$\delta q = c_v\, dT + p\, d\alpha$$

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Special Significance in Meteorology!!

Hint for Homework Problem 5.2