Read Chptrs 1-3 of text

ER1

Electromagnetic Radiation

$c = f\lambda =$ speed of light $= 3 \times 10^8$ m/s
f - frequency (hz - cycles/sec)
λ - wavelength (m)

Commercial power $f = 60$ Hz $\lambda = 5000$ km low freq, long λ
Visible light $f = 6 \times 10^{14}$ Hz $\lambda = .5\ \mu m$ high freq, short λ

WSR-88D radar $f = 2.85$ GHz S Band $\lambda = .1 m \cong 10$ cm

Radars send out series of pulses, each of which contain many cycles. The pulse repetition frequency - PRF - for the WSR-88D can be varied by the operator as a function of weather condition from 318 Hz to 1304 Hz

The volume coverage pattern (VCP) defines the characteristics of the operational mode of the WSR-88D. Low PRF is used to get a broad indicator of wx conditions while high PRF is used when severe wx is more likely. Will come back to the various standard VCP's later.

The pulse duration (τ) in the case of the WSR-88D is either 1.57 μs or 4.5 μs. Then the pulse length (h) where $h = c\tau$ is $h_s \cong .5$ km (1.57 μs) or for $\tau = 4.5\ \mu s$ then $h_e \cong 1.5$ km. So in low PRF mode, then the number of waves transmitted in a single pulse is $2.85 \times 10^9 \frac{\text{cycles}}{\text{sec}}\ 1.57 \times 10^{-6}$ sec or $\sim 4.5 \times 10^3$ cycles

Antenna Properties

Radars send pulse out along a beam. If a radar sent equal energy in all directions, then it would be an isotropic antenna system

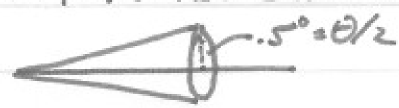
$$\text{Gain} = g = \frac{\text{power on beam axis}}{\text{power } \cancel{\text{in}} \text{ same direction from isotropic antenna}} = \frac{P_b}{P_{iso}}$$

Radar power is focussed so strongly along the beam axis that convention is to express the gain in log base 10. So

$$\text{Gain} = G = 10 \log_{10}\left[\frac{P_b}{P_{iso}}\right] \quad \text{in dB (decibels)}$$

The gain of the WSR-88D is 45db[5], which means that the power along the beam axis is 31622 times that of an isotropic antenna

The gain is related directly to the beamwidth of the antenna, θ, in degrees



$$g = \frac{180^2}{\theta^2}$$

So, the beam width of the WSR-88D is around 1° (actually specified as .93°)

While the radar is designed to send all of the energy in a particular direction some energy will be transmitted along other angles (sidelobes). See the reading for details

Refractivity

Radiation travels slower through media. Sharp changes in the density of the media can ~~be~~ cause the radiation to bend up or down

$$n = \text{refractive index} = \frac{c}{v} = \frac{\text{speed of radiation in vacuum}}{\text{speed of radiation in medium}}$$

$n \geq 1$

In the atmosphere, $n \sim 1.0003 \pm .0001$ in lower atmosphere so radiation travels .03% slower or so than in a vacuum

Define N = refractivity $= (n-1)10^6$ (more convenient)

n & N usually decrease with height but depends on density & water vapor

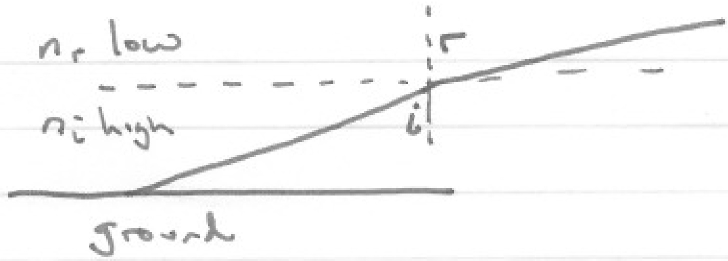
$$N = 77.6\frac{P}{T} + 3.73 \times 10^5 \frac{e}{T^2} \quad (p, e \text{ in mb}) \; T - K$$

Since $\frac{P}{T} \propto \rho$ then $N \propto$ density & water vapor content

Radar propagation is strongly dependent on the gradient of refractivity $\frac{\partial N}{\partial z}$, which is typically around $\frac{-39}{\text{km}}$

Sharp changes in density, either in the vertical or horizontal, can cause the direction of the beam to change

Snell's law defines how a radar will bend through the interface between two media with different refracitivity



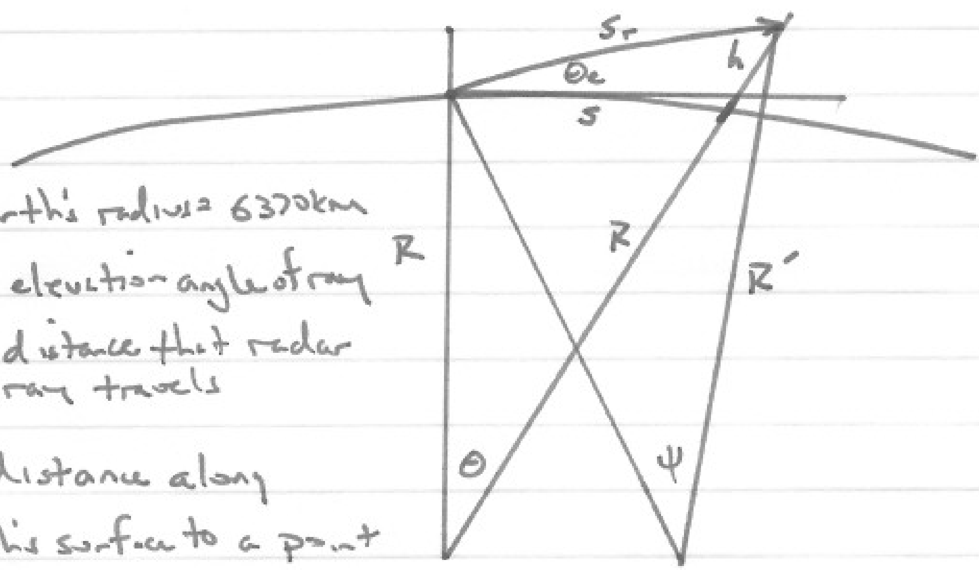
$$\frac{v_i}{v_r} = \frac{\frac{c}{n_i}}{\frac{c}{n_r}} = \frac{n_r}{n_i} = \frac{\sin i}{\sin r}$$

where i - incident angle measured from the ⊥ to the interface
r - angle of refraction

For $n_i > n_r$ then $r > i$ which means the ray bends down. The faster speeds normally aloft tend to cause the ray to bend down

Impact of Earth's Curvature

While the radar beam is directed along a straight radial, the earth's curvature causes the ray to appear to curve up relative to the earth's surface. The geometry is a bit complicated but can be summarized as follows:



R - earth's radius = 6370 km
θ_e - elevation angle of ray
s_r - distance that radar ray travels
s - distance along earth's surface to a point directly below ray
h - height of ray above surface
R' - effective earth radius for radar ray

ERS

Consider first the distance of the ray as a function of distance s on the earth's surface. Then $s = R\theta$

If you travelled completely around the earth ($\theta = 2\pi$) then the distance travelled would be $s = 2\pi R$

The curvature of the earth's surface $C = \frac{1}{R} = \frac{\theta}{s}$

The following requires a lot of geometry & algebra but the curvature of a radar ray depends both on the curvature of the earth & the change of refractivity with height

$$C_{ray} = \frac{1}{R} + \frac{\partial n}{\partial z} = \frac{1}{R'} = \frac{\psi}{s_r}$$

Since $\frac{\partial N}{\partial z} = \frac{-39}{km}$ under "normal" conditions

Then since $n = \frac{N}{10^6} + 1$ $\quad \frac{\partial n}{\partial z} = -39 \times 10^{-6} \frac{1}{km}$

& $\frac{1}{R} = \frac{1}{6370\ km} = 1.57 \times 10^{-4} \frac{1}{km}$ so $C_{ray} = 1.18 \times 10^{-4} \frac{1}{km}$

or $R' = 8474\ km = 1.33 R = 4/3 R$

So, normally a radar ray will have a path with radius larger than that of the earth

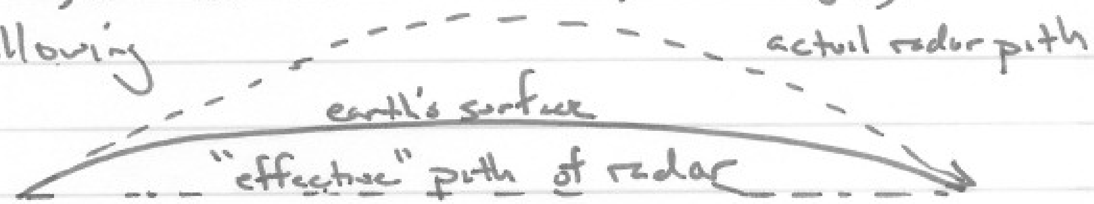
The most extreme case would be when the radar ray follows a straight path, then $R' \to \infty$ & $\frac{1}{R'} = 0$

That could only happen when $\frac{\partial n}{\partial z} = -\frac{1}{R} = -1.57 \times 10^{-4} \frac{1}{km}$

or $\frac{\partial N}{\partial z} = -157 \frac{1}{km}$

This happens when the density decreases more sharply with height than normal and/or water vapor decreases sharply with height

Now, the text handles this very confusingly, so consider the following



In this case, the radar beam is sharply bent downwards (superrefraction) since the speed aloft is faster than that near the ground. The net result is that the "effective" path of the radar is a straight line so $\frac{1}{R'} = 0$

So, the beam is NOT following the earth's surface. That would be when $\frac{1}{R'} = \frac{1}{R}$ or $\frac{\partial n}{\partial z} = 0$

To recap, normally a radar beam is bent downwards due to vertical density variations so that ~~but~~ because the earth's surface is curved, the effective curvature is less (or radius of radar beam is broader).

When the radar beam is bent more towards the surface, then it is experiencing superrefraction (even stronger vertical density variations)

When the radar beam is bent less towards the surface, then it is experiencing subrefraction (less strong vertical density variations)

Superrefraction

Superrefraction can be a serious problem, because it can enhance ground clutter & lead to "anomalous propagation", AP, which is extended range detection of the ground (or cloud) features. This can be quite a problem when strong surface inversions are present in clear weather. Ducting is when the radar beam becomes trapped in a layer due to superrefraction & causes a propagation path to be maintained.

How is it possible to get $\frac{\partial N}{\partial z} = \frac{-300}{km}$, which can happen?

I'm going to go into this in gory detail, because we'll use similar approaches later to examine sensitivities to parameters.

First, $N = 77.6\frac{p}{T} + 3.73\times10^5 \frac{e}{T^2}$

$$\log N = \log 77.6 + \log p - \log T + \log 3.73\times10^5 + \log e - \log T^2$$

$$\frac{1}{N}\frac{\partial N}{\partial z} = \frac{1}{p}\frac{\partial p}{\partial z} - \frac{1}{T}\frac{\partial T}{\partial z} + \frac{1}{e}\frac{\partial e}{\partial z} - \frac{1}{T^2}\frac{\partial T^2}{\partial z}$$

Assume the atmosphere is hydrostatic. Then $\frac{\partial p}{\partial z} = \frac{100\,\partial p'}{1000\,\partial z'} = -100\rho g$
(p' in Pa) p in mb (z' in m) z in km $= \frac{-100}{1000}\frac{p'}{R_d T} g = \frac{-p}{10^{-1} R_d T} g$

$$\frac{1}{N}\frac{\partial N}{\partial z} = \frac{-g}{10^{-1} R_d T} + \frac{1}{e}\frac{\partial e}{\partial z} - \frac{3}{T}\frac{\partial T}{\partial z}$$

effect of density decrease with height in hydrostatic atm

water vapor effect

temp effect

$\frac{9.8}{287}$

Let's plug in some representative numbers & assume that

$\frac{\partial e}{\partial z} \sim 0$ $p = 850\,mb$ $e = 10\,mb$ $T = 300K$ $\frac{\partial N}{\partial z} = \frac{-300}{km}$

Then $N = 219.9 + 41.4 = 261.3 \frac{1}{km}$

$$\frac{\partial T}{\partial z} = \frac{1}{3}\left[-\frac{g}{R_d} - \frac{T}{N}\frac{\partial N}{\partial z}\right]$$

$$\sim \frac{1}{3}\left[\underbrace{\frac{-10}{1000\,287}}_{\text{negligible}} + \frac{300}{261.3}\,\frac{300}{km}\right]$$

$\sim 115 \frac{°C}{km}$ which seems really high but it is not uncommon to have $\sim \frac{10°C}{100m}$ near the surface

So it takes a really strong inversion to cause superrefraction! Having moisture decrease with height helps but moisture decreases by themselves would have to be pretty big as well

$$\frac{1}{N}\frac{\partial N}{\partial z} \sim \frac{1}{e}\frac{\partial e}{\partial z} \Rightarrow \frac{\partial e}{\partial z} \sim \frac{e}{N}\frac{\partial N}{\partial z} = \frac{10mb\left(\frac{-300}{km}\right)}{261\frac{1}{km}}$$

$$\sim -11.4 \frac{mb}{km}$$

So, the best scenario is to have temperature increase sharply with height & moisture decrease sharply as well