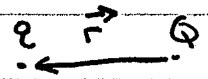
Basic Electrical Concepts

Force between two charges $Q\,(+)$ & $q\,(+\text{ or }-)$



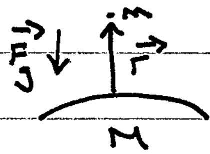
given by Coulomb's law:

$$\vec{F}_e = \frac{qQ}{4\pi\epsilon r^2}\frac{\vec{r}}{r} \quad \text{(newtons)} \qquad q \,\&\, Q \text{ in coulombs (amp-sec)}$$

$$\epsilon - \text{permitivity} = 8.85\times10^{-12}\ \frac{\text{coul}^2}{\text{newt } m^2} \quad \text{in free space}$$

parcels can be accelerated by electric force $\vec{a} = \frac{\vec{F}}{m}$ but really small compared to buoyancy force

Gravitational force analogous



$$\vec{F}_g = -\frac{GmM}{r^2}\frac{\vec{r}}{r}$$

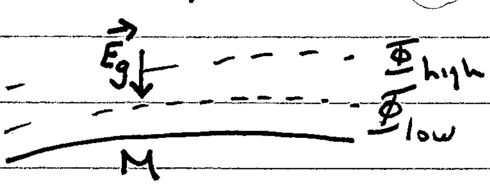
The gravitational force & mass of earth M generate a gravitational field. In the limit as mass of small object m approaches 0 then $\vec{E}_g = \lim_{m\to0}\frac{\vec{F}_g}{m} = \vec{g}$

Similarly, an electric field is generated by charge Q such that in the limit as charge of small object $q\to0$ then

$$\vec{E} = \lim_{q\to0}\frac{\vec{F}_e}{q} = \frac{Q}{4\pi\epsilon r^2}\frac{\vec{r}}{r} \quad \left(\frac{\text{newtons}}{\text{coulomb}}\right)$$

Finally, the gravitational field requires that lifting a mass away from the earth's surface requires work

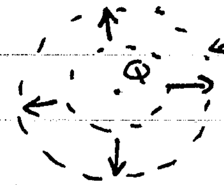
The geopotential Φ is the potential energy of a unit mass in a gravitational field - it's the work required to displace the mass a distance $d\vec{r}$. So $\Phi = \int_0^z \frac{\vec{F}_g}{m}\cdot d\vec{r} = gz$



Similarly, the electrostatic potential (or just potential) is the potential energy of a unit charge in an electric field. It's the work required to bring a charge from infinity to distance r against the field.

$$V \text{ (volts)} = \int_{\infty}^{r} \frac{Q}{4\pi\epsilon r^2}\frac{\vec{r}}{r}\cdot d\vec{r} = \frac{Q}{4\pi\epsilon r} \qquad \left(\text{Volt} = \frac{\text{newton m}}{\text{Coulomb}}\right)$$

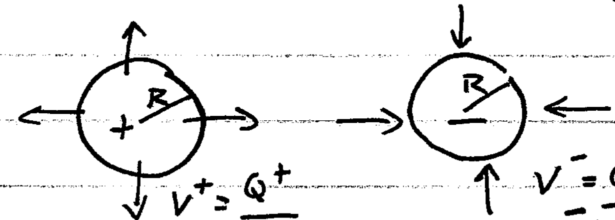
Alternatively $\vec{E} = -\frac{\partial V}{\partial r}\frac{\vec{r}}{r}$



The charge q can move laterally on potential surface without work

Now imagine 2 charged spheres of radius R that are far apart. They have equal but opposite charges

$Q = |Q^-| = |Q^+|$



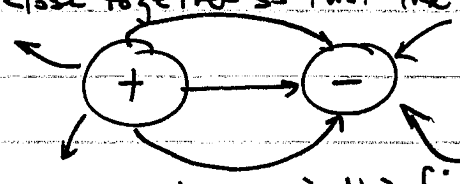
$V^+ = \frac{Q^+}{4\pi\epsilon R}$ $\qquad V^- = \frac{Q^-}{4\pi\epsilon R}$

If the spheres are really large, $R \to \infty$, then $V^+ = V^- = \emptyset$

But, if the spheres are smaller, then the potential difference between the two spheres is $\Delta V = V^+ - V^- = \frac{2Q}{4\pi\epsilon R}$ or $Q = 2\pi\epsilon R\,\Delta V$

We define the capacitance for a charged sphere as $C = 2\pi\epsilon R$, which is a measure of the ease with which charge can travel from 1 conductor to another

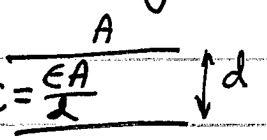
Imagine now that the 2 conducting spheres are close together so that the potential field of one interacts with the other.



The potential difference ΔV has been reduced considerably, as it takes less work to move a charge in this field

So $Q = C\Delta V$ since Q is fixed & ΔV is less, then C is greater

If we have two conducting plates, rather than spheres, then the capacitance is given as shown to the right



$$C = \frac{\epsilon A}{d}$$

Finally, remember from Ohm's law that charge flows at a rate proportional to the potential difference ΔV, i.e.,

$$V = \Delta V = R\frac{dQ}{dt} = RI$$

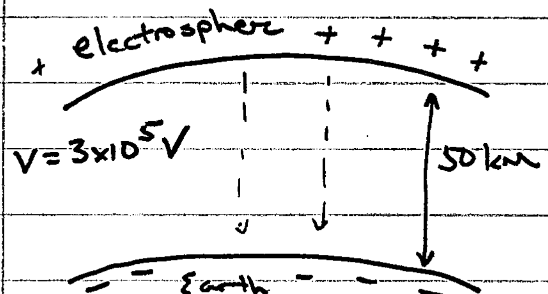
R - resistance (ohms)
I - current (amperes)

where hereafter we simply use V as the potential difference

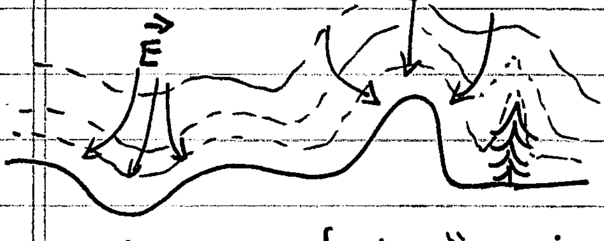
Conductivity is a measure of resistance to current per unit distance. The conductivity of a copper wire $\lambda = .6 \times 10^{8} \frac{1}{\text{ohm-m}}$

Fair Wx Electric Field

The atmosphere is conductive below 50km due to ions created by cosmic rays & natural radioactivity of the earth. Really tiny ions with short lifetimes (100s) are the primary contributors in the lower atmosphere. Above 60km, free electrons are present. There are fewer ions over the ocean than land. Pollution makes conductivity even worse. λ varies from $10^{-14} \frac{1}{\text{ohm-m}}$ at the surface to $10^{-11} \frac{1}{\text{ohm-m}}$ at 20km. Above 60km it is $10^{-9} \frac{1}{\text{ohm-m}}$. Obviously, air is a good insulator & a poor conductor compared to copper wire.



The potential difference between the electrosphere & the earth is $V = 3\times10^{5}$ V. Most of the voltage difference (drop) is in the lowest 20km where the electric field is large.



Remember that $E = \frac{dV}{dz}$

& that $\frac{dV}{dz}$ is largest near the ground

The presence of this "capacitor", i.e. two charged plates, leads to a current of about $4\times10^{-12} \frac{\text{amperes}}{m^2}$ from the electrosphere to the earth's surface.

So, the capacitance between the "plates" per unit area $C = \frac{\epsilon}{d}$

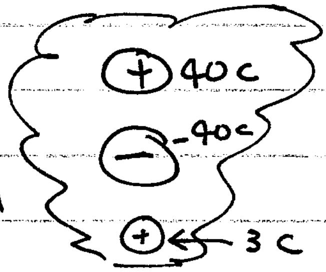
Then $Q = CV = \frac{\epsilon}{d} V = \frac{8.85 \times 10^{-12} \frac{\text{Coulomb}^2}{\text{Nt m}^2}}{50000 \text{ m}} * 3 \times 10^5 \text{ volts} \cong 5 \times 10^{-11} \frac{C}{m^2}$

How long can this charge last if the fair wx current of $4 \times 10^{-12} \frac{\text{amp}}{m^2}$ exists?

$\Delta t = \frac{\left(5 \times 10^{-11} \frac{C}{m^2} - 0\right)}{4 \times 10^{-12} \frac{C\text{-}s}{m^2}} \sim 10 \text{ sec}$. Actually it would take about 10 min

So, something must maintain the fair wx electric field or else it would be destroyed in a matter of minutes. Thunderstorms & lightning do this by adding negative charge to the surface & positive charge is presumed to leak from cloud tops to the electrosphere.

Charge Distribution in Cumulonimbus



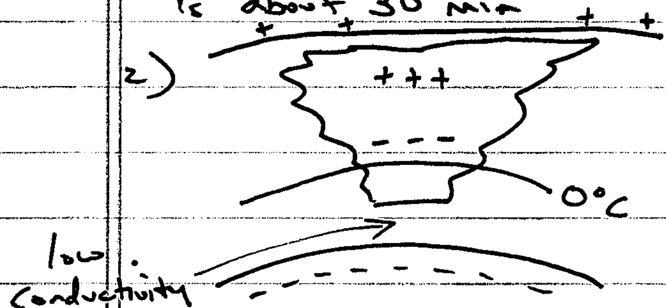
The idealized charge structure of clouds is as a vertical tripole. The positive charge near cloud base is much smaller than the other two.

The negative charge typically occurs around a temperature range of −7°C to −25°C (high based thunderstorms −7°C ; −16°C MCS ; −22°C supercells). There is the presumption that the colder temps are associated with stronger updrafts.

How the charge gets separated is still a matter of research. Any theory must account for following general statements:

1) average duration of ppt & electric activity from 1 thunderstorm is about 30 min

2)



Low conductivity of air (especially dry air) beneath Cb requires huge electric field strength to build up before it is possible for this field to be "broke down" by lightning

Breakdown field in clear dry air is higher 3000 kV/m & only 1000 kV/m in saturated air. BUT, maximum electric fields typically observed in clouds are around 100 kV/m

3) In large Cb, charge generated & separated in a volume bounded by -5 - -40°C within a radius of about 2 km

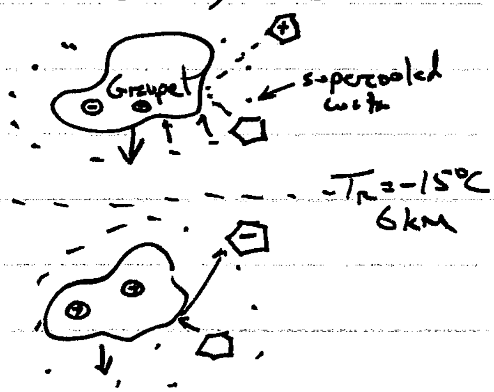
4) Negative charges centered between -10 to -20°C with positive charge several kms above. In mesoscale systems, center of negative charge may be closer to freezing level in trailing stratiform region

5) The Charge generation & separation process is closely associated with the development of precipitation. However, space charge centers appear to be displaced both vertically & horizontally from main precipitation region

6) Sufficient charge must be generated & separated to supply first lightning flash within 20 minutes of the appearance of precipitation particles detectable by radar

7) Charge separation is clearly associated with ice phase & presence of supercooled water in cloud

The dominant theory to explain charge separation in Cb's is the graupel-ice mechanism. The electric charges are produced by collisions between falling precipitation particles (graupel & hail) & small ice crystals in the updraft in the presence of supercooled water droplets. Charge separation appears sensitive to temperature such that above the "reversal" temperature T_R (~-10 to -20°C), the behavior is different. The supercooled water is important to develop significant charge transfer according to lab experiments.



Above T_R (high levels): graupel − ice +

Below T_R (low levels): graupel + ice −

Net result is that negative charge builds in middle of cloud, + charge carried aloft & to cloud base.

Charge simulated to be acquired by riming hail particle during collisions with small ice crystals