Review

Physical Dimensions & Units

SI	Base	Units
Length	meter	m
mass	kilogram	kg
Time	seconds	s
Temperature	Kelvin	K

Conversions

1 nm = 1.15 mi = 1.85 km

1 kt = .5144 ms^{-1}

1 ms^{-1} = 1.944 kt

Derived Units

Frequency	Hertz	Hz	cycles/sec
Force	Newton	N	kg m/s^2
pressure	Pascal	Pa	$\frac{N}{m^2} = \frac{kg}{ms^2}$ 1 mb = 100 Pa
Energy	Joule	J	$Nm = kg/s^2$
Power	Watt	W	J/s
Angular velocity			rad/s

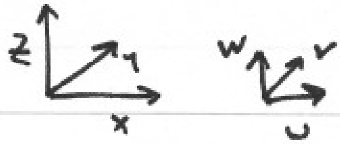
Ideal gas law

$p = \rho R_d T$

$R_d = 287 \frac{J}{kg\,K}$ - gas constant for dry air

ρ = density = $\frac{mass}{Volume}$ $\frac{kg}{m^3}$

Fluid motion



Eulerian perspective

$$\frac{\partial(\ )}{\partial t}$$

Lagrangian perspective

$$\frac{d(\ )}{dt} \qquad \frac{d}{dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

$$\frac{\partial(\ )}{\partial t} = 0$$ steady state

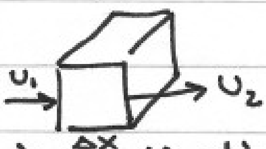
$$\frac{d(\ )}{dt} = 0$$ conservative field

Conservation Laws

1) Conservation of mass (per unit volume)

$$\frac{d\rho}{dt} = -\rho\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right] \quad \text{or} \quad \frac{\partial \rho}{\partial t} = -\frac{\partial \rho u}{\partial x} - \frac{\partial \rho v}{\partial y} - \frac{\partial \rho w}{\partial z}$$

Divergence
~~Flow~~ Contraction or expansion of volume



$$\frac{\partial u}{\partial x} \sim \frac{u_2 - u_1}{\Delta x}$$

2) Conservation of momentum (per unit volume)

$$\frac{du}{dt} = fv - \frac{1}{\rho}\frac{\partial p}{\partial x} - ku$$

k ~ linear dissipation coefficient $\sim 3\times 10^{-5}\ \frac{1}{s}$

$$\frac{dv}{dt} = -fu - \frac{1}{\rho}\frac{\partial p}{\partial y} - kv$$

$$\frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - kw - g$$

Coriolis force | Pressure gradient force | Friction | Gravity

3) Conservation of Energy (per unit volume)

$$\rho \frac{d}{dt}\left[KE + PE + IE\right] = \begin{matrix}\text{rate at which}\\ \text{work done}\\ \text{by forces}\end{matrix} + \begin{matrix}\text{diabatic}\\ \text{heating}\end{matrix} + \begin{matrix}\text{Frictional}\\ \text{effects}\end{matrix}$$

KE - kinetic energy $= \frac{1}{2}\left[u^2 + v^2 + w^2\right]$

PE - potential energy $= gz$

IE - internal energy $= c_v T$ where c_v - specific heat at constant volume $= 717 \frac{J}{kg\,K}$

Work = Force · distance

Rate at which work done = Force · $\frac{\text{Distance}}{\text{Time}}$ ~ Force · velocity

pressure · velocity = $\frac{\text{rate of work}}{\text{unit area}}$ ~ $pu + pv + pw$

$$c_p \frac{d\ln T}{dt} - R\frac{d\ln p}{dt} = \frac{\dot{Q} + \delta}{T}$$

$\dot{Q}$ - diabatic heating
δ - frictional dissipation

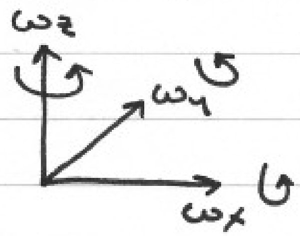
c_p - specific heat at constant pressure $= 1004 \frac{J}{kg\,K}$

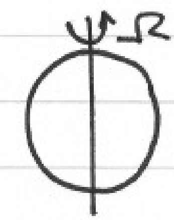
Adiabatic process - no exchange of energy between parcel (volume) & environment $\Rightarrow \dot{Q} \equiv 0$

Define Θ = potential temperature $= T\left(\frac{1000}{p}\right)^{R/c_p}$ then $c_p \frac{d\ln\Theta}{dt} = \frac{\dot{Q} + \delta}{T}$

For adiabatic process (& ignoring friction): $\frac{d\Theta}{dt} = 0$

ω = vorticity - measure of rotation/spin of fluid parcel about center





Earth spins about vertical axis at angular velocity $\Omega = 7.292 \times 10^{-5} \frac{1}{s}$

f - planetary vorticity $= 2\Omega \sin\phi$ $\sim 10^{-4} \frac{1}{s}$ in mid-latitudes

ζ = ~~relative~~ vorticity of fluid on earth, relative to earth's motion, in z direct

$\eta = \zeta + f$ = absolute vorticity in z direction $= \zeta + f$

[redacted]

Moisture parameters

e_s - saturation vapor pressure (Pa or mb) $e_s = f(T)$

e - vapor pressure of water vapor (Pa or mb) $e = f(T_d)$

w = mixing ratio - $\frac{\text{mass of water vapor}}{\text{mass of dry air}}$ (dimensionless or g/kg)

ρ_v - absolute humidity (kg/m^3)

$T_v = T + \frac{w}{6}$ = virtual temperature = temperature at which dry air would have same density as moist air at same pressure (w in $\frac{g}{kg}$)

w_s - saturation mixing ratio $= \frac{\text{maximum mass of water vapor for given temperature}}{\text{mass of dry air}}$

Td - dewpoint - temperature to which air cooled isobarically becomes saturated

Tw - wet bulb - temperature to which a parcel at constant pressure cools through evaporation of water into it