LES of Turbulent Flows: Lecture 11
(ME EN 7960-008)

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Similarity Models

- **<u>Similarity model</u>**: (Bardina et al., AIAA, 1980)

-Bardina et al., (1980) proposed an alternative model to the eddy-viscosity model. They were motivated by the low correlations between $\tau_{ij}^{\Delta}(\vec{x},t)$ and $\tau_{ij}^{\Delta,M}(\vec{x},t)$ in *a priori* studies (more on *a priori* studies in later lectures).

-Based on their analysis (and general reasoning) they hypothesized that near grid-scale energy transfers were the most important (and the most active).

-Recall: $u_i' \equiv u_i - \tilde{u}_i$

and the filtered fluctuating velocity is: $\widetilde{u'}_i = \tilde{u}_i - \tilde{\tilde{u}}_i$

Recall Leonard's decomposition of τ_{ij} is:

$$\tau_{ij} = L_{ij} + C_{ij} + R_{ij}$$

$$= \left(\widetilde{\tilde{u}_i\tilde{u}_j} - \tilde{u}_i\tilde{u}_j\right) + \left(\widetilde{\tilde{u}_i u_j'} + \widetilde{u_i'\tilde{u}_j}\right) + \widetilde{u_i'u_j'}$$

"Resolved" + "Cross" + "Reynolds" stresses

Similarity Models

-using our definition of the filtered velocity fluctuations, and the following assumption shown for R_{ij}, we can write each of our terms as follows:

$$R_{ij} = \widetilde{(u_i - \tilde{u}_i)(u_j - \tilde{u}_j)} \approx (\tilde{u}_i - \tilde{\tilde{u}}_i)(\tilde{u}_j - \tilde{\tilde{u}}_j)$$

$$C_{ij} \approx \tilde{\tilde{u}}_i(\tilde{u}_j - \tilde{\tilde{u}}_j) + \tilde{\tilde{u}}_j(\tilde{u}_i - \tilde{\tilde{u}}_i)$$

$$L_{ij} = \left(\widetilde{\tilde{u}_i\tilde{u}_j} - \tilde{u}_i\tilde{u}_j\right)$$

-if we put all the terms together and do some algebra this reduces to:

$$\tau_{ij} = \left(\widetilde{\tilde{u}_i\tilde{u}_j} - \tilde{\tilde{u}}_i\tilde{\tilde{u}}_j\right)$$

-giving us an estimate for the SGS stress (see Saguat pg 231 for similarity models)

More on Similarity models

• Lui et al (JFM, 1994) examined "bands" around Δ and built a scale-similarity model similar to the model of Bardina et al., (1980)

-They argued that energy in the band at one scale larger than Δ (say 2Δ) and one scale smaller (something like ½Δ) would have the largest contribution to τ_{ij}.

-define: $u_i^n = \tilde{u}_i - \bar{u}_i$ where (~) is is a filter at Δ and (**–**) is a filter at a larger scale 2Δ

-u_i^n can be thought of as the band-pass filtered velocity between Δ and 2Δ. We can do a similar decomposition for u_i^{n+1} and u_i^{n-1}

-With our band-pass filtered decomposition we can build a τ_{ij}^n based on u_i^n and u_i^{n+1} for any other band)

-For example, the stress one level above *n* can be written using another filter at 4Δ (denoted by a **^**) as:

$$\tau_{ij}^{n-1} = \overline{(\tilde{u}_i - \hat{u}_i)(\tilde{u}_j - \hat{u}_j)} - (\overline{\tilde{u}_i - \hat{u}_i})(\overline{\tilde{u}_j - \hat{u}_j})$$

This can be reduced to the following (note that $\hat{u}_i$ is approximately constant to **–** filter)

$$\tau_{ij}^{n-1} = (\overline{\tilde{u}_i \tilde{u}_j} - \bar{\tilde{u}}_i \bar{\tilde{u}}_j)$$

The Similarity and Nonlinear Models

- Lui et al.'s study showed similarity between $\tau_{ij}^{n+1} \rightarrow \tau_{ij}^{n} \rightarrow \tau_{ij}^{n-1}$

1st unresolved band | smallest resolved band | next largest resolved band

- They concluded that because of this the Leonard stress (τ_{ij}^{n-1}) is the best estimate =>

$$\tau_{ij} = C_L L_{ij} \quad \text{where} \quad L_{ij} = (\overline{\tilde{u}_i \tilde{u}_j} - \bar{\tilde{u}}_i \bar{\tilde{u}}_j)$$

- This is the most commonly used form (currently) of the similarity model.

- Another form of the similarity model is **the nonlinear model** (also called the Clark model or the gradient model)

  -idea: approximate $\tilde{u}_i$ by a Taylor series expansion

$$\tilde{u}_i(x) = \tilde{u}_i(x_0) + \tilde{A}_{ik}(x_0)(x_k - x_k^0)$$

  where $\tilde{A}_{ik}$ is the filtered gradient tensor $\tilde{A}_{ik} = \dfrac{\partial \tilde{u}_i}{\partial x_k}$

  We can use this approximation (Taylor series) to estimate Lij and develop another model

$$\tau_{ij} = C_A \Delta^2 \tilde{A}_{ik} \tilde{A}_{jk}$$

Mixed Models

- Both the similarity and nonlinear models exhibit a high level of correlation in *a priori* tests with measured values of τ_{ij}^{Δ} but they underestimate the average dissipation and are numerically unstable

- Typically they are combined with an eddy-viscosity model to provide the proper level of dissipation.

  -an example is (Bardina et al, 1980):

$$\tau_{ij} = C_L \left(\overline{\tilde{u}_i \tilde{u}_j} - \tilde{\bar{u}}_i \tilde{\bar{u}}_j \right) - 2 \left(C_S \Delta \right)^2 |\tilde{S}| \tilde{S}_{ij}$$

  -the similarity term has a high level of correlation with τ_{ij} and the eddy-viscosity term provides the appropriate level of dissipation.

- A few notes on mixed models:

  -Proper justification for the mixed model did not exist at first but a more unified theory has developed in the form of approximate deconvolution or filter reconstruction modeling (Guerts pg 200, Sagaut pg 210)

More on Mixed Models

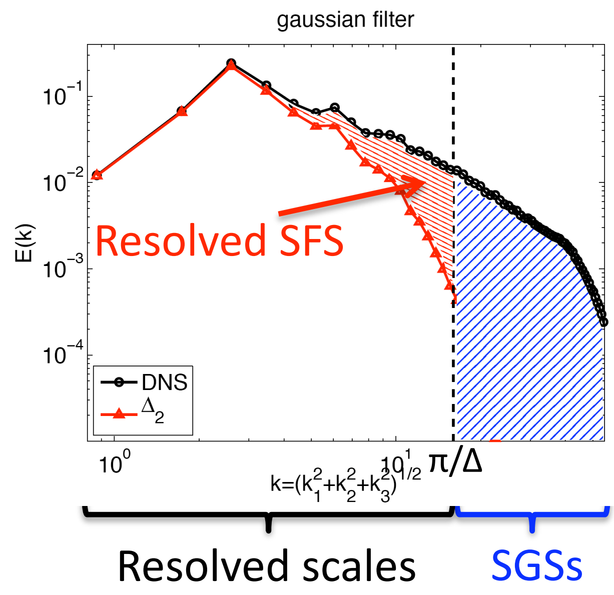
-Idea: a SGS model should be built from 2 parts, **<u>the first part</u>** accounts for the effect of the filter through an approximate reconstruction of the filter's effect on the velocity field (note the similarity model is a zeroth-order filter reconstruction). This is the model for the resolved SFS.

-The **<u>second part</u>** accounts for the SGS component of τ_{ij}

-We then assume that we can build τ_{ij} as a linear combination of these two model components.

-A few last notes on Similarity models:

- Bardina et al.'s model is exactly zero for a spectral cutoff filter.



- Lui et al.'s form of the similarity model also fails. This is credited to the nonlocal structure of the cutoff filter. It breaks the central assumption of the similarity model, that the locally decomposed at different levels is self similar