Assignments

- Give comments on AMS statement to Levi today
- Chapter 3 notes due Feb 15
- Due Feb 17
  - Read the article The 2010 Amazon Drought, including the supplementary material.
  - Pay particular attention to the statistical analysis of drought, not to the carbon part of the paper, and the two figures provided here
  - http://www.sciencemag.org/content/331/6017/554.full.pdf
  - Assume you were a reviewer of this paper
  - You can accept the paper as is, suggest minor or major revisions, or reject
  - Explain your reasoning in 3-4 paragraphs

million km^2 and 1.9 million km^2 in 2010 and 2005, respectively; Fig. 1 and fig. S1). Because dry-season biomass increases over the 2-year drought measurement interval (~0.8 Pg C) and (2) biomass lost adapted species, and drier forests store less carbon (8). If drought events continue, the era of intact Amazon forests buffering the increase in atmospheric carbon dioxide may have passed.

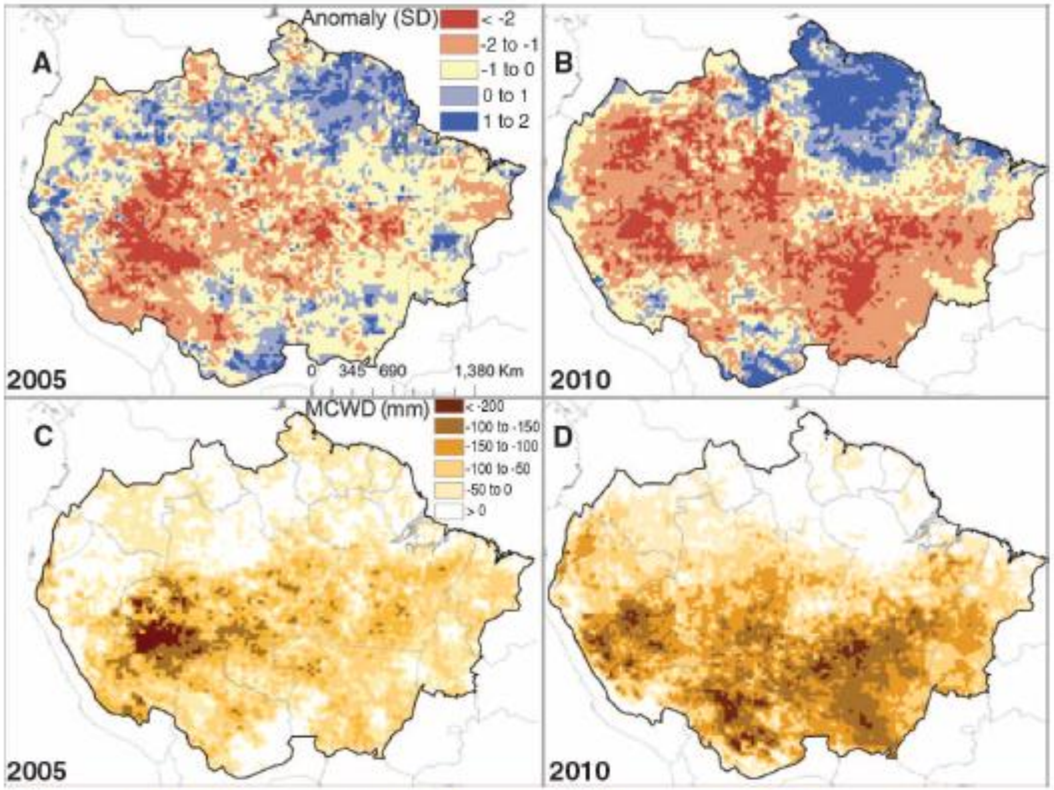

Fig. 1. (**A** and **B**) Satellite-derived standardized anomalies for dry-season rainfall for the two most extensive droughts of the 21st century in Amazonia. (**C** and **D**) The difference in the 12-month (October to September) MCWD from the decadal mean (excluding 2005 and 2010), a measure of drought intensity that correlates with tree mortality. (A) and (C) show the 2005 drought; (B) and (D) show the 2010 drought.

References and Notes

1. Y. Malhi *et al.*, *Science* **319**, 169 (2008); 10.1126/science.1146961.
2. J. A. Marengo *et al.*, *J. Clim.* **21**, 495 (2008).
3. A. Rammig *et al.*, *New Phytol.* **187**, 694 (2010).
4. Material and methods are available as supporting material on *Science* Online.
5. L. E. O. C. Aragão *et al.*, *Geophys. Res. Lett.* **34**, L07701 (2007).
6. O. L. Phillips *et al.*, *Science* **323**, 1344 (2009).
7. S. L. Lewis, *Philos. Trans. R. Soc. London Ser. B* **361**, 195 (2006).
8. E. M. Nogueira, B. W. Nelson, P. M. Fearnside, M. B. França, Á. C. Alves de Oliveira, *For. Ecol. Manage.* **255**, 2963 (2008).
9. We thank T. Baker and L. Aragão for assistance and the Royal Society, Moore Foundation, and NSF for funding.

Supporting Online Material

www.sciencemag.org/cgi/content/full/331/6017/554/DC1
Materials and Methods
Fig. S1
References

23 November 2010; accepted 7 January 2011
10.1126/science.1200807

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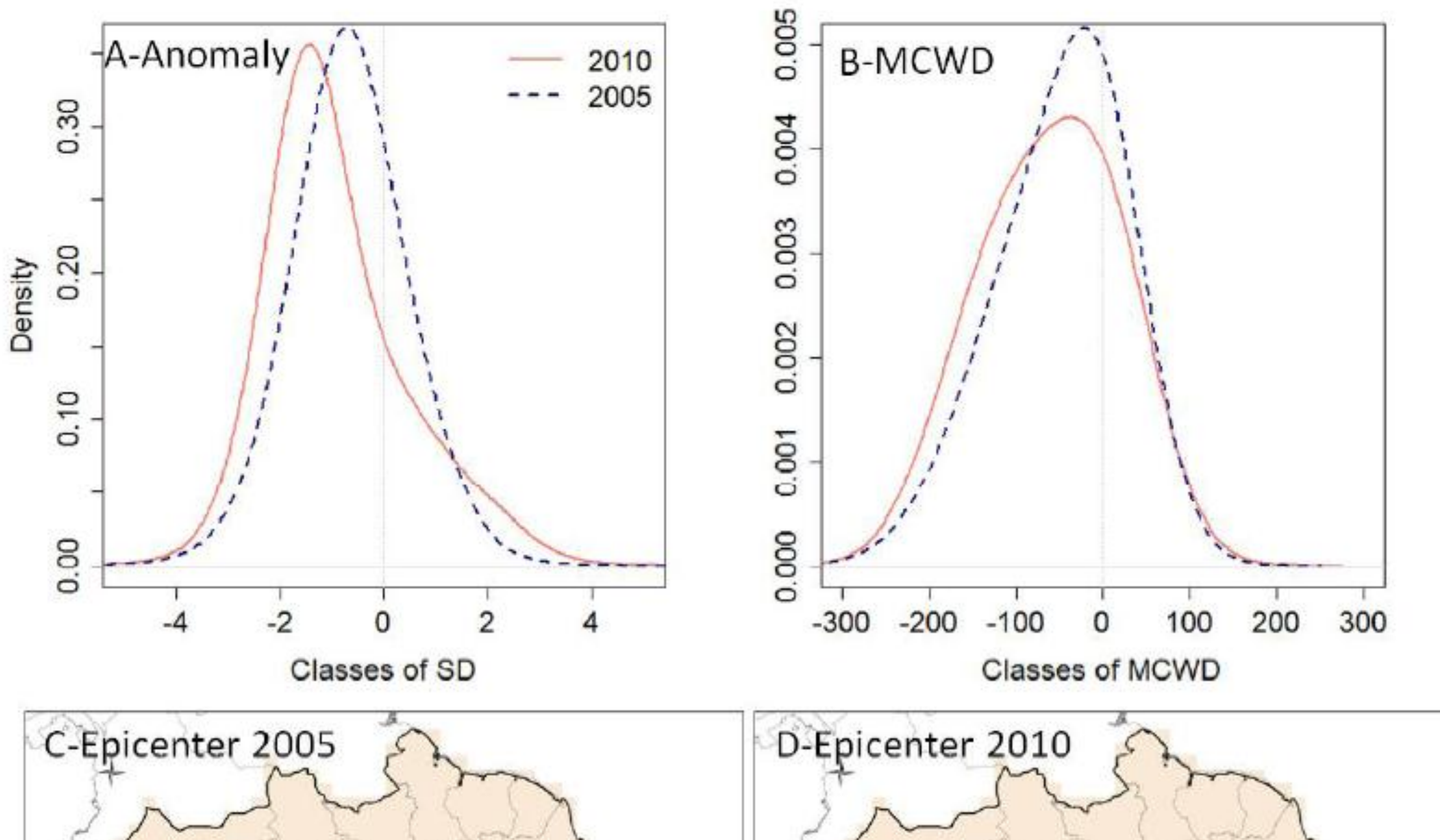
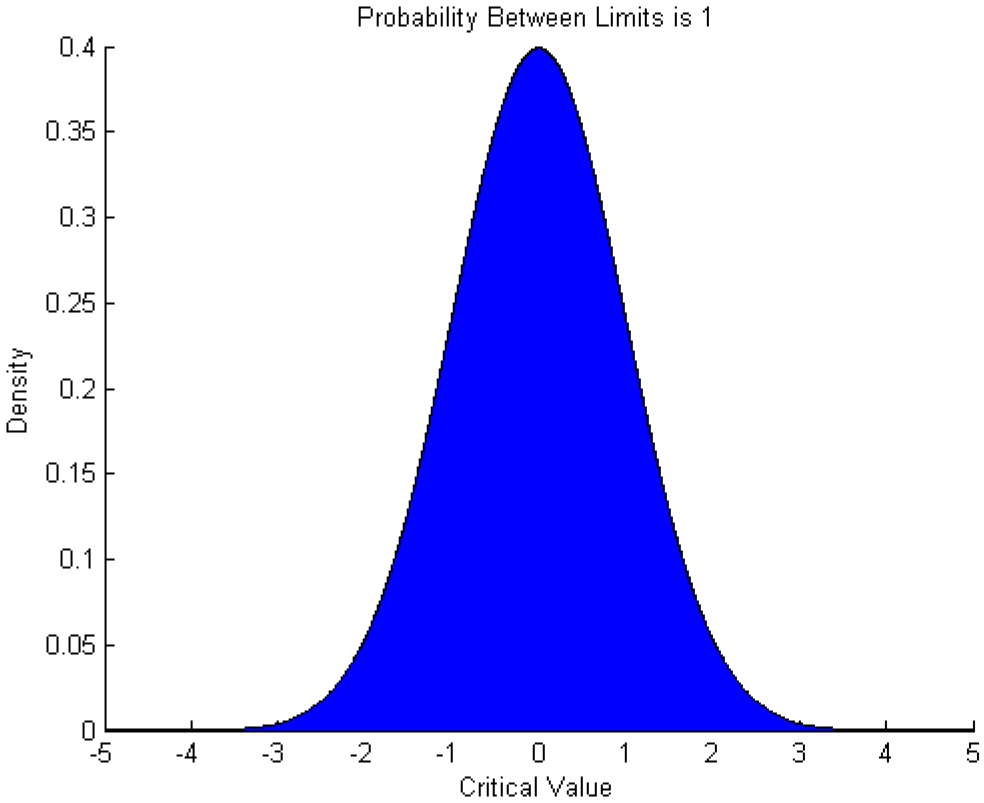
on ΔMCWD <-100 mm. **(D)** Lower right, shows drought epicenter in 2010, based on ΔMCWD <-100 mm.



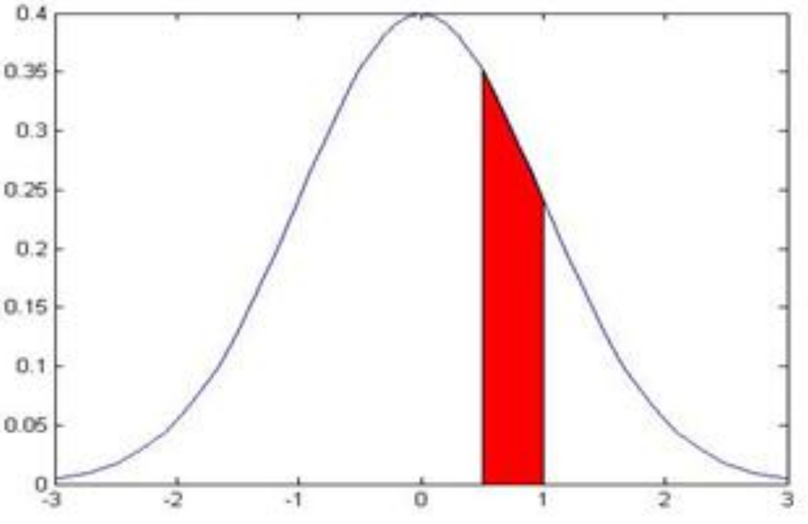
Empirical vs. Parametric Distributions

- Parameteric distributions:
  - Theoretical approach to define populations with known properties
  - Can be defined by a function with couple parameters and assumption that population composed of random events



Random Continuous Variable x

- f(x) probability density function (PDF) for a random continuous variable x
- f(x)dx incremental contribution to total probability

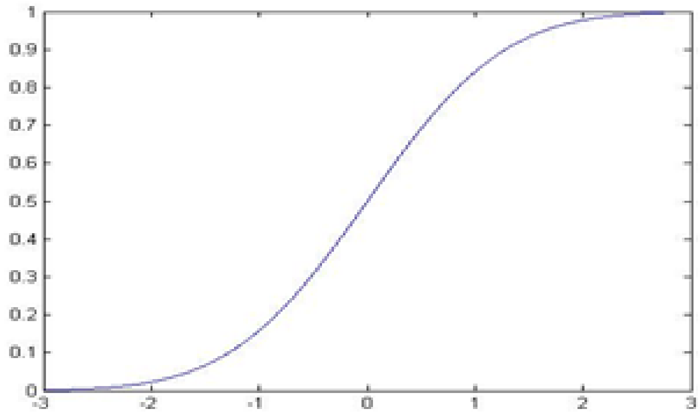


$$\int_{-\infty}^{\infty} f(x)dx = 1$$

Cumulative Density Function of Continuous Variable

- F(X)- total probability below a threshhold
- F(0) = 50%
- F(.66) = 75%
- X(F) – quantile function- value of random variable corresponding to particular cumulative probability
- X(75%) = 0.66

$$F(X) = \Pr\{x \le X\} = \int_{-\infty}^{X} f(x)dx$$



Expected Value

Gaussian Parametric Distribution

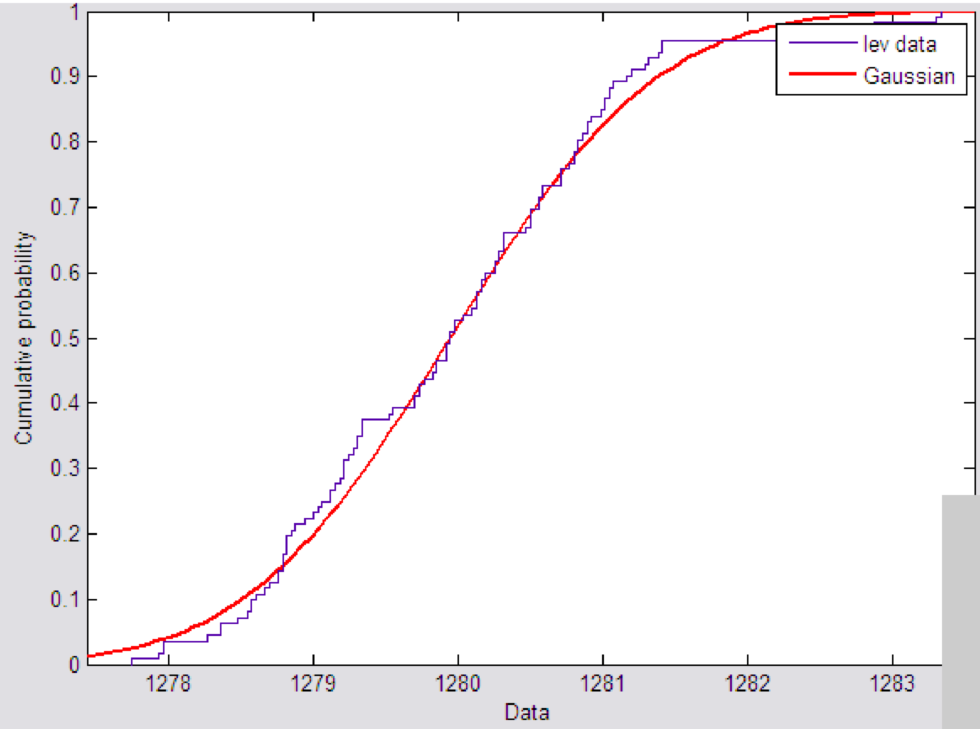
- PDF $$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp(-\frac{(x-\mu)^2}{2\sigma^2})$$
- CDF $$F(X) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{X} \exp(-\frac{(x-\mu)^2}{2\sigma^2})dx$$

- Two parameters define Gaussian distribution: μ and σ
- Nothing magic or “normal” about the Gaussian distribution- it is a mathematical construct

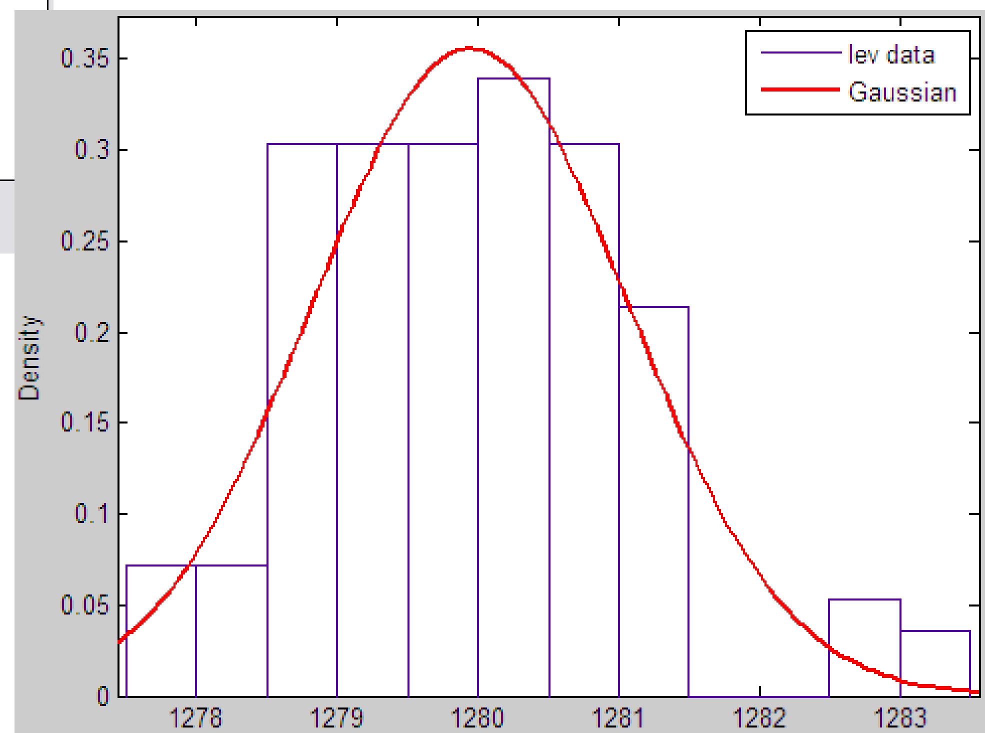
Using parametric distributions

- Generate an empirical cumulative probability (CDF)
- Use dfittool to see if there is a good match between the empirical CDF and a particular parametric distribution
- Use the parameters from that parametric distribution to estimate the probabilities of values above below a threshhold or extreme events

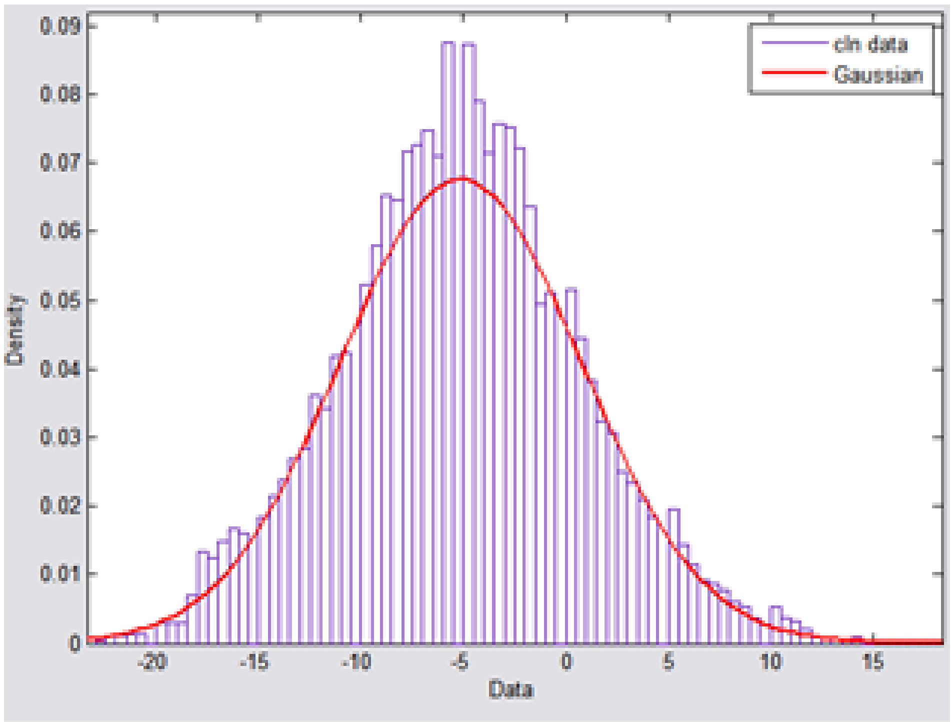
Lake Level

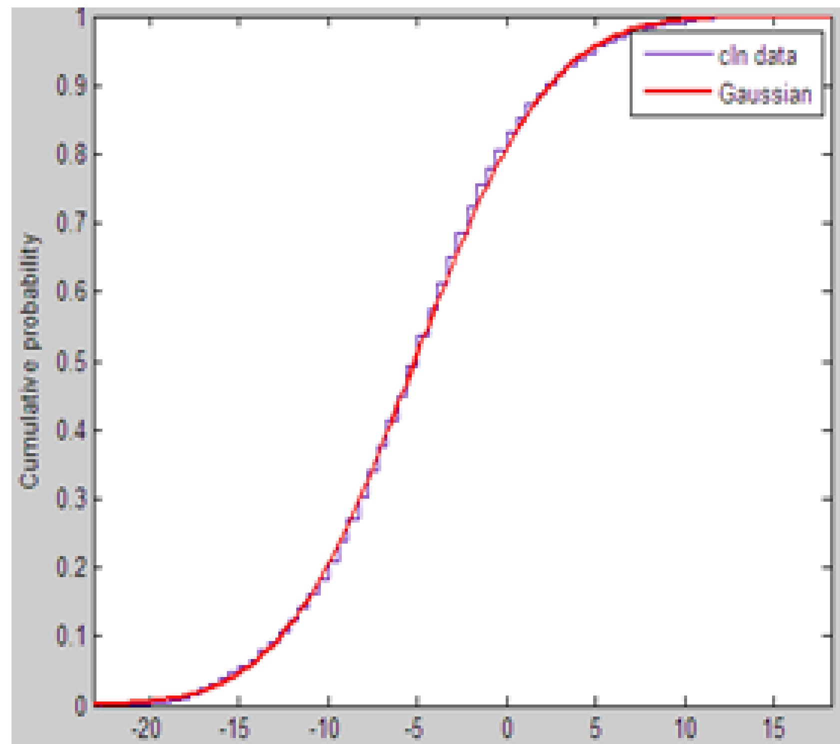


- Fit using sample mean and sample estimate of population std dev



Alta-Collins Temperature

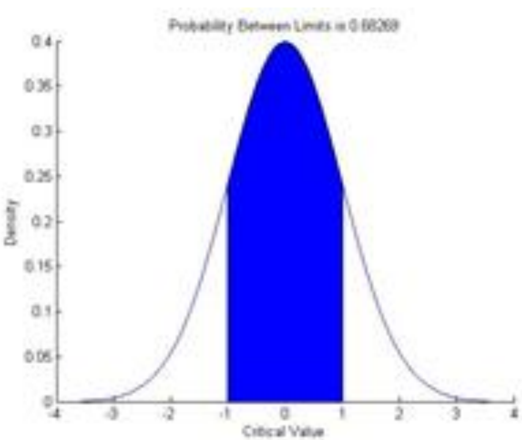


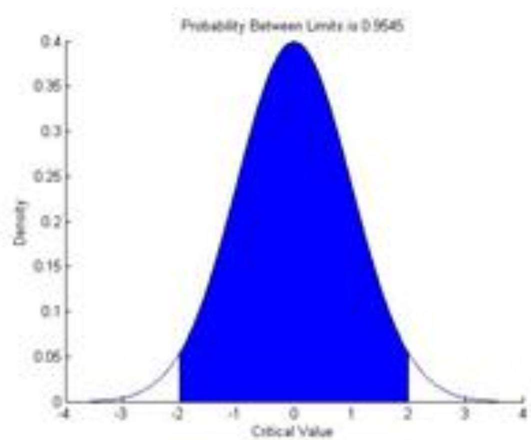


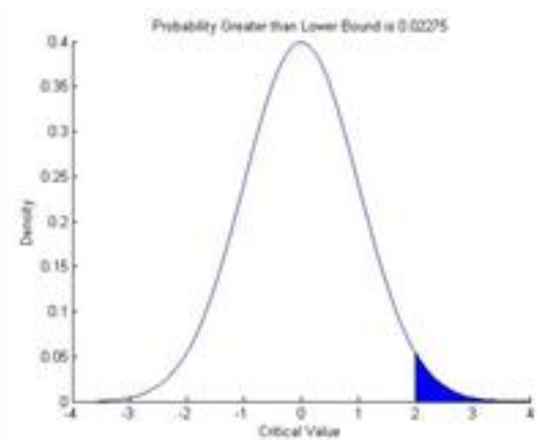
- Mean: -5.1474
- Variance: 34.7724

Using normspec: normal density plot

- 68.3% between -1 and 1
- 95.% between -2 and 2
- 2.3% of time variable explained by Gaussian distribution > 2 std dev of mean

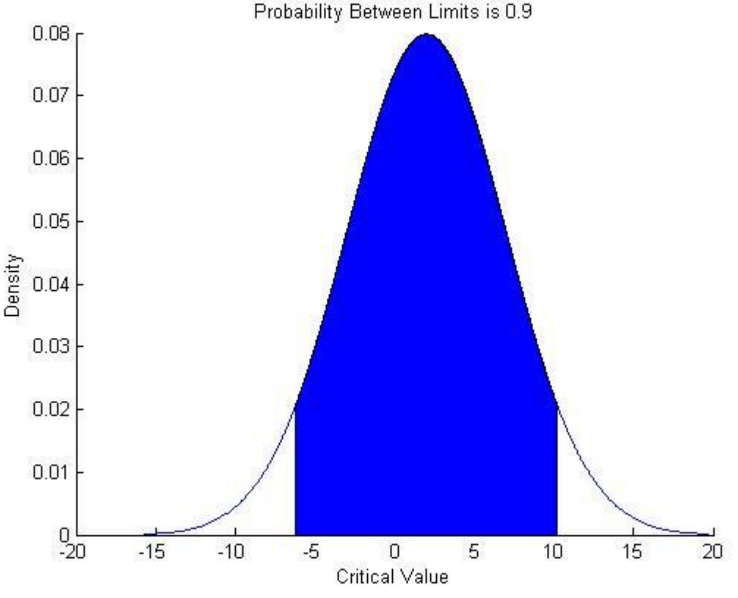






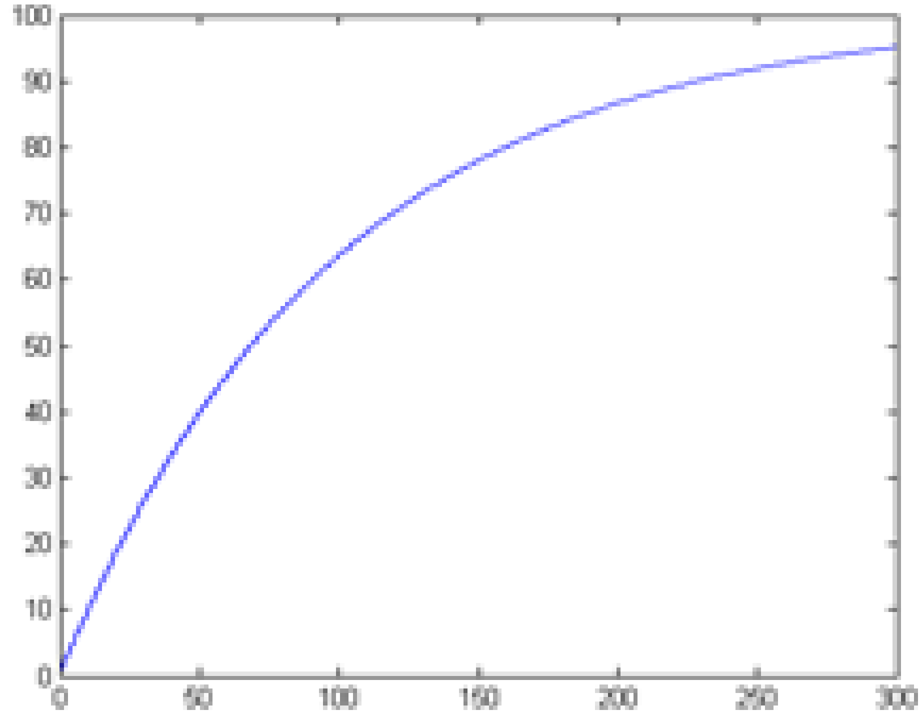
Using norminv: quantile function

- norminv([0.05,0.95],2,5)
- 90% of total variance between -6.2243 10.2243
- normspec([-6.2243,10.2243],2,5)



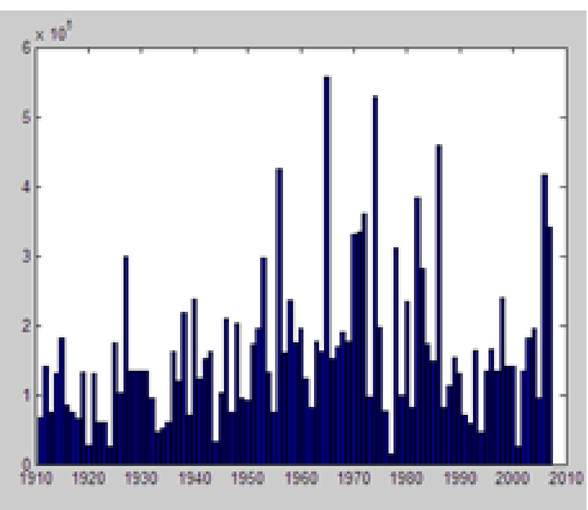
Geometric Distribution

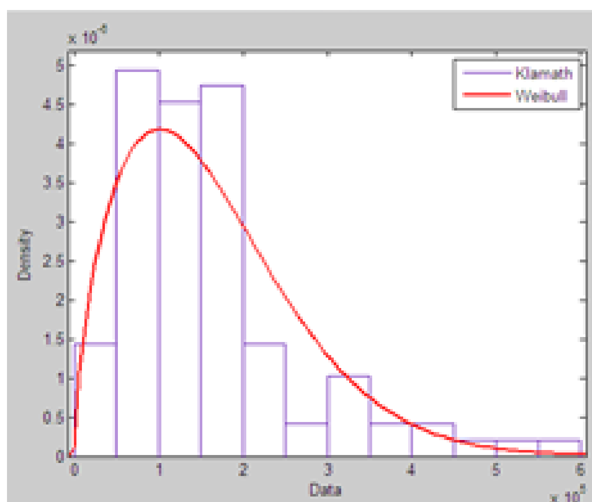
- Estimating how likely rare events can happen by chance
- Pr{0.01}- probability of a 1 in 100 year event
- geocdf(x,0.01)- probability for the next event to happen in 1, 10, 30, 100, 200, 300 years
- 63% chance in next 100 years
- 12% chance not until 200 years

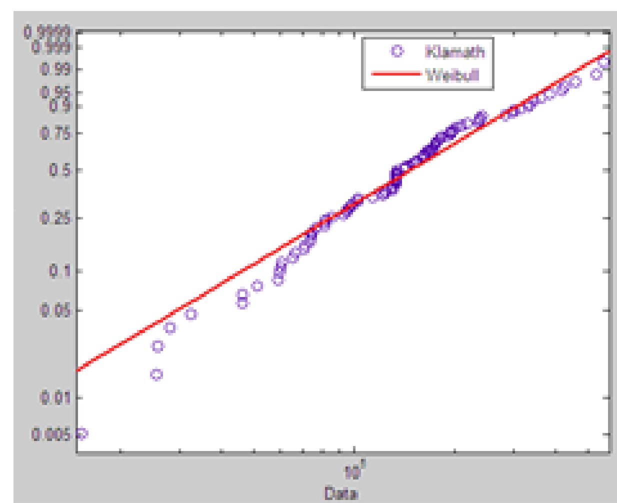


Klamath River Streamflow

- Weibull parametric fit

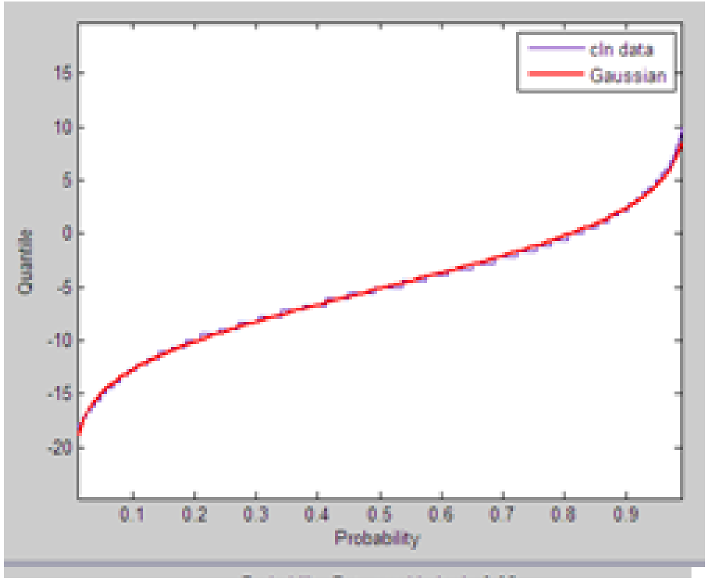






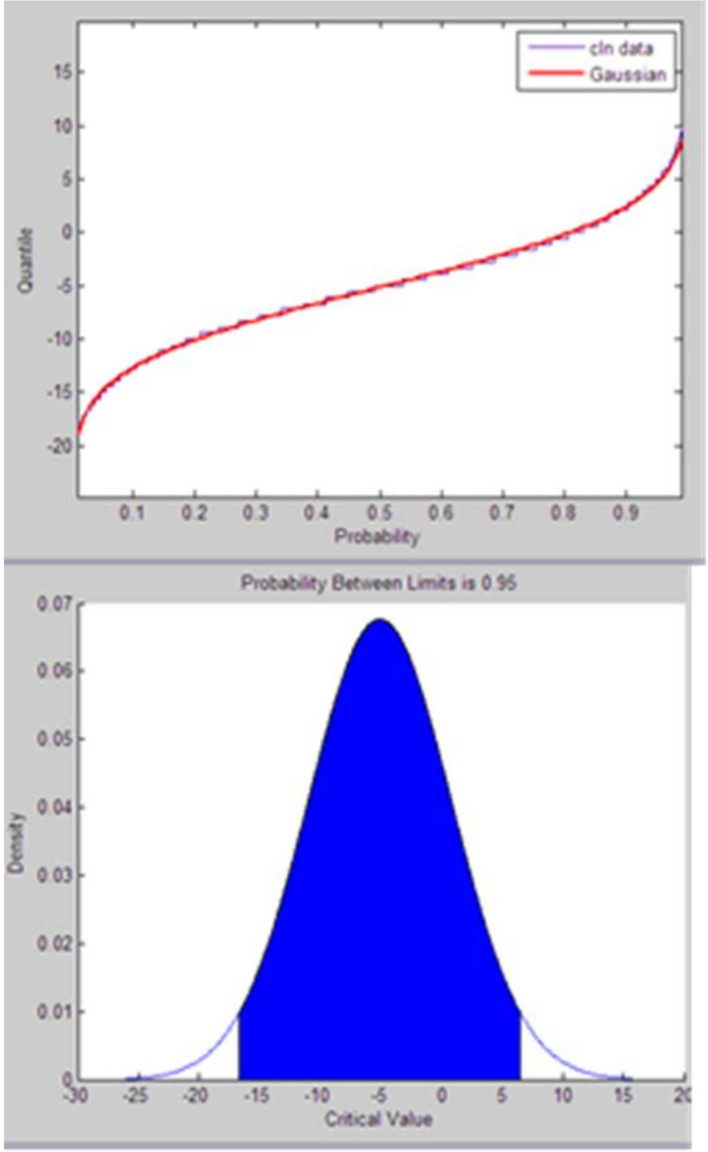
Hypothesis Testing

- Alta temperature:
- Empirically: probability of temperature less than -15 is low
- Empirical estimates:
  - Mean= -5.1C
  - Std dev = 5.9C
- What are chances of getting temp of -20 IF this was a population of random numbers with that mean and std dev?

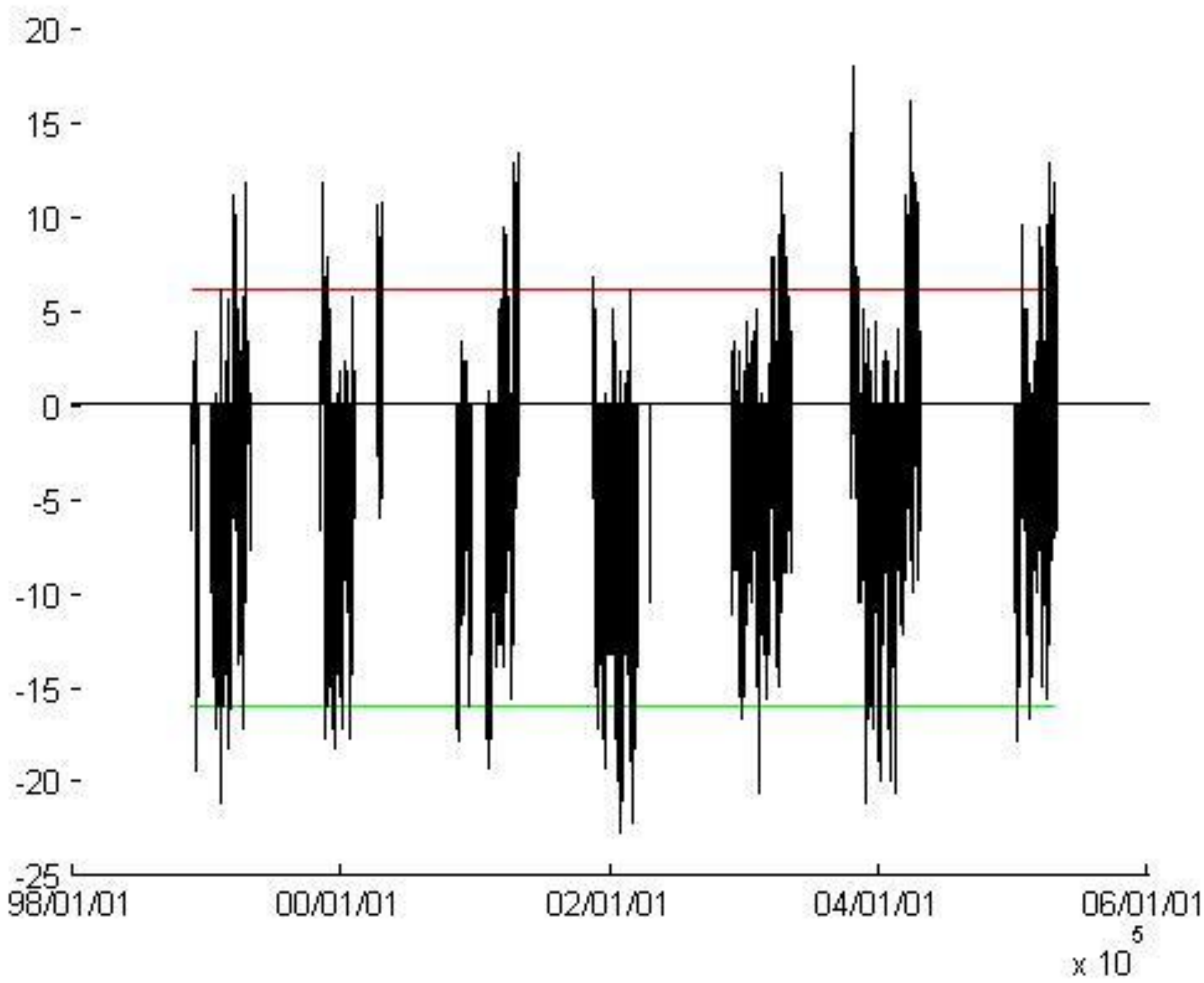


Null hypothesis

- Null hypothesis: Temp of -20C does not differ significantly from mean of -5.1C
- 95% of time, random value would be within -16 and 6C
- So 5% of time, random value would be outside this range
- REJECT the null hypothesis accepting a 5% risk that we are rejecting the null hypothesis incorrectly
- If null hypothesis: Temp of -15C does not differ significantly from mean of -5.1C
- CANNOT reject the null hypothesis since 95% of the time the value could be within -16 and 6C

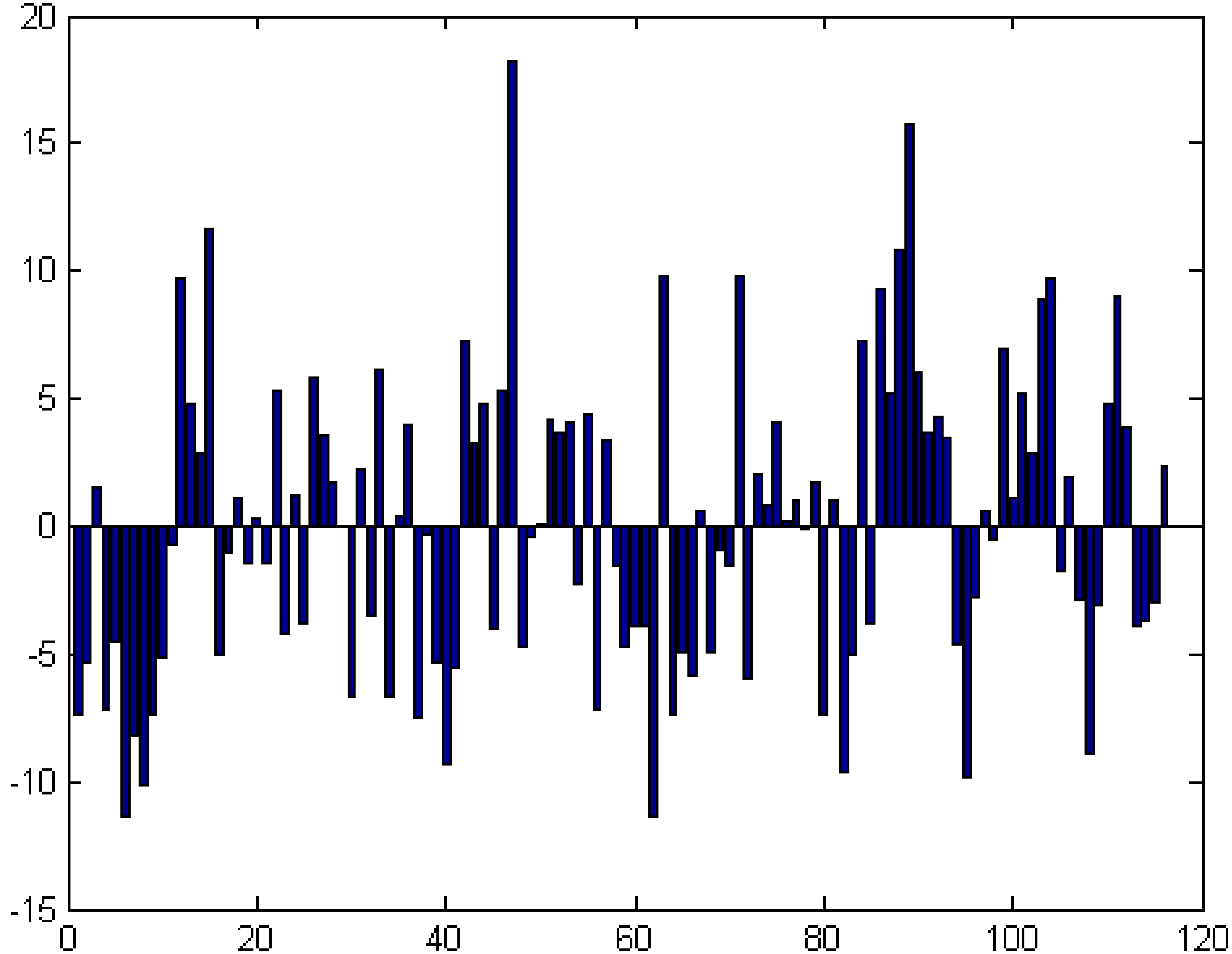


Collins: Confidence Intervals



Annual Precipitation of Utah

- Have we been in a drought the past 5 years?
- Define drought as average precipitation over 5 year period substantively below 0
- Values: 3.9, -3.9,-3.8,-2.9,2.4
- 5 yr mean -0.9, sd = 3.9
- 116 year sample mean 0, sd = 5.8


20
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0
-5
-10
-15
0
20
40
60
80
100
120

Steps of Hypothesis Testing

- Identify a test statistic that is appropriate to the data and question at hand
  - Computed from sample data values. 5 yr sample mean -0.9
- Define a null hypothesis, H_0 to be rejected
  - 5 yr sample mean 0
- Define an alternative hypothesis, H_A
  - 5 yr sample mean < 0
- Estimate the null distribution
  - Sampling distribution of the test statistic IF the null hypothesis were true
  - Making assumptions about which parametric distribution to use (Gaussian, Weibull, etc.)
  - Use sample mean of 0 and 116 yr sd of 5.8
- Compare the observed test statistic (-0.9) to the null distribution. Either
  - Null hypothesis is rejected as too unlikely to have been true IF the test statistic fall in an improbable region of the null distribution
    - Possibility that the test statistics has that particular value in the null distribution is small
    - normspec([-1.96*5.8,1.96*5.8],0,5.8)
    - OR
    - The null hypothesis is not rejected since the test statistic falls within the values that are relatively common to the null distribution

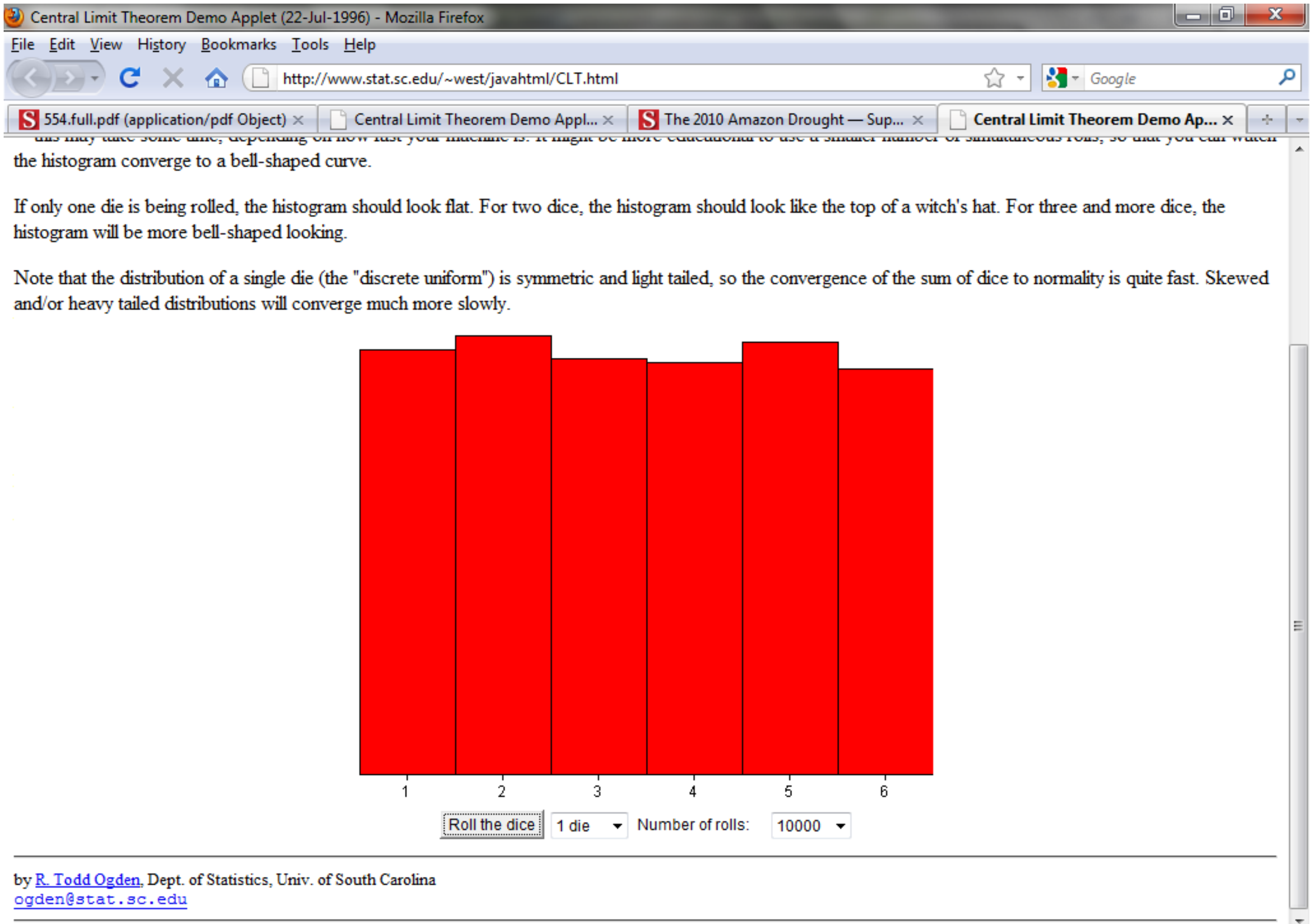
Caution!

- NOT rejecting the null hypothesis is not the same as saying the null hypothesis is true
  - There is insufficient evidence to reject H_0
- H_0 is rejected if the probability p of the observed test statistic is ≤ α significance or rejection level
- If odds of test statistic occurring in the null distribution less than 1 or 5%, then we may choose to reject the null hypothesis
- Rejecting the null hypothesis MAY be same as accepting alternative hypothesis BUT there may be many other possible alternative hypotheses
- You must define ahead of time the α significance or rejection level
  - 1% or 5%, 1 in 100 or 5 in 100 chance that you accept the risk of rejecting the null hypothesis incorrectly
  - Type 1 category error of a false rejection of the null hypothesis

Central Limit Theorem

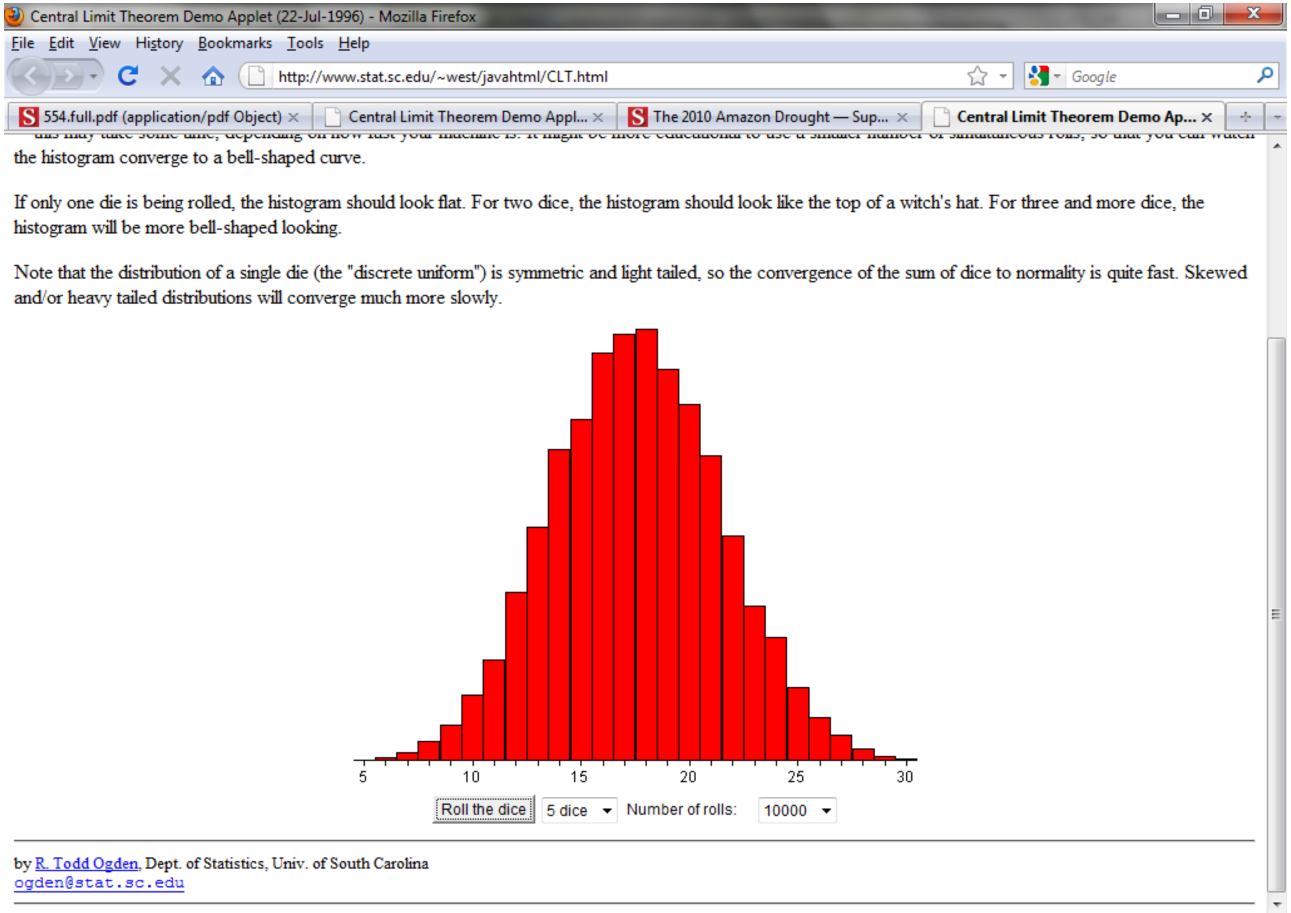
- http://www.stat.sc.edu/~west/javahtml/CLT.html
- http://www.mathcs.org/java/programs/CLT/clt.html
- http://www.stat.tamu.edu/~west/applets/

Roll 1 die 10000 times



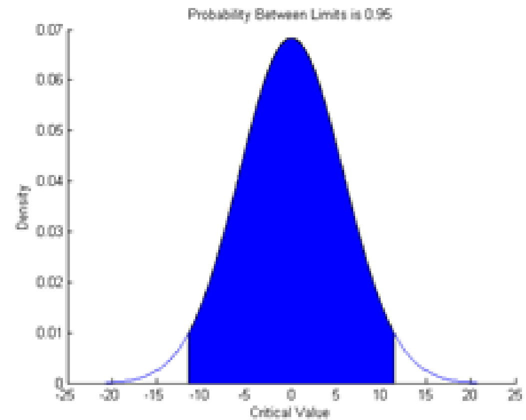
Roll 5 die 10000 times

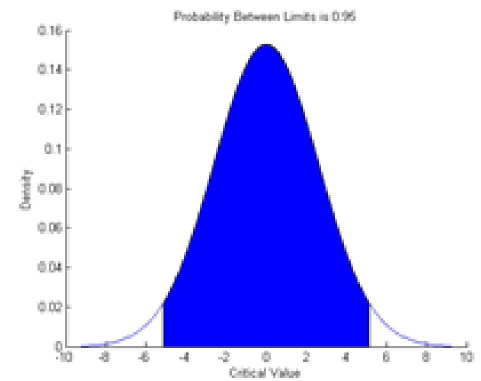
- Getting the sum or mean of 5 numbers



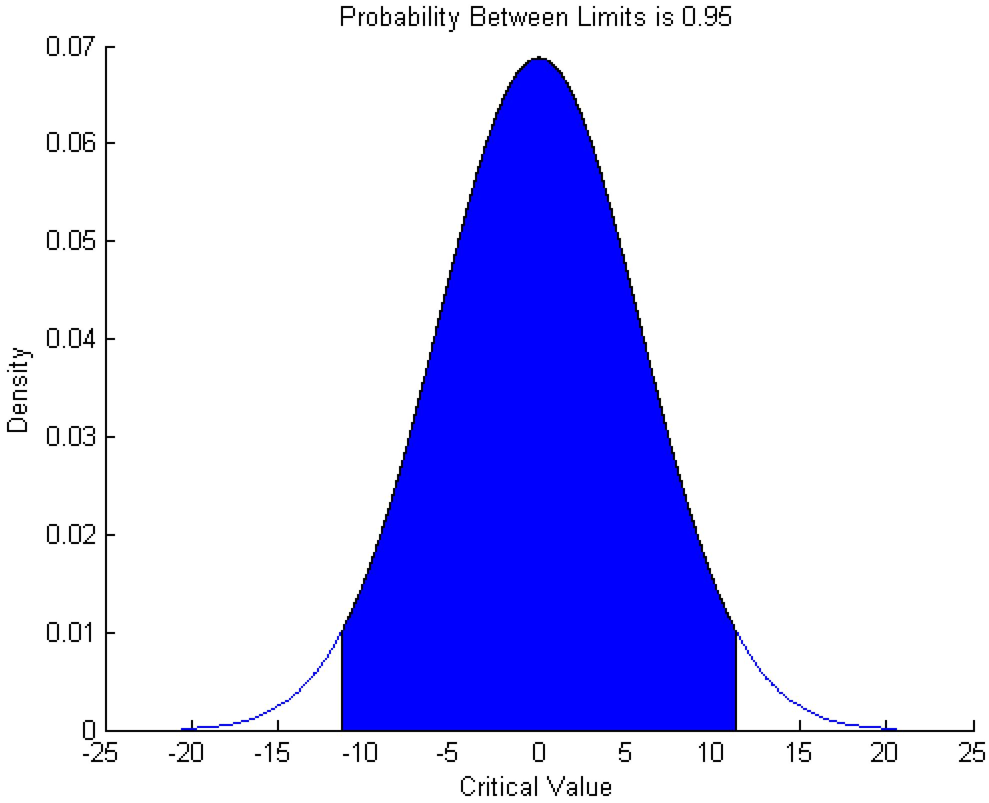
Central Limit Theorem

- Sum (or mean) of a sample (5 dice) will have a Gaussian distribution even if the original distribution (1 die) does not have a Gaussian distribution, especially as the sample size increases
- $\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}}$
- Individually 116 yr sd = 5.8 cm
- 5 yr samples sd = 5.8/sqrt(5)



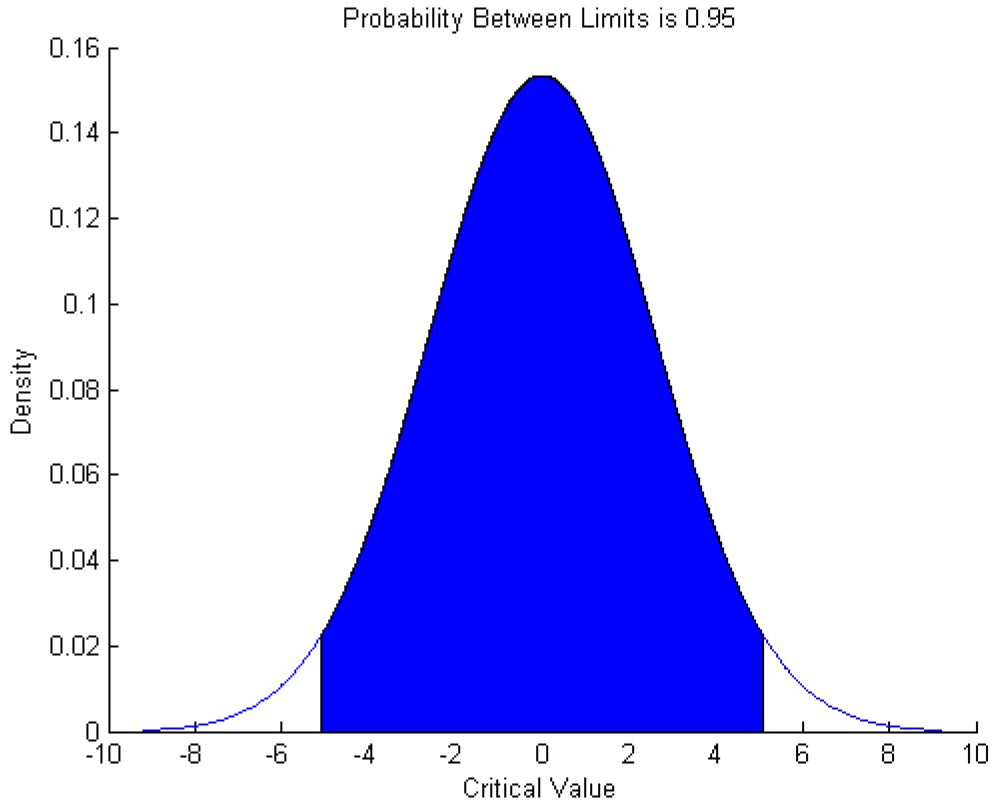


normspec([-1.96*5.8,1.96*5.8],0,5.8)



95% chance that individual anomaly within 11.5 cm

normspec([-1.96*2.6,1.96*2.6],0,2.6)



95% chance that 5 yr sample mean within 5.6 cm
Less likely to have a drought than to have a single dry year

Students' t test

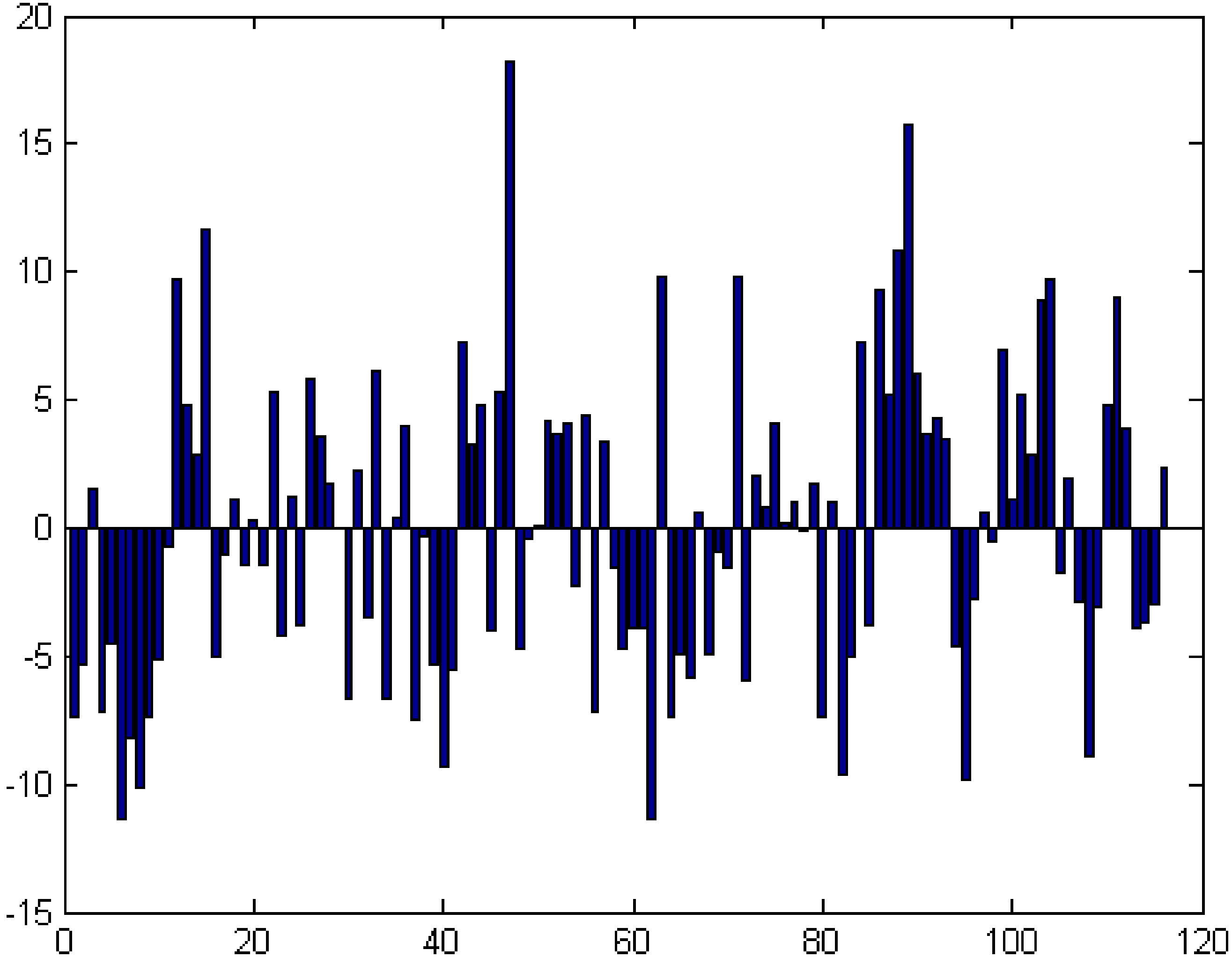
- $\sigma_{\overline{X}} = \frac{s_x}{\sqrt{n-1}}$
- Estimate of population variance from sample
- T value:
- Numerator: signal
- Denominator: noise

$$t = (\bar{x} - \mu)\sqrt{n-1} / s_x$$

- At t gets larger, confidence in rejecting the null hypothesis (sample mean differs from population mean) gets higher
- T large IF:
  - Spread between sample and population means large
  - Degrees of freedom is large
  - Variability in sample is small

Using t test

- **[h,p,ci,stat]= ttest(valy,0,.05,'left');**
- where on input valy is the vector of values in each 5-year sample
- 3.8, -3.9, -3.6, -2.9, 2.3
- 0 is the mean value for the null hypothesis
- .05 is the significance level chosen (5%)
- ‘left’ indicates that we are assuming that we have ruled out that large positive anomalies are relevant (the other options are ‘both’ a two-tailed test and ‘right’ where we rule out large negative anomalies, i.e., look for wet periods)
- Output:
  - h is a flag, 0 means the null hypothesis can not be rejected, 1 means it can be rejected
  - p is the significance level corresponding to the t value, the smaller the number the better
  - ci is the confidence interval
  - stat- is an array that returns the value of the t statistic, the number of degrees of freedom, and the estimated population standard deviation
- in this case, h= 0., p = .31 (really big), ci= -2.6, sample mean needs to be less than -2.6 to reject null hypothesis



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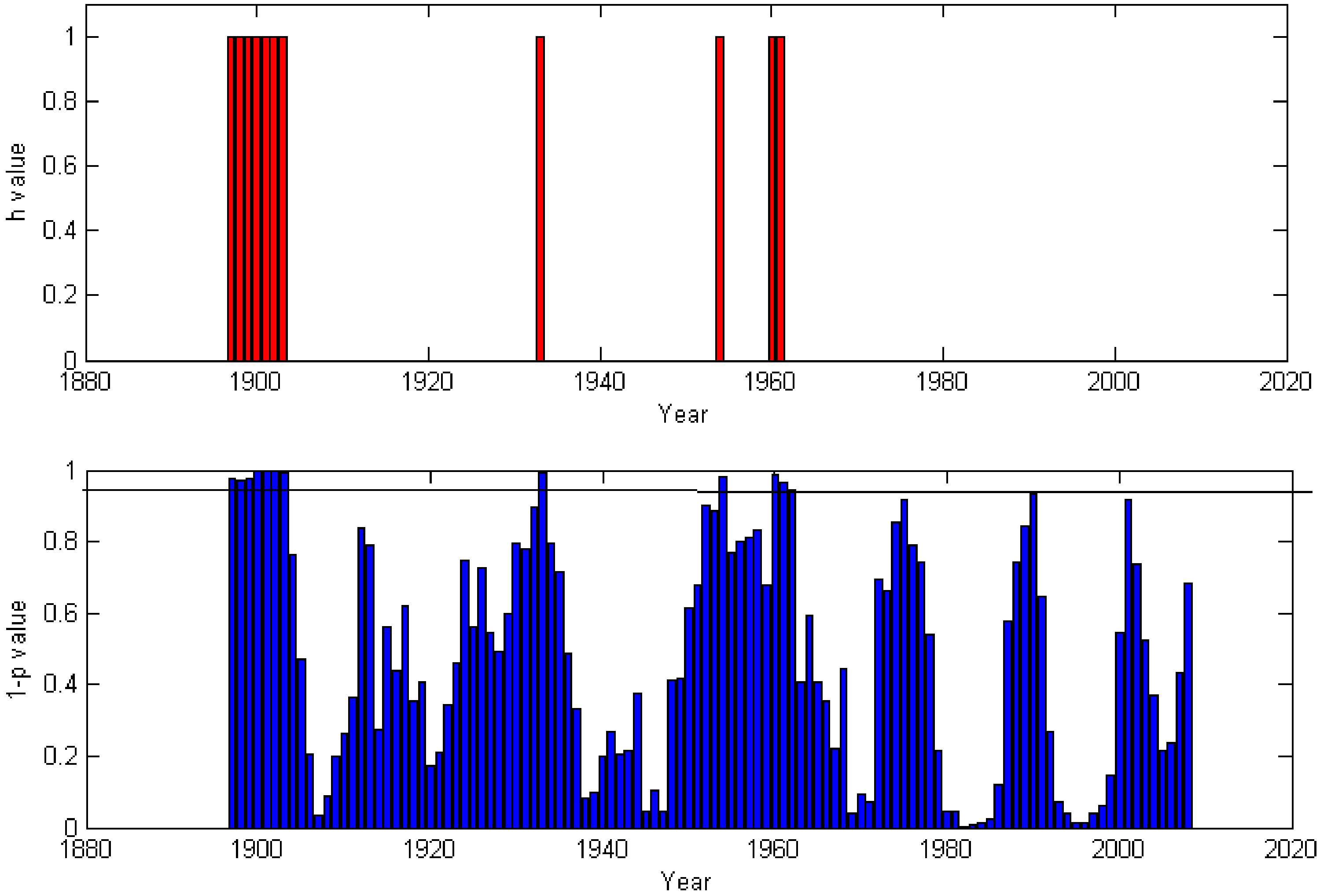
Which 5-yr samples would be considered a drought?



Left, Right, 2 Sided (both)

- Left- alternative hypothesis is drought
- Right- alternative hypothesis is flood
- 2 Sided- either drought or flood
- 2 sided tests are weaker and should be avoided
- [h,p,ci,stat]= ttest(valy,0,.05,'both')
- Sample value must be further from 0 (smaller p value) since α is smaller by 2 (2.5% in each tail)

Summary

- Research involves defining a testable hypothesis and demonstrating that any statistical test of that hypothesis meets basic standards
- Typical failings of many studies include:
  - (1) ignoring serial correlation in environmental time series that reduces the estimates of the number of degrees of freedom and
  - (2) ignoring spatial correlation in environmental fields that increases the number of trials that are being determined simultaneously.
    - Inflates the opportunities for the null hypothesis to be rejected falsely.
- Use common sense
- Be very conservative in estimating the degrees of freedom temporally and spatially
- Avoid attributing confidence to a desired result when similar relationships are showing up far removed from your area of interest for no obvious reason
- The best methods for testing a hypothesis rely heavily on independent evaluation using additional data not used in the original statitiscal analysis