## Assignments

- Assignment: due Feb. 8
- Read AMS draft policy statement on communicating science
- http://www.ametsoc.org/policy/draftstatements/comm unicating science draftstatement.pdf
- Summarize and critique the draft statement in a couple paragraphs
- Comment in a couple paragraphs on the role of appropriate use of statistics for communicating science
- Add what you would consider an appropriate summation of the use of statistics to communicate science in a couple of sentences
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lity of each event ation and location. aticians involved, oms", lemmas, etc. my event is
${ }_{1}$ English: an event or else it is not an
he compound event The probability will happen is 1 .
one or the other of two mutually exclusive events is the sum of their

|  |  |
| :---: | :---: |
| Temperature below <br> Precipitation below | Temperature above <br> Precipitation below |
| Temperature below <br> Precipitation above | Temperature above <br> Precipitation above |

Figure 4.2. MECE possibilities for seasonal forecasts temperature and precpitation anomalies for a specific location.

## Storm Reports: Salt Lake County 1993-2005



Number of Opportunities: 2340 (180 days * 13 years)

## More concepts

- $\{E\}^{c}$ - complement of $\{E\}$, event does not occur
- $\operatorname{Pr}\{E\}\}^{c}=1-\{E\}$
- $\operatorname{Pr}\left\{\mathrm{E}_{1} \cap \mathrm{E}_{2}\right\}$ - joint probability that $\mathrm{E}_{1} \& \mathrm{E}_{2}$ occur
- $\operatorname{Pr}\left\{E_{1} \cap E_{2}\right\}=0$ if $E_{1} \& E_{2}$ are mutually exclusive
- $\operatorname{Pr}\left\{E_{1} \cup E_{2}\right\}$ - probability that $E_{1}$ OR $E_{2}$ occur
- $\operatorname{Pr}\left\{\mathrm{E}_{1} \cup \mathrm{E}_{2}\right\}=\operatorname{Pr}\left\{\mathrm{E}_{1}\right\}+\operatorname{Pr}\left\{\mathrm{E}_{2}\right\}-\operatorname{Pr}\left\{\mathrm{E}_{1} \cap \mathrm{E}_{2}\right\}$


## Conditional Probability

- Conditional probability: probability of $\left\{\mathrm{E}_{2}\right\}$ given that $\left\{\mathrm{E}_{1}\right\}$ has occurred
- $\operatorname{Pr}\left\{\mathrm{E}_{2} \mid \mathrm{E}_{1}\right\}=\operatorname{Pr}\left\{\mathrm{E}_{1} \cap \mathrm{E}_{2}\right\} / \operatorname{Pr}\left\{\mathrm{E}_{1}\right\}$
- $\mathrm{E}_{1}$ is the conditioning event
- If $\mathrm{E}_{1}$ and $\mathrm{E}_{2}$ are independent of each other, then

$$
\operatorname{Pr}\left\{\mathrm{E}_{2} \mid \mathrm{E}_{1}\right\}=\operatorname{Pr}\left\{\mathrm{E}_{2}\right\} \text { and } \operatorname{Pr}\left\{\mathrm{E}_{1} \mid \mathrm{E}_{2}\right\}=\operatorname{Pr}\left\{\mathrm{E}_{1}\right\}
$$

- Fair coin- $\operatorname{Pr}\{$ heads $\}=0.5$
- chance of getting heads on second toss is independent of the first

Pr\{heads $\mid$ heads $\}=0.5$
Pr\{heads\} twice $=0.5^{*} 0.5=025$

## Bayes Theorem

- $\operatorname{Pr}\left\{\mathrm{E}_{2} \mid \mathrm{E}_{1}\right\}=\operatorname{Pr}\left\{\mathrm{E}_{1} \cap \mathrm{E}_{2}\right\} / \operatorname{Pr}\left\{\mathrm{E}_{1}\right\}$ or
- $\operatorname{Pr}\left\{\mathrm{E}_{1} \cap \mathrm{E}_{2}\right\}=\operatorname{Pr}\left\{\mathrm{E}_{2} \mid \mathrm{E}_{1}\right\}^{*} \operatorname{Pr}\left\{\mathrm{E}_{1}\right\}$
- $\operatorname{Pr}\left\{E_{1} \cap E_{2}\right\}=\operatorname{Pr}\left\{E_{1} \mid E_{2}\right\} * \operatorname{Pr}\left\{E_{2}\right\}$ then
- $\operatorname{Pr}\left\{\mathrm{E}_{1} \mid \mathrm{E}_{2}\right\}=\operatorname{Pr}\left\{\mathrm{E}_{2} \mid \mathrm{E}_{1}\right\}{ }^{*} \operatorname{Pr}\left\{\mathrm{E}_{1}\right\} / \operatorname{Pr}\left\{\mathrm{E}_{2}\right\}$
- What is the advantage? Probability of conditioning event $E_{2}$ only computed once


## Bayesian Application: how a rational person responds to evidence

$\left.\begin{array}{l|l|l|l|}\hline & \text { Pos Test } & \text { Neg Test } & \text { TOTAL } \\ \hline \text { DRUG USER } & 0.495 \% & 0.005 \% & 0.5 \% \\ \hline \begin{array}{l}\text { NOT DRUG } \\ \text { USER }\end{array} & .995 \% & 98.505 \% & 99.5 \% \\ \hline \text { TOTAL } & 1.49 \% & 98.51 \% & \\ \hline\end{array} \begin{array}{l}\text { What are odds of falsely accusing non drug user? }\end{array}\right]$

## Bayesian Application:

|  | COLD | WARM | TOTAL |
| :--- | :--- | :--- | :--- |
| DRY | 20 | 60 | 80 |
| WET | 20 | 0 | 20 |
| TOTAL | 40 | 60 | 100 |
| $E_{1}-$ cold |  |  |  |
| $E_{2}-$ dry |  |  |  |
| $\operatorname{Pr}\left\{E_{1}\right\}-0.4$ |  |  |  |
| $\operatorname{Pr}\left\{E_{2}\right\}-0.8$ |  |  |  |
| $\operatorname{Pr}\left\{E_{2} \mid E_{1}\right\}-0.5$ |  |  |  |

We can't tell if it is cold or warm
But we know it is dry

$$
\begin{aligned}
\operatorname{Pr}\left\{\mathrm{E}_{1} \mid \mathrm{E}_{2}\right\} & =\operatorname{Pr}\left\{\mathrm{E}_{2} \mid \mathrm{E}_{1}\right\} * \operatorname{Pr}\left\{\mathrm{E}_{1}\right\} / \operatorname{Pr}\left\{\mathrm{E}_{2}\right\}= \\
& =0.5 * 0.4 / 0.8=0.25
\end{aligned}
$$

## NAME

$\qquad$
ATMOS 5040/6040 in Class Assignment
Number of opportunities: 2340
$\left\{\mathrm{E}_{3}\right\}=$ occurrence of winter storms ( $142+79=221$ )
$\left\{E_{3}\right\}=$ occurrence of convective storms |25+83=108)
$\left\{E_{3}\right\}=$ occurrence of property damage
$(79+113+25=217)$

| $\operatorname{Pr}\left[\mathrm{E}_{1}\right]=$ | $\operatorname{Pr}\left\{\mathrm{E}_{3}\right\}=$ | $\operatorname{Pr}\left(\mathrm{E}_{k}\right]=$ | $\operatorname{Pr}\left\{\mathrm{E}_{3} \cap \mathrm{E}_{3}\right\}=$ | $\operatorname{Pr}\left[E_{3} \cap E_{2}\right]=$ | $\operatorname{Pr}\left[\mathrm{E}_{1} \cap \mathrm{E}_{3}\right]=$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\operatorname{Pr}\left[\mathbf{E}_{1} \cup \mathbf{E} \mathbf{E}_{2}\right]=$ | $\operatorname{Pr}\left[\mathrm{E}_{2} \cup \mathrm{E} \mathrm{E}_{2}\right]=$ | $\operatorname{Pr}\left[\mathrm{E}_{1} \cup \mathrm{E} \mathrm{E}_{3}\right]=$ | $\operatorname{Pr}\left[E_{2} \mid E_{1}\right]=$ | $\operatorname{Pr}\left\{\mathrm{E}_{3} \mid E_{3}\right\}=$ |  |

For as standard deck of 52 cards, desl out all cards in pairs. Pr $\{\mathrm{ace}\}=\quad \operatorname{Pr}\{10-\mathrm{K}\}=\operatorname{Pr}\{2-9\}=$ What is the probsbility of getting a blsckjack for any pair of cards: $\operatorname{Pr}\{$ ace $\cap 10-\mathrm{K}\}=$ What is the probsbility of getting blackjack twice: $\operatorname{Pr}[$ ace $\cap 10-\mathrm{K}\}{ }^{*} \operatorname{Pr}\{$ ace $\cap 10-\mathrm{K}\}=$

Now, play at least 20 hands of blackjack with 3 other people Summarize in a table below your own relative frequencies (a/n) of getting an ace, 10-K, 2-9, blackjack, and two blackjacks. Who in your group was resily lucky?

|  | n | s | $\mathrm{s} / \mathrm{n}$ |
| :--- | :--- | :--- | :--- |
| ace |  |  |  |
| $10-\mathrm{K}$ |  |  |  |
| $2-9$ |  |  |  |
| blackjack |  |  |  |
| Two blackjocks |  |  |  |

## Empirical vs. Parametric Distributions

- Empirical distributions derived from a sample of a population


## Empirical vs. Parametric Distributions

- Parameteric distributions:
- Theoretical approach to define populations with known properties
- Can be defined by a function with couple parameters and assumption that population composed of random events



## Discrete Uniform Distribution

- 6 sided die
- Probability density function:
- $f(1,2,3,4,5$, or 6$)=1 / 6$
- $f(x)=1 / N ;$
$-x$ - one of the sides of the die, random variable
-N - total possible values
$-\mathrm{f}(\mathrm{x}) \mathrm{dx}$ incremental contribution to total probability

$$
\int_{1}^{6} f(x) d x=1
$$

## Cumulative Density Function of Discrete Uniform Variable

- $F(X)$ - total probability below a threshhold
- $F(X)=x$ * $1 / N$
- $F(1)=1 / 6$

$$
F(X)=\operatorname{Pr}\{x \leq X\}=\int_{1}^{x} f(x) d x
$$

- $F(2)=2 / 6 \ldots$
- X(F) - quantile function- value of random variable corresponding to particular cumulative probability
- $\mathrm{X}(50 \%)=3$


## Random Continuous Variable x

- $f(x)$ probability density function (PDF) for a random continuous variable $x$
- $\mathrm{f}(\mathrm{x}) \mathrm{dx}$ incremental contribution to total probability



## Cumulative Density Function of Continuous

## Variable

- $F(X)$ - total probability below a threshhold
- $F(0)=50 \%$
- $F(.66)=75 \%$

$$
F(X)=\operatorname{Pr}\{x \leq X\}=\int_{-\infty}^{x} f(x) d x
$$

- X(F) - quantile function- value of random variable corresponding to particular cumulative probability
- $X(75 \%)=0.66$



## Gaussian Parametric Distribution

- PDF $f(x)=\frac{1}{\sigma \sqrt{2 \pi}} \exp \left(-\frac{(x-\mu)^{2}}{2 \sigma^{2}}\right)$
- CDF $\quad F(X)=\frac{1}{\sigma \sqrt{2 \pi}} \int_{-\infty}^{x} \exp \left(-\frac{(x-\mu)^{2}}{2 \sigma^{2}}\right) d x$
- Two parameters define Gaussian distribution: $\mu$ and $\sigma$
- Nothing magic or "normal" about the Gaussian distribution- it is a mathematical construct


## Using parametric distributions

- Generate an empirical cumulative probability (CDF)
- Use dfittool to see if there is a good match between the empirical CDF and a particular parametric distribution
- Use the parameters from that parametric distribution to estimate the probabilities of values above below a threshhold or extreme events


## Lake Level



## Alta-Collins Temperature




- Mean: -5.1474
- Variance: 34.7724


## Using normspec: normal density plot

- 68.3\% between -1 and 1
- 95.\% between -2 and 2
- $2.3 \%$ of time variable explained by Gaussian distribution > 2 std dev of mean





## Using norminv: quantile function

- norminv([0.05,0.95],2,5)
- $90 \%$ of total variance between -6.2243 10.2243
- normspec([-6.2243,10.2243],2,5)



## Geometric Distribution

- Estimating how likely rare events can happen by chance
 100 year event
- geocdf(x,0.01)- probability for the next event to happen in $1,10,30,100,200,300$ years
- $63 \%$ chance in next 100 years
- $12 \%$ chance not until 200 years



## Klamath River Streamflow

- Weibull parametric fit



## Hypothesis Testing

- Alta temperature:
- Empirically: probability of temperature less than -15 is low
- Empirical estimates:
- Mean= -5.1C
- Std dev = 5.9C

- What are chances of getting temp of -20 IF this was a population of random numbers with that mean and std dev?


## Null hypothesis

- Null hypothesis: Temp of -20C does not differ significantly from mean of 5.1C
- $95 \%$ of time, random value would be within -16 and 6C
- So $5 \%$ of time, random value would be outside this range
- REJECT the null hypothesis accepting a $5 \%$ risk that we are rejecting the null hypothesis incorrectly
- If null hypothesis: Temp of -15C does not differ significantly from mean of 5.1C
- CANNOT reject the null hypothesis since $95 \%$ of the time the value could be within -16 and 6 C



## Collins: Confidence Intervals



