LES of Turbulent Flows: Lecture 1
(ME EN 7960-008)

Prof. Rob Stoll

Department of Mechanical Engineering

University of Utah

Spring 2009

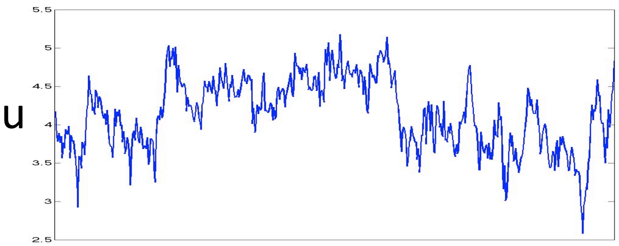
Turbulent Flow Properties

- Why study turbulence? Most real flows in engineering applications are turbulent.

Properties of Turbulent Flows:

1. Unsteadiness:

u

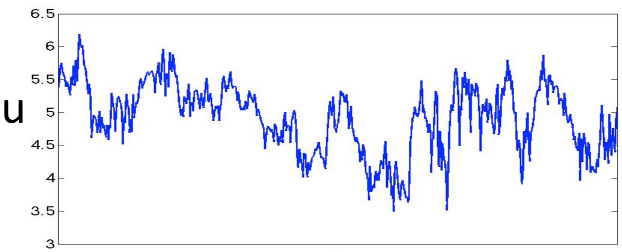


u=f(**x**,t)

time

2. 3D:

u



contains random-like variability in space

x_i (all 3 directions)

3. High vorticity:
Vortex stretching → mechanism to increase the intensity of turbulence
(we can measure the intensity of turbulence with the turbulence intensity => $\frac{\sigma_u}{\langle u \rangle}$)

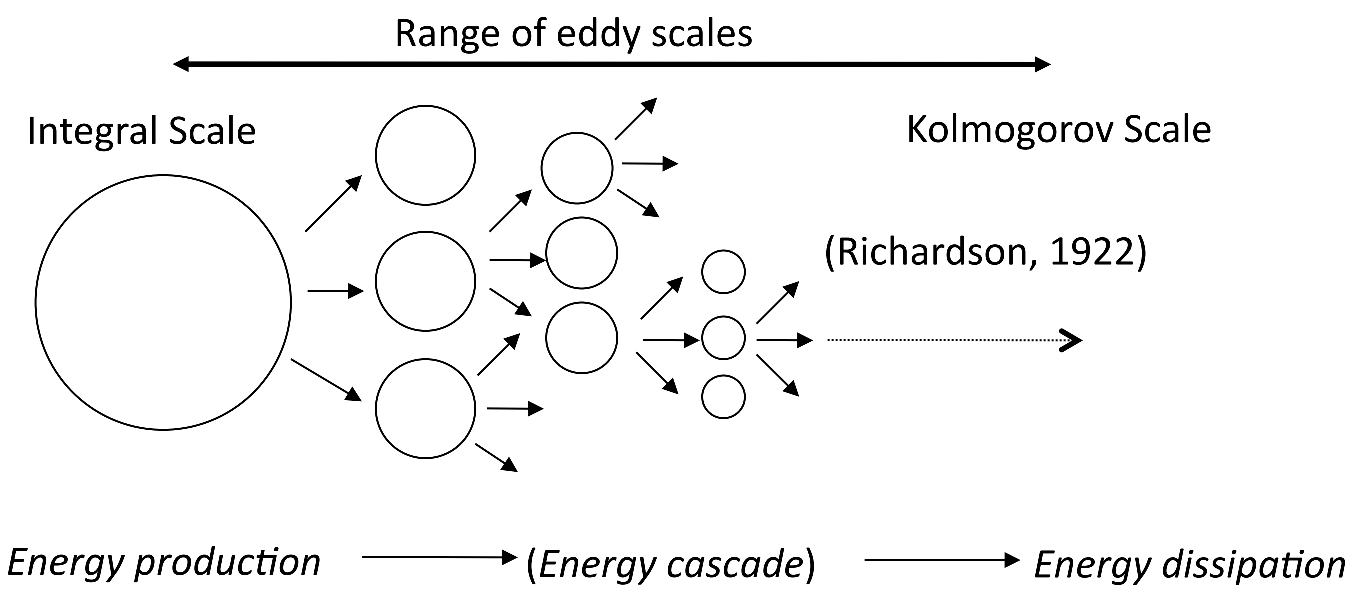
Vorticity: $\omega = \vec{\nabla} \times \vec{u}$ or $\omega_k = \epsilon_{ijk} \frac{\partial}{\partial x_i} u_j \hat{e}_k$

Turbulent Flow Properties (cont.)

Properties of Turbulent Flows:

4. Mixing effect:
   Turbulence mixes quantities with the result that gradients are reduced (e.g. pollutants, chemicals, velocity components, etc.). This lowers the concentration of harmful scalars but increases drag.

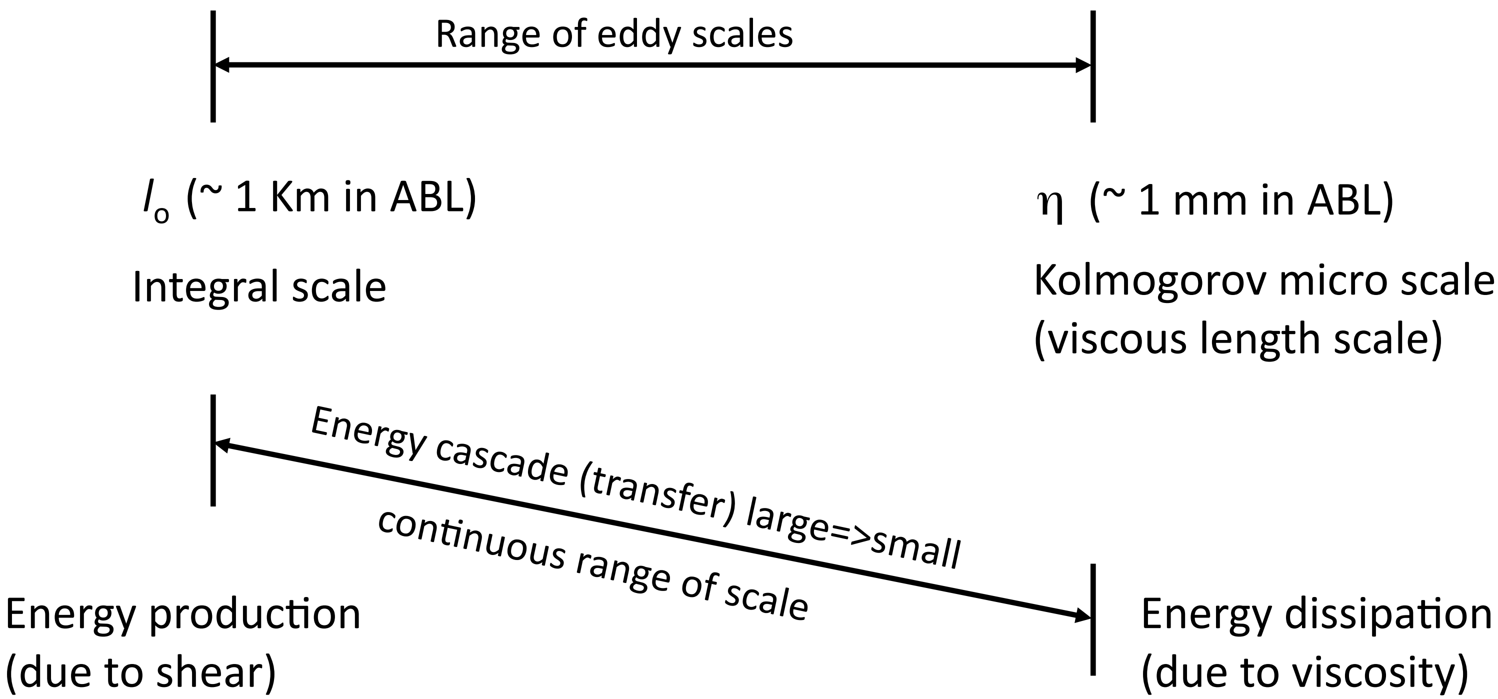
5. A continuous spectrum (range) of scales:



Turbulence Scales

- The largest scale is referred to as the Integral scale (l_o). It is on the order of the autocorrelation length.

- In a boundary layer, the integral scale is comparable to the boundary layer height.



Kolmogorov's Similarity hypothesis (1941)

- smallest scales receive energy at a rate proportional to the dissipation rate ( )

- motion of the very smallest scales in a flow depend only on:

  a) rate of energy transfer from small scales: $\epsilon \left[\dfrac{L^2}{T^3}\right]$

  b) kinematic viscosity: $\nu \left[\dfrac{L^2}{T}\right]$

With this he defined the Kolmogorov scales (dissipation scales):

- length scale: $\eta = \left(\dfrac{\nu^3}{\epsilon}\right)^{\frac{1}{4}}$

- time scale: $\tau = \left(\dfrac{\nu}{\epsilon}\right)^{\frac{1}{2}}$

- velocity scale: $v = (\nu\epsilon)^{\frac{1}{4}}$

Re based on the Kolmolgorov scales => Re=1

Kolmogorov's Similarity hypothesis (1941)

From our scales we can also form the ratios of the largest to smallest scales in the flow (using $\ell_o,\ U_o,\ t_o$).
Note: dissipation at large scales => $\epsilon \sim \dfrac{U_o^3}{\ell_o}$

- length scale:
$$\eta = \left(\frac{\nu^3}{\epsilon}\right)^{\frac{1}{4}} \sim \left(\frac{\nu^3 \ell_o}{U_o^3}\right)^{\frac{1}{4}} \Rightarrow \frac{\eta}{\ell_o^{1/4}} \sim \frac{\nu^{3/4}}{U_o^{3/4}} \Rightarrow \boxed{\frac{\eta}{\ell_o} \sim \frac{\nu^{3/4}}{U_o^{3/4}\ell_o^{3/4}} \sim Re^{-3/4}}$$
- velocity scale:
$$v = (\nu\epsilon)^{\frac{1}{4}} \sim \left(\frac{\nu U_o^3}{\ell_o}\right)^{\frac{1}{4}} \Rightarrow \frac{v}{U_o^{3/4}} \sim \frac{\nu^{1/4}}{\ell_o^{1/4}} \Rightarrow \boxed{\frac{v}{U_o} \sim Re^{-1/4}}$$
- time scale:
$$\tau = \frac{\eta}{v} \Rightarrow \boxed{\frac{\tau}{t_o} \sim Re^{-1/2}}$$

For very high-Re flows (e.g., Atmosphere) we have a range of scales that is small compared to ℓ_o but large compared to η. As *Re* goes up, η/ℓ_o goes down and we have a larger separation between large and small scales.

Kolmogorov's Similarity hypothesis (1941)

Kolmolgorov also hypothesized:

In Turbulent flow, a range of scales exists at very high Re where statistics of motion in a range ℓ (for $\ell_o >> \ell >> \eta$) have a universal form that is determined only by ϵ (dissipation) and independent of ν (kinematic viscosity).