ATMOS 5130

Lecture 8

- The First Law and Its Consequences
  - Dry Adiabatic Processes
    - Poisson’s Equation
    - Potential temperature
    - Dry adiabats
  - Heat Engines

Special Cases of the First Law

$$\delta q = c_v \, dT + p \, d\alpha$$

$$\delta q = c_p \, dT - \alpha \, dp$$

Special Significance in Meteorology!!

Dry Adiabatic Process: (δq = 0) negligible change of heat between the system and environment

$$c_v \, dT = -p \, d\alpha$$

$$c_p \, dT = \alpha \, dp$$

Potential Temperature (Poisson's Equation)

$$c_p\, dT = \alpha\, dp = \frac{R_d T}{p} dp$$

$$\frac{1}{T} dT = \frac{R_d}{c_p} \frac{1}{p} dp$$

$$\int_{T_0}^{T} \frac{1}{T} dT = \frac{R_d}{c_p} \int_{p_0}^{p} \frac{1}{p} dp$$

$$\boxed{\frac{T}{T_0} = \left(\frac{p}{p_0}\right)^{\kappa}}$$

$$\kappa = \frac{R_d}{c_p} \approx 0.286$$

Potential Temperature - Poisson's Equation

$$\frac{T}{T_0} = \left(\frac{p}{p_0}\right)^{\kappa}$$

Set p_0 = 1000 hPa

$$\frac{T}{\theta} = \left(\frac{p}{1000\ hPa}\right)^{\kappa}$$

$$\theta = T\left(\frac{1000\ hPa}{p}\right)^{\kappa}$$

Potential temperature of a parcel at pressure (p) is the temperature that the parcel would acquire if adiabatically brought to 1000 hPa.

Potential Temperature

- Potential temperature is **more** dynamically important quantity than the actual temperature.
- Not affected by physical lifting or sinking associated with flow over obstacles or large-scale atmospheric turbulence.
- Parcel of air moving over a mountain will expand and cool as it ascends, then compress and warm as it descends on the other side- but the potential temperature will **not change** in the absence of heating, cooling, evaporation, or condensation (i.e. dry adiabatic processes).
- Since parcels with the same potential temperature can be exchanged without work or heating being required, lines of constant potential temperature are natural flow pathways.

Potential Temperature

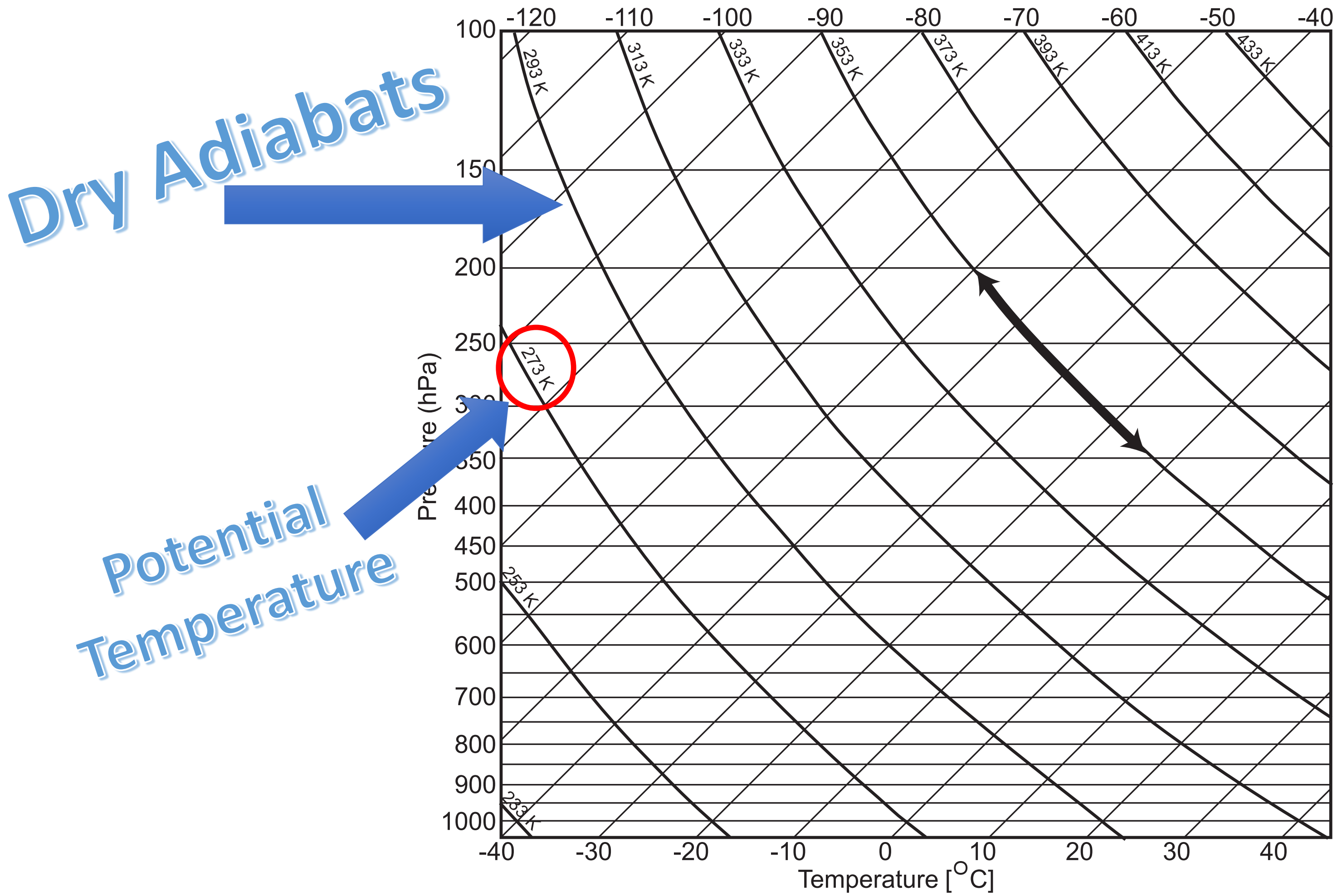
Important quantity in the planetary boundary layer (often very close to dry adiabatic).

Potential temperature is a useful measure of the static stability of the unsaturated atmosphere.

Under normal, stably stratified conditions, the potential temperature increases with height, $\frac{\delta\theta}{\delta z} > 0$, vertical motions are suppressed.

If the potential temperature decreases with height, $\frac{\delta\theta}{\delta z} < 0$,

atmosphere is unstable to vertical motions, and convection is likely.



Dry Adiabats
Potential Temperature
-120
-110
-100
-90
-80
-70
-60
-50
-40
100
150
200
250
300
350
400
450
500
600
700
800
900
1000
293 K
313 K
333 K
353 K
373 K
393 K
413 K
433 K
273 K
253 K
233 K
Pressure (hPa)
-40
-30
-20
-10
0
10
20
30
40
Temperature [°C]

Fig. 5.3

In class problem

Problem 5.7:

For a potential temperature of 310 K, compute the corresponding temperature T at 700 hPa, 300 hPa, and 100 hPa.

Plot your results at the appropriate location on skew T diagram and sketch in the corresponding dry adiabat.

In class problem

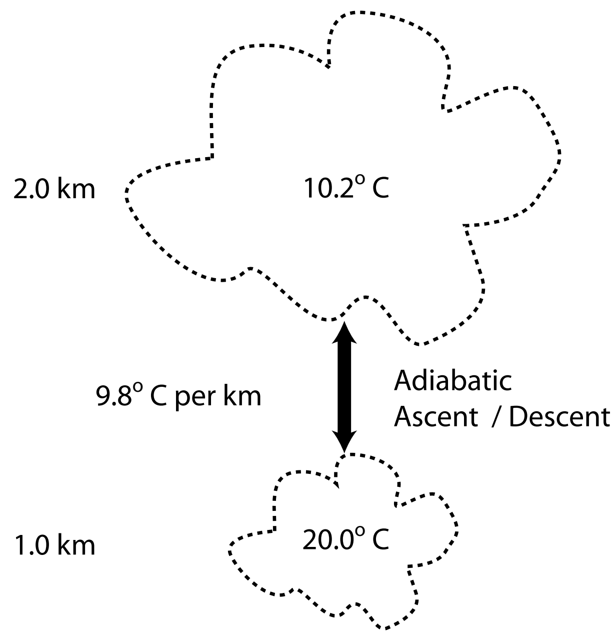
Problem 5.7:

For a potential temperature of 310 K, compute the corresponding temperature T at 700 hPa, 300 hPa, and 100 hPa.

Plot your results at the appropriate location on skew T diagram and sketch in the corresponding dry adiabat.

Answer:
T(700) = 280 K
T(300) = 219.8 K
T(100) = 160.6 K

Dry Adiabatic Lapse Rate



Dry Adiabatic Lapse Rate

$$c_p\, dT = \alpha\, dp = \frac{R_d T}{p} dp$$

$$\frac{dT}{dz} = \frac{dT}{dp}\frac{dp}{dz} = \frac{R_d T}{c_p p}\frac{dp}{dz}$$

The change in pressure in environment is same as parcel with height

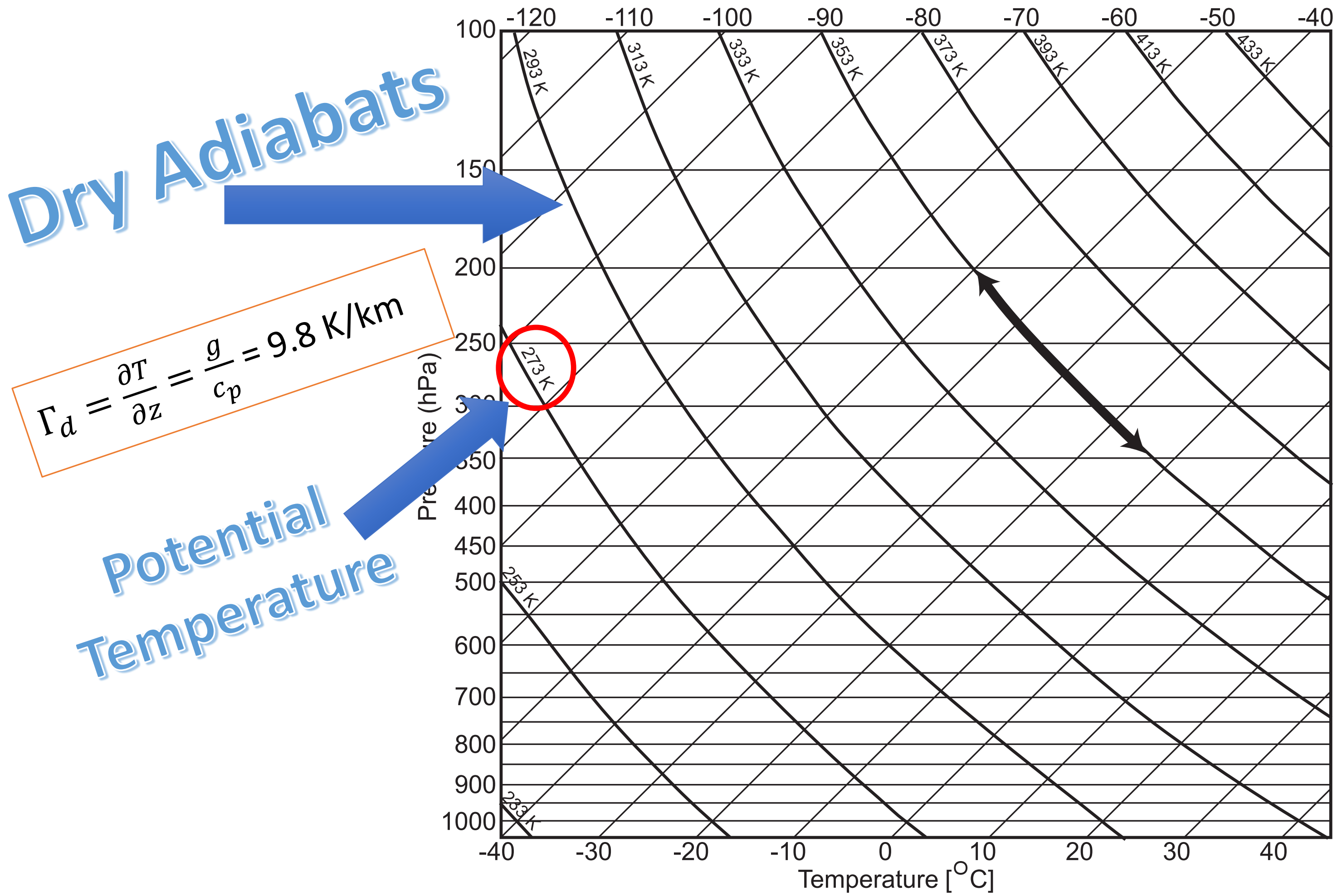
$$\frac{dp}{dz} = \rho' g = \frac{p'}{R_d T'} \mathrm{g}$$

Where the prime denotes the ambient/environment condition

$$\frac{dT}{dz} = \frac{R_d T}{c_p p}\frac{p\prime}{R_d T\prime} g$$

$$\frac{dT}{dz} = \frac{g}{c_p}$$

Approximation…
Poisson's equation better, if you know the pressure at each level.



$$\Gamma_d = \frac{\partial T}{\partial z} = \frac{g}{c_p} = 9.8 \text{ K/km}$$

Fig. 5.3

Diabatic Process

- Heat does not equal zero ($\delta q \neq 0$)
- Examples of diabatic heating process include
  - Radiation
  - Molecular conduction
  - Evaporation/Condensation (latent heat release)
  - Frictional dissipation of kinetic energy

Heat Engine

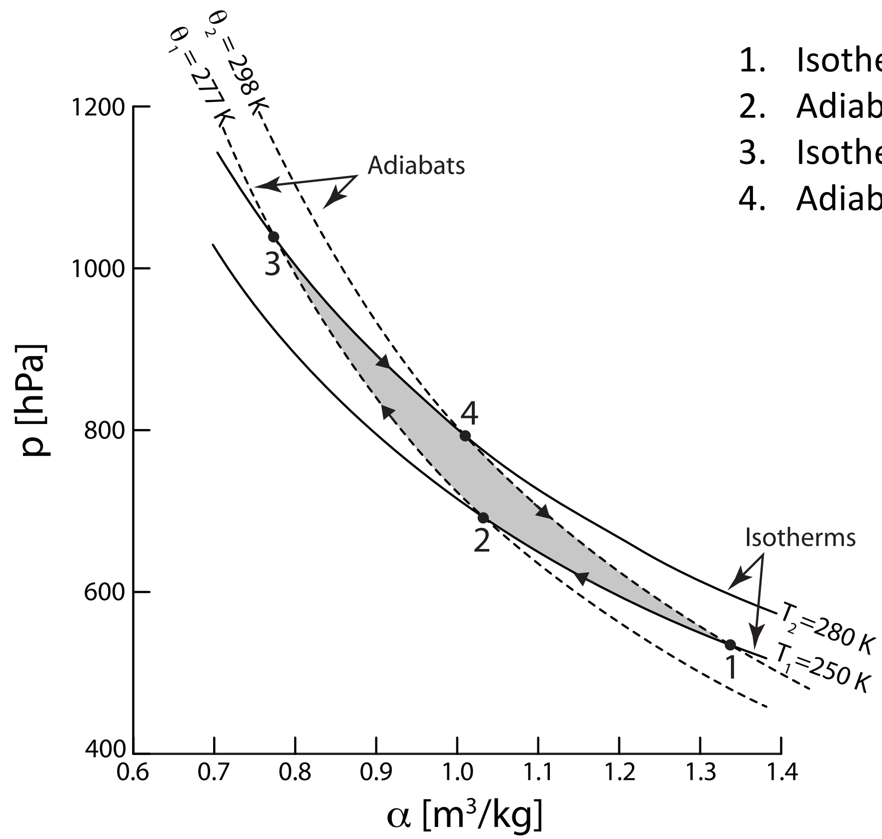
- Cyclic process (zero net change in internal energy).
- Converts heat supplied by the environment in to mechanical work performed by the system.
- Heat transfers from warm to cold
- Less heat leaves the system than is put into the system.

Efficiency of heat engine is ratio of work produced to heat supplied

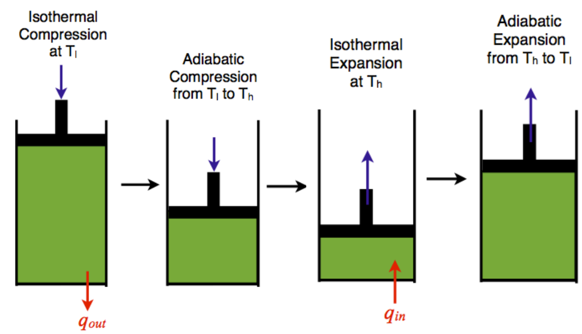
Waste heat

$$\eta = \frac{w}{q_{in}} = \frac{q_{in} - q_{out}}{q_{in}}$$

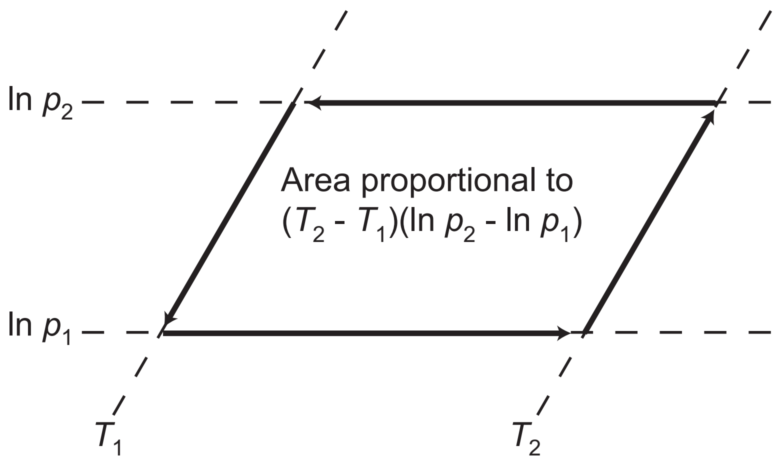
Carnot Cycle



1. Isothermal compression at cooler, T_1
2. Adiabatic compression to warmer, T_2
3. Isothermal expansion at T_2
4. Adiabatic expansion back to T_1



Work and Energy on skew-T



Recall

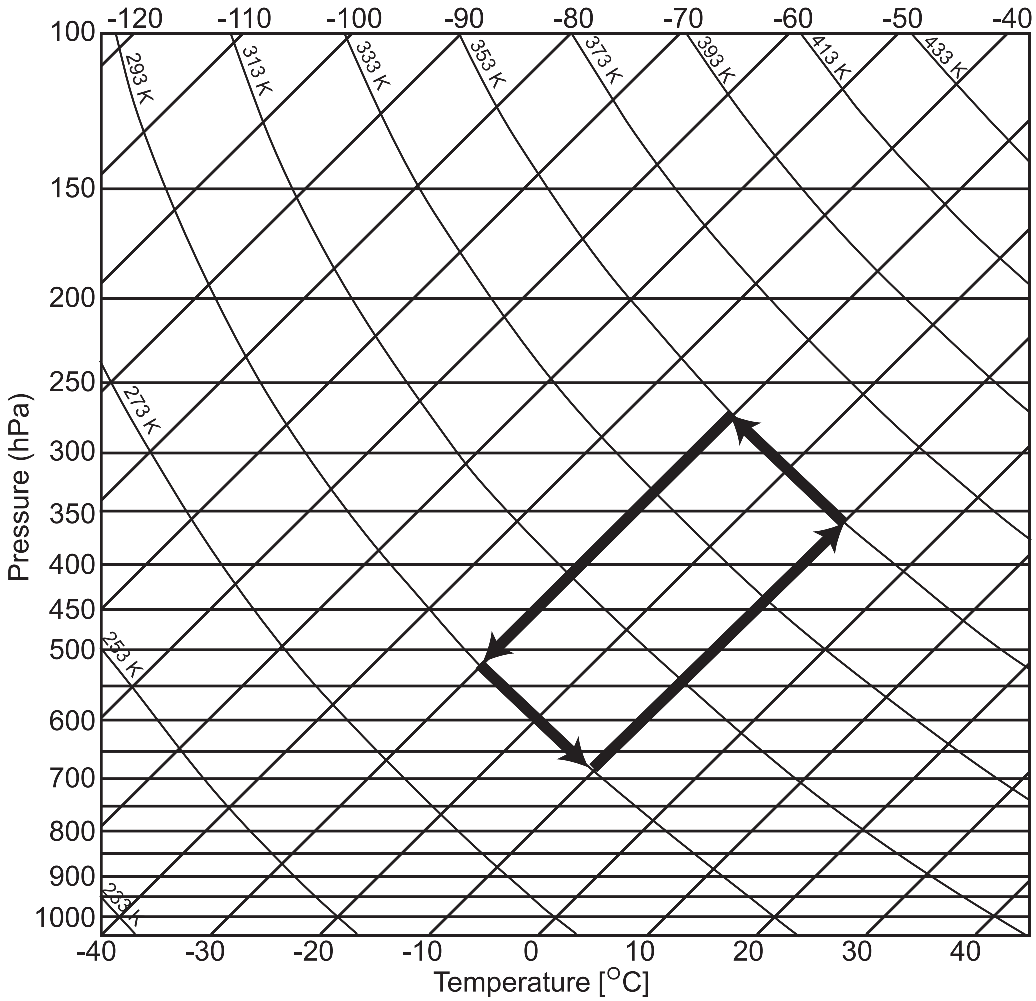
$$w = \int \delta w \equiv \int_{\alpha_0}^{\alpha_1} p\, d\alpha$$

<u>Isobaric</u> $w = \pm \int_{\alpha_0}^{\alpha_1} p\, d\alpha = \pm R_d (T_2 - T_1)$

<u>Isothermal</u> $w = \pm \int_{\alpha_0}^{\alpha_1} p\, d\alpha = \pm R_d T \int_{\alpha_0}^{\alpha_1} \frac{1}{\alpha}\, d\alpha = \pm R_d T \ln\left(\frac{p_1}{p_2}\right)$

Add up all the leg to get net work

$$w_{net} = R_d (T_1 - T_2) \ln\left(\frac{p_1}{p_2}\right) \propto Area$$



-120
-110
-100
-90
-80
-70
-60
-50
-40
100
150
200
250
300
350
400
450
500
600
700
800
900
1000
Pressure (hPa)
293 K
313 K
333 K
353 K
373 K
393 K
413 K
433 K
273 K
253 K
-40
-30
-20
-10
0
10
20
30
40
Temperature [°C]

Fig. 5.7