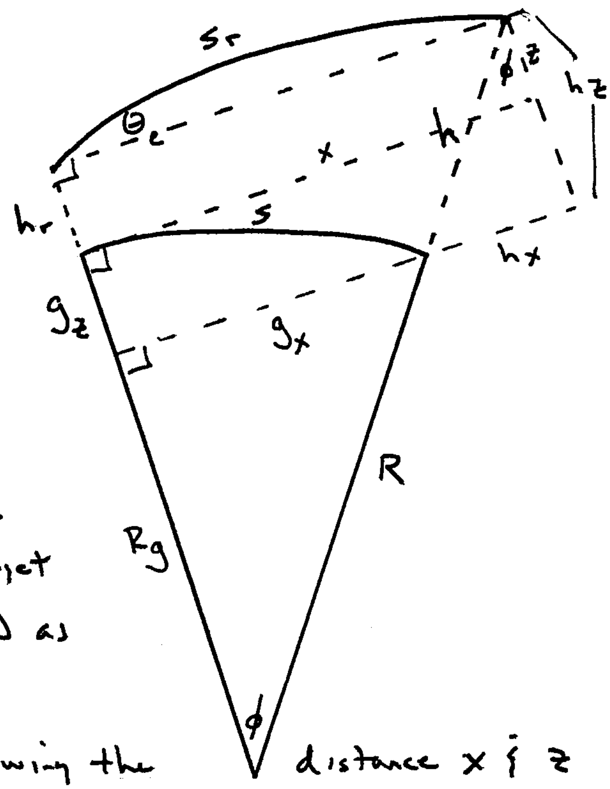
ERS

The standard RHI displays ignore the perspective of the curvature of the earth, which leaves the impression that radar rays tilt strongly upwards when they actually don't. The radar returns 2 pieces of information: for a ray sent at an elevation angle of θ_e; the distance from the target to the radar is s_r.



A standard RHI displays h (the height of the target above the earth's surface) as a function of θ_e & s_r.

We are interested in knowing the distance x & z relative to the ground beneath the radar. So, let's consider the geometry of the situation beginning from the texts eqn 3.12:

$$h(s_r, \theta_e) = \sqrt{s_r^2 + R'^2 + 2 s_r R' \sin\theta_e} - R' + h_r$$

where $R' = k_e R$ & $k_e = 4/3$ for the standard atmosphere

k_e increases if we have a strong inversion & greater potential for superrefraction. So k_e depends on the stability & moisture profile.

h_r - distance above the surface of the radar (i.e. the height of Prom Point above the valley floor)

With some geometry, it is possible to define the distance on the earth's surface directly beneath the target:

$$s(s_r, \theta_e) = R' \sin^{-1}\left[\frac{s_r \cos\theta_e}{R' + h}\right]$$

The angular distance ϕ between the radar & the spot directly under the target is $\phi = \frac{s}{R}$

So, the distance g_x between the radar & the point on the ground under the target is $g_x = R \sin\phi$

The projection of the horizontal distance of the target above ground, h_x, is $h_x = h \sin\phi$

So, the total distance horizontally from the radar to the target is

$x = g_x + h_x$

Since $R_g = R\cos\phi$, then the vertical distance between the ground point below the target & the ground under the radar is $g_z = R - R_g = R(1-\cos\phi)$

Since $h_z = h\cos\phi$, then $z = h\cos\phi - g_z$

In Assignment 5, you will plot $h(s_r, \theta_e)$ vs plotting $z(s_r, \theta_e)$ to see the different perspectives.

Radar Equation

I won't go through all the gory details, but will hit a number of critical points.

P_b - power transmitted by radar on beam axis $= P_t g$
where P_t is the power if it was isotropic & g is the radar gain
At a distance r, the power is distributed over a spherical surface $4\pi r^2$ but intercepted by only a small target area A_σ. So

$$P_\sigma = \frac{P_t g A_\sigma}{4\pi r^2}$$ where σ denotes the target

If the target reradiates isotropically, only a small fraction is measured by the radar with antenna area A_r

$$P_r = \frac{P_\sigma A_r}{4\pi r^2} \quad \& \quad A_r = \frac{g\lambda^2}{4\pi}.$$ The target has a backscattering cross-sectional area σ_t

So $$P_r = \frac{P_t g^2 \lambda^2 \sigma_t}{64\pi^3 r^4}$$ RE1.1

The backscattering cross-sectional area of the target depends on the wavelength of the radar relative to the size of the target:

(1) Very large targets $(D > 10\lambda)$ $\sigma_t = \pi \frac{D^2}{4}$ (birds, etc.)
(2) Intermediate targets $(10\lambda > D > .1\lambda)$ is complicated. This is the Mie region (see Fig 4.2) Hailstones fall into this category
(3) Small targets $(D < .1\lambda)$ Rayleigh regime (raindrops, etc.)
$\sigma_t \propto D^6$

There is rarely a single target in a radar volume. Imagine a $\tau = 1.57\,\mu s$ pulse from the WSR-88D ($h_s = 500$ m). At a distance of 100 km from the radar, the cross-sectional area of the beam is ~ 2.4 km^2 (Assignment 5).

Then, the volume being scanned is $1.2\times10^9 m^3$. Since there is roughly 1 raindrop of size .1 mm / liter (see Wallace & Hobbs Fig 6.18), then there are 1000 raindrops per m^3 or 1.2×10^{12} raindrops in the scanned volume! (see also Fig 8.1 of text)

The total back-scattering cross-sectional area can be expressed as the sum of all the contributing targets $\sigma_t = \sum_{i=1}^{\wedge} \sigma_i$

It is very important to recognize that it is generally assumed that the returned power comes from the equivalent of small spherical rain drops, even when it isn't. We define the effective radar reflectivity factor or "reflectivity" as

$$Z_e = \sum_{i=1}^{\wedge} D_i^6$$

Effective means we are assuming that the target is coming from spherical raindrops of varying diameters

Then $$P_r = \frac{Z_e l}{C_r r^2}$$ where l is the attenuation factor $(0 \le l \le 1)$ due to power lost as a result of the beam travelling through a medium (see Chapter 8)

C_r - radar constant $= 6.91831\times10^5 \frac{mm^6}{m^3} mW^{-1} km^{-2}$, which takes into consideration all of the radar parameters for the WSR 88D

What we're really interested in is Z_e, the radar measures P_r so

$$Z_e = \frac{C_r r^2 P_r}{l}$$

So, a given returned power P_r measured by a radar is equivalent to larger reflectivity when the signal comes from greater distance r. As attenuation gets worse $l \to 0$, there must be a bigger target at distance r for us to be able to see it

Since z_e can vary from $10^{-3}\frac{mm^6}{m^3}$ (fog) to $10^{7}\frac{mm^6}{m^3}$ (hail),

then define $Z_e = 10\log_{10}\left[\frac{z_e}{1\frac{mm^6}{m^3}}\right]$ the logarithmic reflectivity or usually just "reflectivity" too

This is scaled relative to a $z_e = 1\frac{mm^6}{m^3}$, i.e., when $z_e = 1\frac{mm^6}{m^3}$ then $Z_e = 0$

when $z_e = 10^{-3}\frac{mm^6}{m^3}$, $Z_e = -30$ dBZ ; when $z_e = 10^{7}\frac{mm^6}{m^3}$ $Z_e = 70$ dBZ

It is critical to recognize that the same power returned by a radar on 2 scans may come from completely different targets

Imagine 729 drops diameter 1mm } in $1m^3$
1 drop " " 3mm }

$$z_e = 729\left[\frac{1mm^6}{m^3}\right] + 1\left[\frac{(3mm)^6}{m^3}\right] = 1458\frac{mm^6}{m^3}$$

$Z_e = 31.6$ dBZ which would be equivalent to 1 3.4mm drop

Z-R Relationships

Empirically, the reflectivity can be related to rain/precipitation rate using eqns of the form: $z_e = AR^b$ where R is the rain rate in $\frac{mm}{h}$, z $\frac{mm^6}{m^3}$ & A & b are empirical constants.

The traditional estimate is from Marshall-Palmer distribution of precipitation from years ago $z_e = 200R^{1.6}$

Z-S Relationships

Snow ppt rate is generally less than rain rate & the dielectric constant of ice $|K|^2 = .197$ is smaller than that of water $|K|^2 = .93$ (See Probert-Jones eqn). So, power received from snow/ice ~ 7 dB less than if radar looking at rain, even though ice/snow tends to have larger diameter (but density less)

As text discusses, snow is even more difficult to measure because cloud heights tend to be much less

Bright Band

A layer of high reflectivity is often observed immediately below the 0°C isotherm as snowflakes descend into this layer, become wet on the outside & coalesce into larger, wet flakes. As it continues to fall & completely melt, the fall speed is faster meaning fewer hydrometeors are present. The result is a bright band resulting ~~from~~ the following changes:

	Dielectric	Fall Velocity	Coalescence	Shape	Growth	Total
Snow to bright band	+4	-1.5	+3	+1.5	0	+7
Bright band to rain	+1	-5.5	-1	-1.5	.5	-6.5

So, the reflectivity can vary by as much as 13-15 dBZ simply as a result of the phase change. This becomes particularly evident in stratiform precipitation & decaying thunderstorms, & less obvious in convective situations because of turbulence

Hail

PPT in form of ice with diameter > 5 mm (5mm-10cm)
Reflectivity depends on whether outer surface is wet or dry & large hail typically is in the Mie region. See chapter 8 for more info.

Doppler Velocity

Consider a single target at distance r from radar

The total distance travelled $= 2r = c\Delta t$ if the object is stationary

However, if the target is moving towards the radar, then the radar beam travels a distance that is $2V_r \Delta t$ less since it has a velocity V_r towards the radar & it doesn't have to go all the way to the target & back, or dis $= c\Delta t + 2V_r \Delta t$

where V_r − when object moving towards the radar & V_r + when moving away

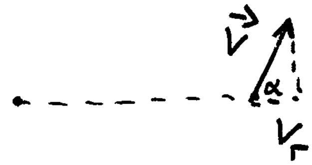
or $(\lambda f')\Delta t = (c + 2V_r)\Delta t$ where λ is the wavelength of the radar & f' is the apparent frequency of the radar beam.

If the object were stationary then $f\lambda = c$

So $f' - f = \dfrac{2V_r}{\lambda}$ = Doppler or frequency shift due to object motion

If the wind is $\vec{V}$, then the radial velocity is

$V_r = \vec{V} \cos \alpha$



So the target can appear stationary when it is moving $\perp$ to the radar beam

We are limited by how much frequency shift can be unambiguously detected because the radar has a PRF $\left(\frac{\text{cycles}}{\text{sec}}\right)$ defined. You can only detect properly the phase of waves with $\frac{PRF}{2}$, which is the Nyquist frequency

So $f' - f$ can not exceed $\frac{PRF}{2}$

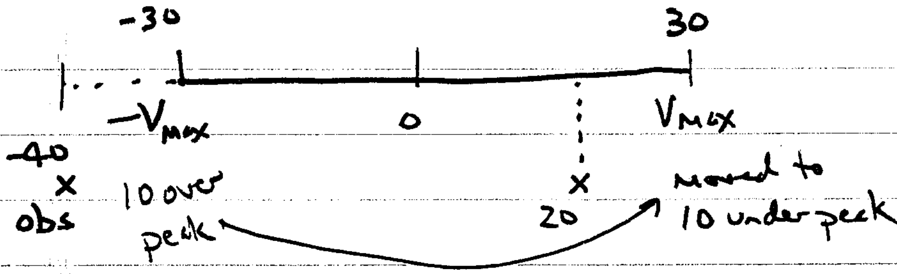
$PRF = 318\,Hz \Rightarrow V_{r_{max}} = 8\ m/s$

$1304\,Hz \Rightarrow V_{r_{max}} = 33\ m/s$

That limits $V_{r_{max}} = \dfrac{PRF\,\lambda}{4}$

So, to detect high wind speeds, the PRF must be high, the wavelength of the radar is fixed (10 cm for WSR-88D)

If winds are greater than $V_{r\,max}$, then they get "folded" over to a different radial speed from the opposite direction as follows



In other words, the phase of the signal becomes ambiguous & the velocity becomes interpreted as from the opposite direction from that actually observed.

We are also limited in terms of the maximum possible range. First, there are pragmatic limits. Most targets are less than 10 km above the surface, so even high thunderstorms are missed by radar beams at long distances. Further, the power returned from an object decreases as $\frac{1}{r^2}$ so only really strong signals can be detected at long distances.

Radar pulse durations are short $\tau \simeq 1.57$ or $4.5\,\mu s$. The pulse repetition time $PRT = \frac{1}{PRF}$ is long $PRT = 3000\,\mu s$ or $766\,\mu s$

So the listening period is $PRT - \tau \sim PRT$

We have to wait that long so that the returned signal is clearly from the current pulse. So $2\,r_{max} = c\,PRT$ since the pulse has to travel to the target & back

Then: $r_{max} = \frac{c\,PRT}{2} = \frac{c}{2\,PRF}$

The maximum distance then is entirely a function of the PRF

For the WSR 88D $PRF = 318\,Hz \Rightarrow r_{max} = 470\,km$

$PRF = 1304\,Hz \Rightarrow r_{max} = 115\,km$

So, you should see the "Doppler dilemma" cropping up. We can either measure a strong wind close to the radar or see lighter winds further away, but we can't sense strong winds far from the radar.

Combining the two limits $V_{max} r_{max} = \frac{c\lambda}{8}$
See Fig 6.4 in the text for this limitation imposed on all radars.

Range Aliasing

As shown in the radar modules & handouts, a further complication is that we send out many pulses & an early one may travel past r_{max}, hit a strong reflector, & return power to the radar that will arrive as if it was at a range $r < r_{max}$. This kind of second (or third) trip echo is quite common. The actual location of the target would be at $r_{possible} = r_{displayed} + n\, r_{max}$ (where $n = 1, 2 \ldots$)
See the reading & modules for further info on velocity & range aliasing in Chapter 6.