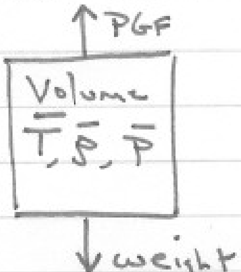
Convection

Fundamental to this course are the concepts associated with convection. Once those building blocks are established, then we can examine how convection becomes organized on the mesoscale.

Begin by performing a simple scale analysis. If a parcel accelerated upward on the order of $10\,m/s$ in 10 min or less then the acceleration $\frac{U}{t} \sim \frac{10\,m/s}{10\,\text{min}\; 60\,\frac{\text{sec}}{\text{min}}} \sim 10^{-2}\,m/s^2$

What are the forces acting on both the environment & a parcel rising at this rate?



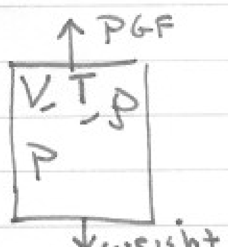
Environmental air of volume V has a downward force due to its weight $= V\bar{\rho}g$ where $\bar{\rho}$ is the enviromental density

The PGF acting upward is $= -V\frac{\partial \bar{p}}{\partial z}$

If we assume that the environment is in hydrostatic balance then

$$\bar{\rho}V\frac{d\bar{w}}{dt} = 0 = -V\frac{\partial \bar{p}}{\partial z} - V\bar{\rho}g \quad \text{or} \quad \frac{\partial \bar{p}}{\partial z} = -\bar{\rho}g$$

Now consider a parcel's volume that is displacing that environmental air



$$\rho V\frac{dw}{dt} = -V\frac{\partial p}{\partial z} - V\rho g$$

We don't make the assumption that the parcel's pressure exactly matches that of the environment so $p = \bar{p} + p'$ where p' is the deviation of the parcel's pressure from that of the surrounding environment

Then $\frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g$

$$\frac{\partial p}{\partial z} = \frac{\partial \bar{p}}{\partial z} + \frac{\partial p'}{\partial z} = -\bar{\rho} g + \frac{\partial p'}{\partial z}$$

or $$\frac{dw}{dt} = +\frac{\bar{\rho}}{\rho} g - \frac{1}{\rho}\frac{\partial p'}{\partial z} - g = -\frac{1}{\rho}\frac{\partial p'}{\partial z} + \frac{\bar{\rho}-\rho}{\rho} g \qquad \underline{4.1}$$

The first term on the right is the small but not insignificant acceleration that arises from pressure perturbations arising from the displacement of the environmental air by the parcel. We will return to this term a bit later.

The second term on the right is the buoyancy acceleration

$\underline{4.2}$ $b = \dfrac{\bar{\rho}-\rho}{\rho} g$ when the parcel is lighter than the environment

$\rho < \bar{\rho}$ $b+ \Rightarrow \dfrac{dw}{dt} + \Rightarrow$ accelerates up

$\rho > \bar{\rho}$ parcel heavier then $b-$ $\dfrac{dw}{dt}-$ acc. down

$\rho = \bar{\rho}$ neutral buoyancy - parcel in motion stays in motion

From the ideal gas law $p = \rho R_d T$; $\bar{p} = \bar{\rho} R_d \bar{T}$

If we ignore the small differences between p ; $\bar{p}$ then

$$\cancel{b = \frac{\frac{R_d \bar{T}}{\bar{p}} - \frac{R_d T}{\bar{p}}}{R}} \qquad b = g\,\frac{\frac{\bar{p}}{R_d \bar{T}} - \frac{\bar{p}}{R_d T}}{\frac{\bar{p}}{R_d T}} = \frac{T-\bar{T}}{\bar{T}} g \qquad \underline{4.3}$$

when the parcel is warmer than the environment $T > \bar{T}$ $b+$ $\dfrac{dw}{dt}+$

" " " " colder " " " $T < \bar{T}$ $b-$ $\dfrac{dw}{dt}-$

Since $\Theta = T\left(\dfrac{1000}{p}\right)^{R/c_p}$ $\bar{\Theta} = \bar{T}\left(\dfrac{1000}{\bar{p}}\right)^{R/c_p}$; if $p \sim \bar{p}$

Then $b = \dfrac{\Theta - \bar{\Theta}}{\bar{\Theta}} g$ $\underline{4.4}$

If parcel's potential temperature greater than that of the environment then $b + \Rightarrow \dfrac{dw}{dt}+$ accelerates upward

Buoyancy acceleration for a moist parcel

The total pressure of the parcel p equals the sum of the partial pressures of dry air & water vapor

$$p = p_d + e = \rho_d \frac{R^*}{m_d} T + \rho_v \frac{R^*}{m_v} T \qquad \underline{5.1}$$

where R^* - universal gas constant m_d - molecular weight of dry air $\cong 29$
m_v = molecular weight of $H_2O \cong 18$

We consider a fixed volume, so when we replace a molecule of dry air with one of water vapor, the density of the mixture (ρ) decreases. $\rho = (M_d + M_v)/V$ &

$$\rho_d = \frac{M_d}{V} \quad \& \quad \rho_v = \frac{M_v}{V}$$ where M_d & M_v are the masses of dry air & H_2O_v respectively

Since w = mixing ratio $= \frac{M_v}{M_d} = \frac{\rho_v}{\rho_d}$ then $\underline{5.1}$ becomes

$$p = \frac{R^* T}{V}\left[\frac{M_d}{m_d} + \frac{M_v}{m_v}\right] = \rho R^* T \left[\frac{M_d}{m_d} + \frac{M_v}{m_v}\right]\left(\frac{1}{M_d + M_v}\right)$$

Since $R_d = \frac{R^*}{m_d}$ then $p = \rho R_d T \dfrac{\left[M_d + M_v \frac{m_d}{m_v}\right] \frac{1}{M_d}}{1 + \frac{M_v}{M_d}}$

define $\epsilon = \frac{m_v}{m_d} = .62$ & $w = \frac{M_v}{M_d}$

then $p = \rho R_d T \dfrac{\left[1 + \frac{w}{\epsilon}\right]}{1 + w} \sim \rho R_d T \left[1 + .61 w\right] \qquad \underline{5.2}$

This is the ideal gas law for a moist parcel of air with density ρ & temperature T

Define $T_v = T\left[1 + .61 w\right]$ = virtual temperature

Then $p = \rho R_d T_v$ for a moist parcel $\qquad \underline{5.3}$

T_v is the temperature of dry air having the same density as a sample of moist air at the same pressure

Physically, the more water vapor present, then the lighter the volume. Then a dry parcel with the same density must have a higher temperature

Since the vertical momentum equation for a moist parcel is

$$\frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g$$

Then, we consider the impact of water vapor by using the virtual temp

$$b = \frac{\bar{\rho} - \rho}{\rho} g \doteq \frac{T_v - \bar{T}_v}{\bar{T}_v} g = \frac{\theta_v - \bar{\theta}_v}{\bar{\theta}_v} g \qquad 6.1$$

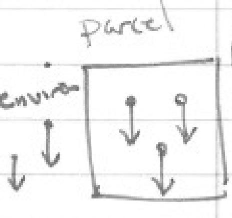
where θ_v - virtual potential temperature

Or, consider water vapor as a correction to the "dry" buoyancy

$$b = \frac{T_v - \bar{T}_v}{\bar{T}_v} g = \frac{T[1+.61\omega] - \bar{T}[1+.61\bar{\omega}]}{\bar{T}[1+.61\bar{\omega}]} g \cong \underbrace{\frac{T-\bar{T}}{\bar{T}}}_{\text{dry buoyancy}} + \underbrace{.61[\omega - \bar{\omega}]}_{\text{water vapor correction}}$$

If the water vapor mixing ratio of the parcel exceeds that of the environment, then the buoyancy of the moist air is larger than that if the water vapor is absent

Effects of precipitation on parcel buoyancy



Adding condensate adds a downward force equal to the weight of the condensate inside the parcel as well as outside it

M_c - mass condensate inside parcel
$\bar{M}_c$ - mass condensate in environment

Then $\bar{\rho}_T = \dfrac{\bar{M} + \bar{M}_c}{V}$; $\rho_T = \dfrac{M + M_c}{V}$ where $\bar{M}$, M are the masses of the "air"

Since $b = \dfrac{\bar{\rho}_T - \rho_T}{\rho_T} g \sim \dfrac{\bar{\rho} - \rho}{\rho} g + \bar{\omega}_c - \omega_c$

where $\bar{\omega}_c$ - condensate loading in environment $\left(\dfrac{\text{kg condensate}}{\text{kg dry air}}\right)$

ω_c - condensate loading in parcel $\left(\frac{\text{kg condensate}}{\text{kg dry air}}\right)$

Hence the more condensate in the parcel relative to that in the environment, then the greater reduction in buoyancy

Summary

Buoyancy acceleration can be considered to have 3 parts

$$b = \frac{T-\bar{T}}{\bar{T}}g + .61[\omega-\bar{\omega}] + \bar{\omega}_c - \omega_c$$

temperature of dry air effect — water vapor effect — hydrometeor effect

So parcel experiences acceleration upward when:

(1) parcel is lighter/warmer than environment
(2) parcel has more water vapor " " "
(3) parcel has fewer hydrometeors " " "

$$\boxed{\begin{array}{c} T > \bar{T} \\ \rho < \bar{\rho} \\ \omega > \bar{\omega} \\ \omega_c < \bar{\omega}_c \end{array}}$$

Remember: ω's here are mixing ratios NOT vertical velocities

CAPE & CIN

Convective Available Potential Energy (CAPE) is a measure of the maximum kinetic energy that an unstable parcel can acquire neglecting nonadiabatic effects such as precipitation loading, entrainment, & phase changes

Convective INhibition (CIN) - measure of kinetic energy required to lift air to level of free convection (LFC). It is a measure of strength of cap or lid that inhibits convection

From 6.1, $\frac{dw}{dt} = \frac{T_v - \bar{T}_v}{\bar{T}_v} g = \frac{\theta_v - \bar{\theta}_v}{\bar{\theta}_v} g$

To be positively buoyant, $T_v > \bar{T}_v$, i.e. the parcel is warmer than environment

Assume that as the parcel ascends, that the local vertical velocity remains unchanged, i.e. steady state. Then $\frac{\partial w}{\partial t} = 0$

Further, neglect horizontal motion since the parcel is ascending rapidly. Then

$$\frac{dw}{dt} = \underbrace{\cancel{\frac{\partial w}{\partial t}}}_{0} + \underbrace{\cancel{u}}_{0}\frac{\partial w}{\partial x} + \underbrace{\cancel{v}}_{0}\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = g\frac{[T_v - \bar{T}_v]}{\bar{T}_v}$$

$$d\left[\tfrac{1}{2}w^2\right] = d\left[\begin{matrix}\text{kinetic energy}\\ \text{due to vertical}\\ \text{motion}\end{matrix}\right] = g\int_{LFC}^{EL} \frac{T_v - \bar{T}_v}{\bar{T}_v}\, dz \equiv CAPE$$

Begin from LFC & continue to equilibrium level (EL)

$$\int_{w_{LFC}}^{w_{EL}} d\left[\tfrac{1}{2}w^2\right] = CAPE \Rightarrow \tfrac{1}{2}w_{EL}^2 - \tfrac{1}{2}\underbrace{w_{LFC}^2}_{0\text{ small}} = CAPE$$

So $w_{EL} = \sqrt{2\,CAPE}$ = maximum velocity likely at top of cloud. Entrainment typically cuts w_{EL} by factor of 2 or more

$$\int_{W_{SFC}}^{W_{LFC}} d\left[\frac{1}{2}w^2\right] = \cancel{-}\,CIN = g\int_{SFC}^{LFC} \frac{T_v - \overline{T_v}}{\overline{T_v}}\,dz$$

Since parcel will decelerate as rises $\Rightarrow w_{LFC} \sim 0$

Then $-\frac{1}{2}w_{SFC}^2 = \cancel{-}\,CIN$ or $w_{SFC} = \sqrt{-2\,CIN}$ - velocity required to lift parcel to LFC

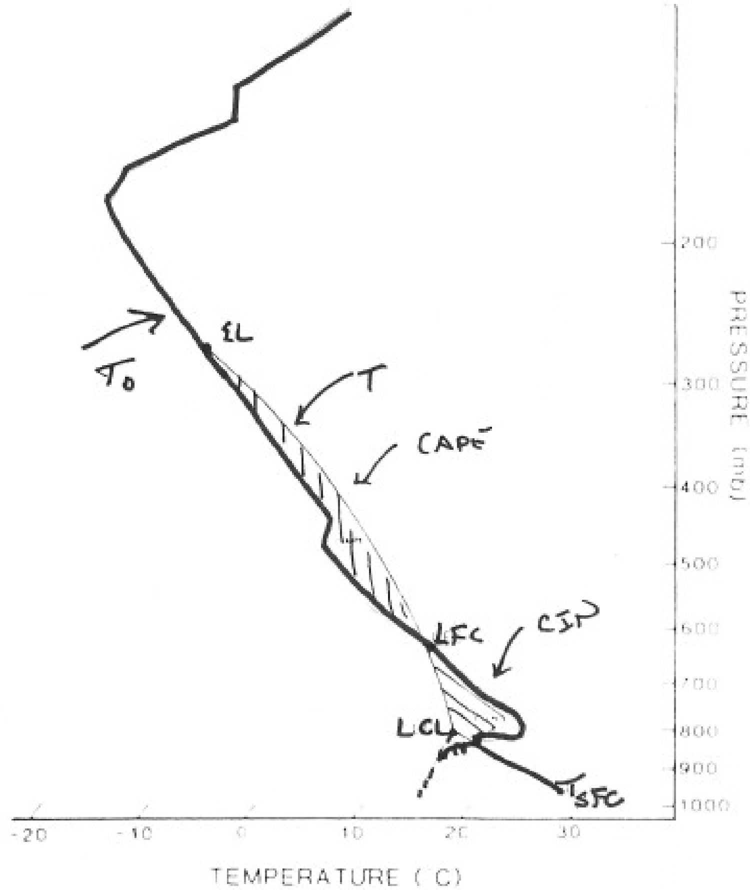


Figure 3.7 Idealized illustration of convective available potential energy (CAPE) and convective inhibition (CIN). Skewed abscissa and logarithmic ordinate are temperature (°C) and pressure (mb), respectively. Surface temperature (T_0), lifting condensation level (LCL), level of free convection (LFC), equilibrium level (EL). Temperature profile (thin solid line), dew point in the moist, well-mixed boundary layer (dashed line); line of constant saturation water-vapor mixing ratio (dotted line). CAPE proportional to the area (hatched area) formed by the temperature curve and the moist adiabat passing through the LFC (thick solid line); thick solid line between the surface temperature and the LCL represents a dryadiabat. CIN is proportional to the area (hatched area) formed by the temperature curve and (above the LCL) the moist adiabat passing through the LCL and (below the LCL) the dry adiabat connecting the LCL to T_0 (thick solid line). In this example the lapse rate between the surface and a little below the LCL is dryadiabatic (after Bluestein Vol. 2, 1993)

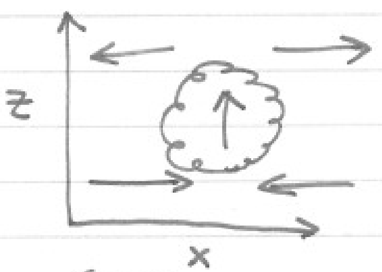
For CAPE $= 2500 \frac{J}{kg} = 2500 \frac{m^2}{s^2}$ typical severe storm $\Rightarrow w_{EL} \sim 70$ m/s

CIN $= 200 \frac{J}{kg} = 200 \frac{m^2}{s^2}$ typical cap $\Rightarrow w_{SFC} = 20$ m/s

Maximum CAPE is $5000 - 7000 \frac{J}{kg}$ but in West much less as result of higher surface ~~&~~ less buoyancy

Parcel pressure perturbation

Buoyancy can't exist without simultaneous disruption of the mass (& pressure) field. As a buoyant plume rises, it pushes air laterally away near the top & sucks air back in below in its wake as shown below:



Let's treat the problem in 2-D

$$\frac{du}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} \qquad \underline{11.1}$$

$$\frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g \qquad 11.2$$

As before (4.1) & assuming the environment is at rest (with no lateral pressure gradients), then

$$\frac{du}{dt} = -\frac{1}{\rho}\frac{\partial p'}{\partial x} \quad \& \quad \frac{dw}{dt} = b - \frac{1}{\rho}\frac{\partial p'}{\partial z}$$

Using the anelastic assumption that $\rho = \bar{\rho} = f(z)$ only in terms other than the buoyancy term, then

$$\frac{du}{dt} = -\frac{1}{\bar{\rho}}\frac{\partial p'}{\partial x} \qquad \frac{dw}{dt} = b - \frac{1}{\bar{\rho}}\frac{\partial p'}{\partial z} \qquad \underline{11.3}$$

& the continuity equation $\frac{\partial}{\partial x}\bar{\rho}u + \frac{\partial}{\partial z}\bar{\rho}w = 0$ $\underline{11.4}$

Taking $\frac{\partial}{\partial x}\bar{\rho}\frac{du}{dt} = -\frac{\partial^2 p'}{\partial x^2}$ & $\frac{\partial}{\partial z}\bar{\rho}\frac{dw}{dt} = \frac{\partial}{\partial z}\bar{\rho}b - \frac{\partial^2 p'}{\partial z^2}$

Taking another inelegant step (there are better ways to do it), then

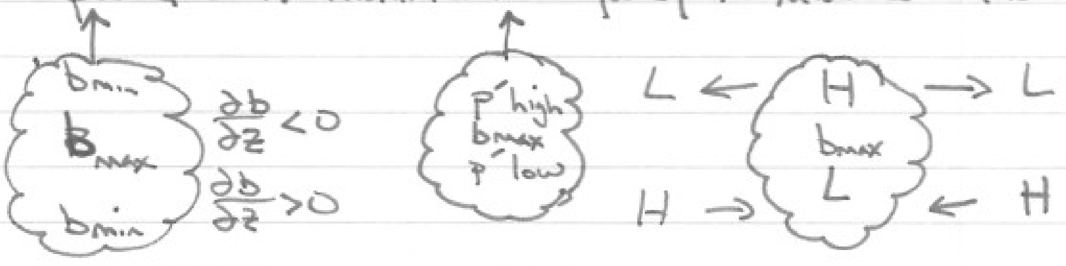
$$\frac{d}{dt}\Big(\underbrace{\frac{\partial \bar{\rho}u}{\partial x} + \frac{\partial \bar{\rho}w}{\partial z}}_{0 \text{ from continuity}}\Big) = \frac{\partial}{\partial z}\bar{\rho}b - \frac{\partial^2 p'}{\partial x^2} - \frac{\partial^2 p'}{\partial z^2}$$

Then $\frac{\partial^2 p'}{\partial x^2} + \frac{\partial^2 p'}{\partial z^2} = \frac{\partial}{\partial z}\bar{\rho}b$

Since the left hand side is the laplacian of p', then a crude sinusoidal assumption is that the LHS $\sim -p'$

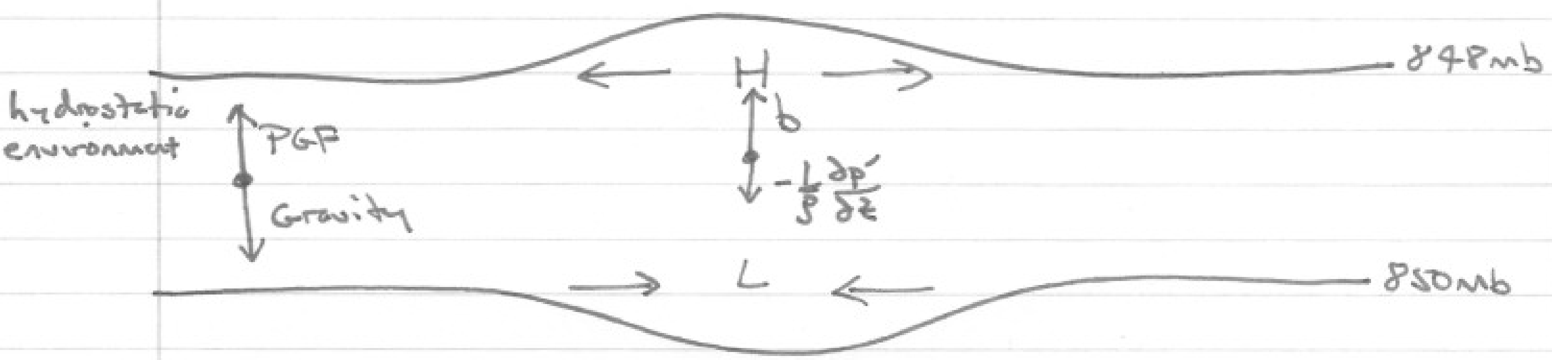
So $p' \propto -\frac{\partial \bar{\rho} b}{\partial z}$ 12.1

Imagine a parcel with maximum buoyancy in middle of the parcel



The vertical gradients in buoyancy within the parcel define the perturbation pressures inside the parcel. Then, through continuity, the pressure perturbation outside the parcel become established. Note that the pressure perturbation tend to oppose the buoyancy, hence weakening it.

Now viewing the buoyant plume in the context of the total environment:



Upward buoyancy acceleration is counteracted to some degree by downward perturbation pressure gradient force. This must occur because some of buoyancy is used to push environmental air out of the way

As parcel gets broader, then must push more air away laterally $\Rightarrow$ less buoyancy so becomes closer to hydrostatic

$\therefore$ convective plume broader than deep tends to be quasi-hydrostatic
" " narrower than deep $\Rightarrow$ nonhydrostatic;
perturbation vertical pressure gradient force can't counteract buoyancy