LES of Turbulent Flows: Lecture 10
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1-Equation Eddy viscosity Models

- Evolution of τ_{ij}: Deardorff (ASME J. Fluids Eng., 1973)

-Deardorff (1973) suggested that the evolution of τ_{ij} should be modeled (see Sagaut pg. 243 for equations)

-To close this model, equations for the components of τ_{ij} and for $\tilde{k}_r$ (SGS kinetic energy) are both needed (similar to the a 2^{nd}-order RANS closure, see Speziale, ARFM, 1991 for a review of RANS closures including 2^{nd}-order models)

-Deardorff (BLM, 1980) suggested a simpler 1-equation approach to avoid the need to solve prognostic equations for τ_{ij} (in addition to the N-S equations). This model is equivalent to a k-*l* ,1-equation RANS model (see Speziale, 1991)

-This was also proposed earlier for the isotropic part of a 2-part eddy-viscosity model by Schumann (J. Comp. Physics, 1975) although the model is usually credited to Deardorff (especially in the atmospheric community).

1-Equation Eddy viscosity Models

- The model takes the eddy-viscosity as: (see Sagaut pg. 128 or Guerts pg. 227)

$$\nu_T = (C_1\Delta)\,\tilde{k}_r^{1/2}$$

our length scale l^* is $C_1\Delta$ (same as Smagorinsky) and the velocity scale u^* is $\tilde{k}_r^{1/2}$

so that our model is now: $\tau_{ij} = -2\,(C_1\Delta)\,\tilde{k}_r^{1/2}\tilde{S}_{ij}$

and $\tilde{k}_r$ is found from an SGS kinetic energy equation of the form:

$$\frac{\partial \tilde{k}_r}{\partial t} = -\frac{\partial\left(\tilde{u}_i\tilde{k}_r\right)}{\partial x_j} + \Pi - \epsilon + \frac{\partial}{\partial x_j}\left(\frac{C_1}{C_2}\Delta\tilde{k}_r^{1/2}\frac{\partial \tilde{k}_r}{\partial x_j}\right) + \frac{1}{Re}\frac{\partial^2\tilde{k}_r}{\partial x_j^2}$$

-viscous dissipation is typically modeled based on isotropy assumption as $\epsilon \approx \frac{C_k}{\Delta}\tilde{k}_r^{3/2}$

-C_2 is an order 1 constant and C_k is the Kolmogorov constant ≈ 1.7

-Note the transport terms (for pressure and SGS kinetic energy) have been modeled by the 4th term on the RHS of the equation.

-This model is popular in the ABL due to the ability to include SGS transport or energy drain effects as extra parameters in the SGS TKE equation (e.g. for SGS canopy drag, bouyancy forces etc.)

2-point Eddy viscosity Models

• **2-point eddy-viscosity model**: (Metais and Lesieur, JFM, 1992)
(see Sagaut pg 124 or Lesieur et al., 2005 "Large-Eddy Simulation of Turbulence")

-This model is an attempt to go beyond the Smagorinsky model while keeping, in physical space, the same scaling as the spectral eddy-viscosity model of Kriachnan (JAS, 1976).

-Idea: in physical space build an eddy-viscosity normalized by

$$\sqrt{E_{\vec{x}}(k_c)/k_c} \quad \text{with} \quad k_c = \frac{\pi}{\Delta}$$

and where $E_{\vec{x}}(k_c)$ is the local kinetic energy spectrum at point $\vec{x}$.

-$E_{\vec{x}}(k_c)$ must be evaluated in terms of physical space quantities. The best candidate for this is the **2nd-order structure function**:

$$F^{iso}(r) = \langle [\vec{u}(\vec{x},t) - \vec{u}(\vec{x}+\vec{r},t)]^2 \rangle$$

-Note, the isotropic 2nd-order structure function spectrum (Fourier transform) is equivalent to the the Kolmogorov $k^{-5/3}$ energy spectrum (see Pope sections 6.2 and 6.4 for the relationship between structure functions and energy spectrum).

2-point Eddy viscosity Models

-For the 2-point eddy-viscosity model, the local structure function is used:

$$F_2(\vec{x},\Delta) = \left\langle \left[\tilde{\vec{u}}(\vec{x},t) - \tilde{\vec{u}}(\vec{x}+\vec{r},t)\right]^2 \right\rangle_{\|\vec{r}\|}$$

where we now have a local statistical average over the nearest 6 points (or 4 points in a boundary layer).

-Assuming a $k^{-5/3}$ spectrum from zero to k_c, we get

$$\nu_T(\vec{x},\Delta,t) = 0.105 C_k^{-3/2} \Delta \left[F_2(\vec{x},\Delta)\right]^{1/2}$$

where C_k is the Kolmogorov constant

- **Relating the structure function model to the Smagorinsky model**:

-If we replace the velocity increments by 1st order spatial derivative we can show that

$$\nu_T \approx 0.777 (C_S \Delta)^2 \sqrt{2\tilde{S}_{ij}\tilde{S}_{ij} + \tilde{\omega}_i\tilde{\omega}_i}$$

where $\tilde{\omega} \equiv \text{ filtered vorticity } = \vec{\nabla} \times \tilde{\vec{u}}$

-We can imagine the 2-point (or structure function) model as the Smagorinsky model in a strain/vorticity formulation