LES of Turbulent Flows: Lecture 13
(ME EN 7960-008)

Prof. Rob Stoll

Department of Mechanical Engineering
University of Utah

Spring 2009

Statistical Conditions for a SGS model

- <u>What conditions should a SGS model satisfy?</u>

-Specifically we are interested in answering the question what **statistical** properties should τ_{ij} and τ_{ij}^{mod} share?

-We know a "good" model should adhere to our equations of motion:

- Invariance to translation, rotation, and reflection (in the absence of boundaries)
- Hopefully, invariance to Re
- Ideally, invariant to Δ

-To get more specific than this, we need to talk about **statistics of SGS models** (Meneveau, Physics of Fluids, 1994).

- **To obtain correct 1st and 2nd order moments** of our resolved field, our **model must** at least be able to **produce average modeled stresses** that match the real stresses everywhere.
- **This doesn't guarantee that our 2nd order moments** are correct it is only a **necessary condition**.
- **To produce 2nd order moments**, we need to have our model reproduce 2nd and 3rd order SGS stats including stresses and correlations (e.g. stresses with velocity or gradients). This includes **matching <Π> everywhere**.
- For even higher order moments we need to match higher order SGS stats...

THE UNIVERSITY OF UTAH

Computing SGS quantities

• Procedurally, **How do we compute these SGS stats from data** (DNS or Experiments)?
Here is a "quick" list, also see the handout Project_apriori_study.pdf on the web.

-Select your data (after quality control) and identify missing velocity or gradient terms

-Separate the data into resolved and SGS scales by calculating $\tilde{u}_i$ and $\widetilde{u_i u_j}$ with an appropriate LES filter (see lecture 5 for the most common examples).

- •At this point, a decision must be made: to **down-sample or not** (see Liu et al.,JFM 1994)
  - -Down-sampling means removing points from the field that are separated (spatially) by < our filter scale Δ (denoted by the ~). Effectively this means we keep less points than we started with (e.g. from 128^3 to 32^3) after filtering.
  - -**Pros:** we get a "true" representation of the effect of gradient estimates on our SGS models and avoid enhanced correlations due to filter overlap.
  - -**Cons:** we lose data points (important if we have limited data) and we now need to consider the above gradient estimation errors!

-Calculate **<u>local values</u>** of all the components of $\tau_{ij}^\Delta = \widetilde{u_i u_j} - \tilde{u}_i \tilde{u}_j$ and $\tilde{S}_{ij} = \frac{1}{2}\left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i}\right)$
you can (you may need approximations here based on your data!

-For some models you may need to calculate other parameters (e.g., mixed and nonlinear models) but the general procedure is the same

THE UNIVERSITY OF UTAH

Computing SGS quantities

-Recall that some filtering examples are given on the website under data/mfiles

-Once you have these basic quantities calculated you can calculate model values $\tau_{ij}^{\Delta,M}$ and statistics of the actual (from data) and modeled SGS stresses including average values, correlation coefficients and variances (see project handout).

-We can also calculate other SGS statistics like $\langle \Pi^\Delta \rangle = -\langle \tau_{ij}^\Delta \tilde{S}_{ij} \rangle$ and $\langle \Pi^{\Delta,M} \rangle$ or any model coefficients of interest (see handout for an example).

• The following pages give some examples of SGS statistics and model coefficients calculated form various references (discussed in class).

THE UNIVERSITY OF UTAH

SGS Dissipation

- SGS Energy transfer from experiments in the Utah desert (Carper and Porté-Agel , 2004)

$$\Pi = -\tau_{ij}\tilde{S}_{ij}$$

$\langle\Pi\rangle \times 10^3 = 7.13$ m^{2}s^{-3}
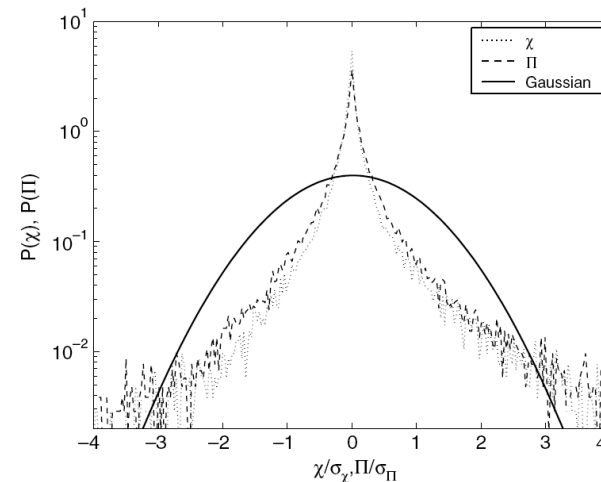$\sigma_{\Pi} \times 10^2 = 18.02$ m^{2}s^{-3}
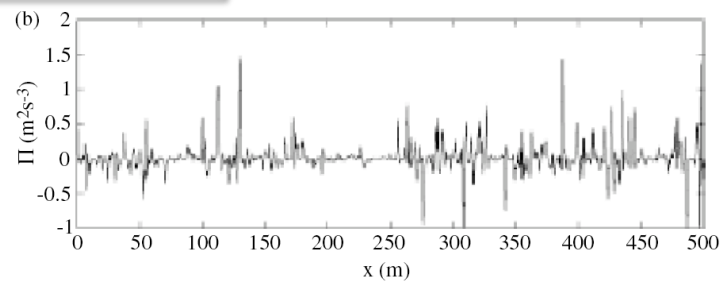
Example of time series of Π from the ABL (late afternoon)

Example PDF of Π from the ABL (late afternoon)

Experimental setup

THE UNIVERSITY OF UTAH

SGS Dissipation

- SGS Energy transfer from wind tunnel experiments in a round jet (Liu et al., 1994)

$$\Pi = -\tau_{ij}\tilde{S}_{ij}$$
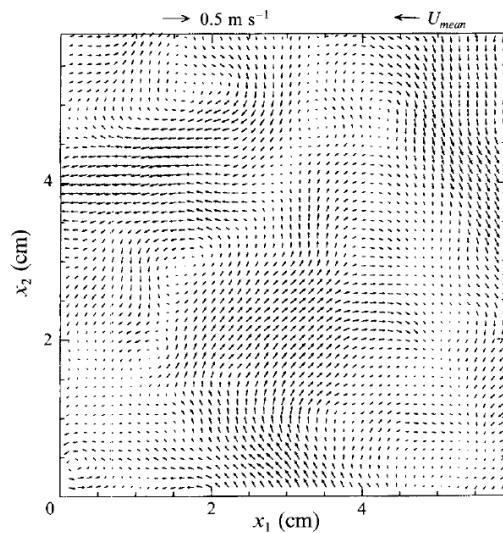
Top-hat filtered PIV field
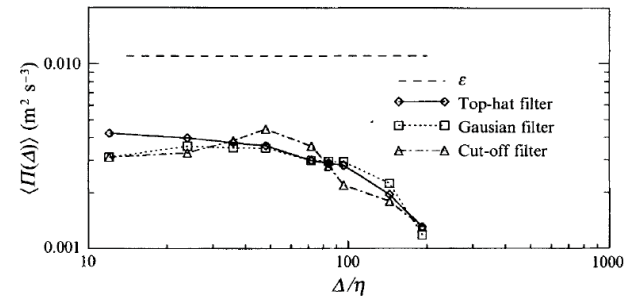
Average Π from the wind tunnel experiment compared to molecular dissipation
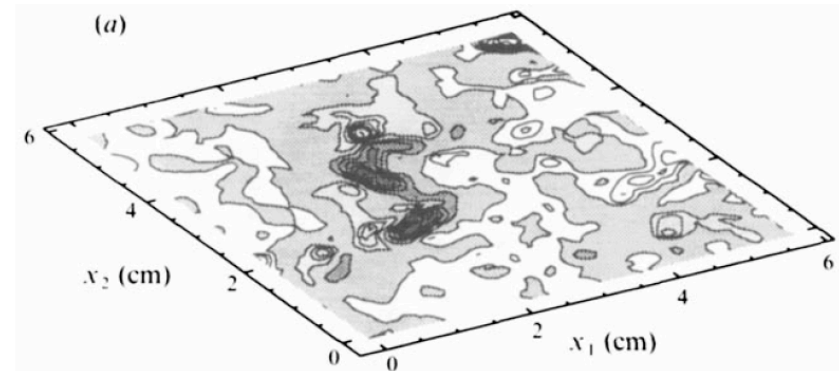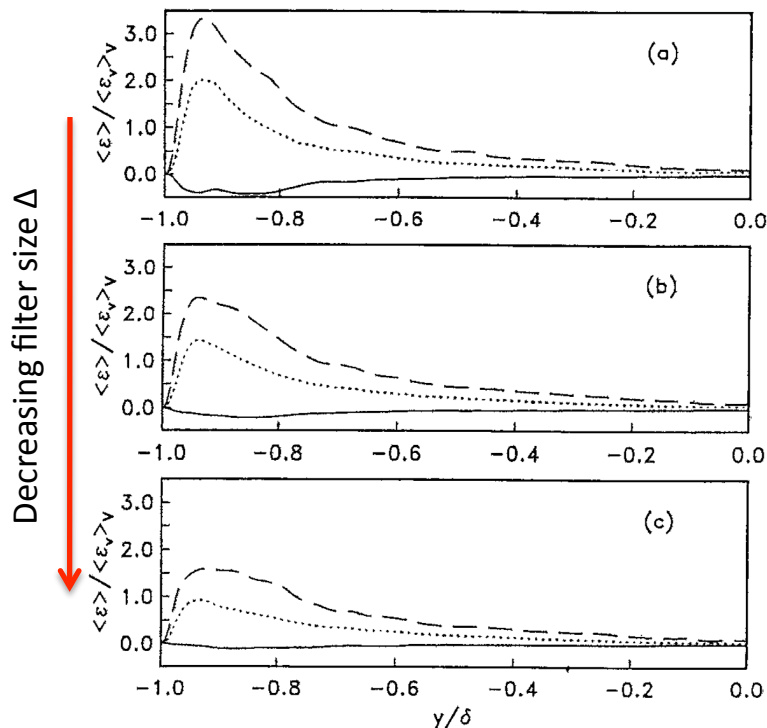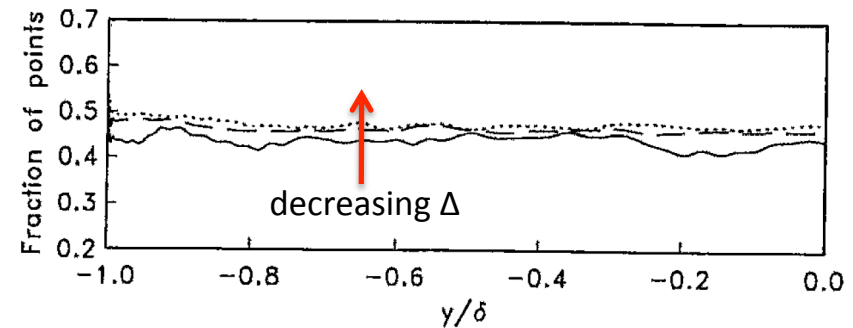

Spatial distribution of Π from PIV

SGS Dissipation

- SGS Energy transfer from DNS of turbulent channel flow Re=3300 (U_c) (Piomelli et al., 1991)

$$\Pi = -\tau_{ij}\tilde{S}_{ij}$$

Decreasing filter size Δ

Π normalized by the total dissipation
___ average; ---- rms and .... backscatter

decreasing Δ

Fraction of points in channel flow with backscatter for 3 different filter widths

- backscatter increases with Re
- fraction of backscatter points decreases for a Gaussian filter (cutoff results shown) to about 30%.

THE UNIVERSITY OF UTAH

SGS Model Correlation Coefficients

Correlation coefficients from Clark et al, (1979) for different models

*Evaluation of subgrid-scale models*   13

Term	Model	Correlation $\frac{17}{8}\Delta$	$\frac{9}{8}\Delta$	Model constant $\frac{17}{8}\Delta$	$\frac{9}{8}\Delta$
τ_{ij} (tensor)	Smagorinsky	0·366	0·277	0·270	0·247
	Vorticity	0·344	0·260	0·294	0·275
	Turbulent kinetic energy	0·363	0·303	0·196	0·175
	Eddy viscosity	0·352	0·295		
$\frac{\partial \tau_{ij}}{\partial x_j}$ (vector)	Smagorinsky	0·425	0·346	0·240	0·264
	Vorticity	0·408	0·327	0·220	0·247
	Turbulent kinetic energy	0·434	0·362	0·138	0·155
	Eddy viscosity	0·426	0·356		
$u_i \frac{\partial \tau_{ij}}{\partial x_j}$ (scalar)	Smagorinsky	0·710	0·580	0·186	0·171
	Vorticity	0·700	0·582	0·202	0·191
	Turbulent kinetic energy	0·723	0·606	0·085	0·095
	Eddy viscosity	0·716	0·605		

TABLE 2. Summary of correlations between exact subgrid-scale Reynolds stresses and models.

- Eddy-viscosity- $\tau_{ij} = -2\nu_T \tilde{S}_{ij}$
- Smagorkinsky- $\nu_T = (C_S \Delta)^2 |\tilde{S}|$
- Kinetic energy- $\nu_T = (C_1 \Delta) \tilde{k}_r^{1/2}$
- Vorticity- $\nu_T = (C\Delta)^2 (\omega_i \omega_i)^{1/2}$
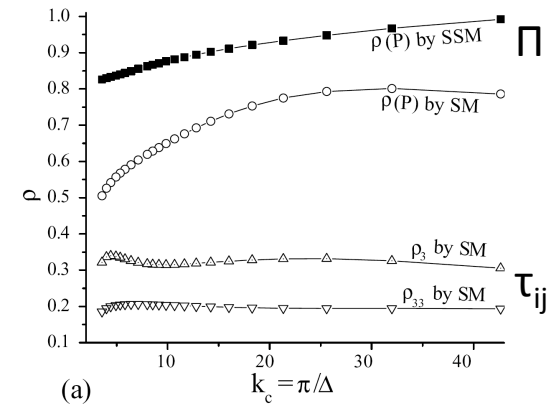
Correlation coefficients from Lu et al (2007) for Smagorinsky and Similarity models
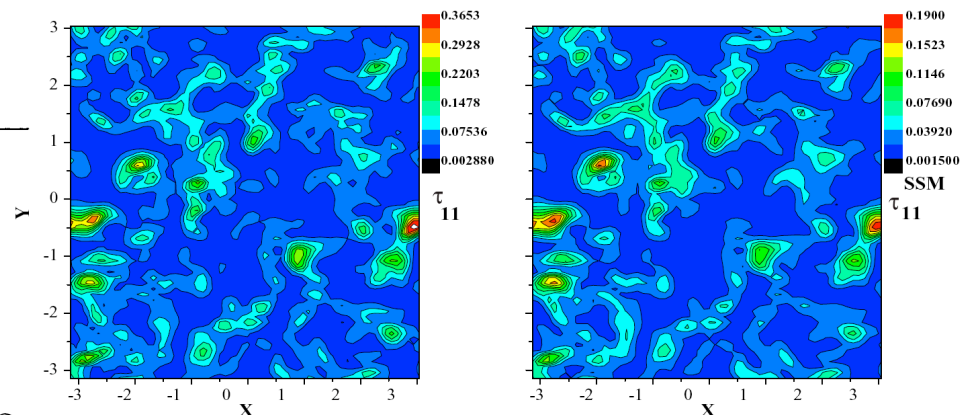
Measured (left) and modeled (right) with the similarity model τ_{11} from Lu et al (2007).
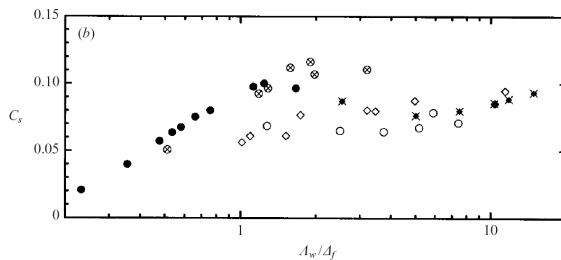
THE UNIVERSITY OF UTAH

SGS Model Coefficient Estimates

Model coefficients evaluated by matching Π from ABL study of Sullivan et al (2003).
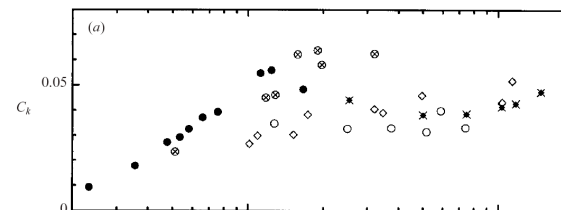
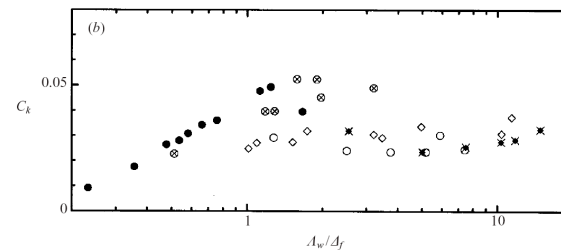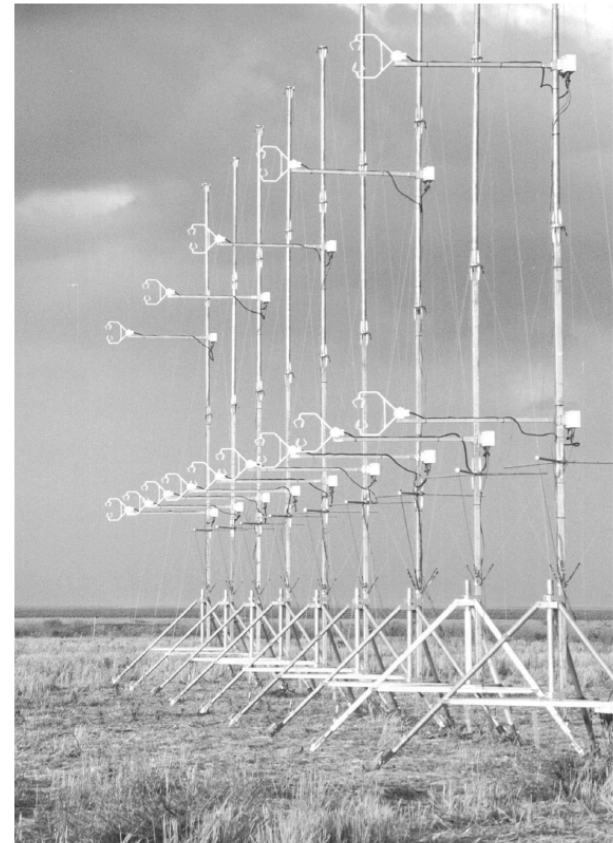Λ_w is the peak in the w velocity spectra and Λ_f is the filter scale
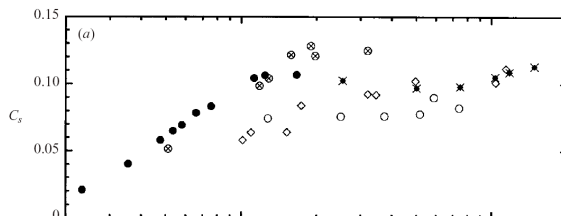
Smagorinsky

Mixed model

Kinetic **E**nergy

Mixed KE

Experimental setup in Colorado

THE UNIVERSITY OF UTAH

SGS Model Coefficient Estimates

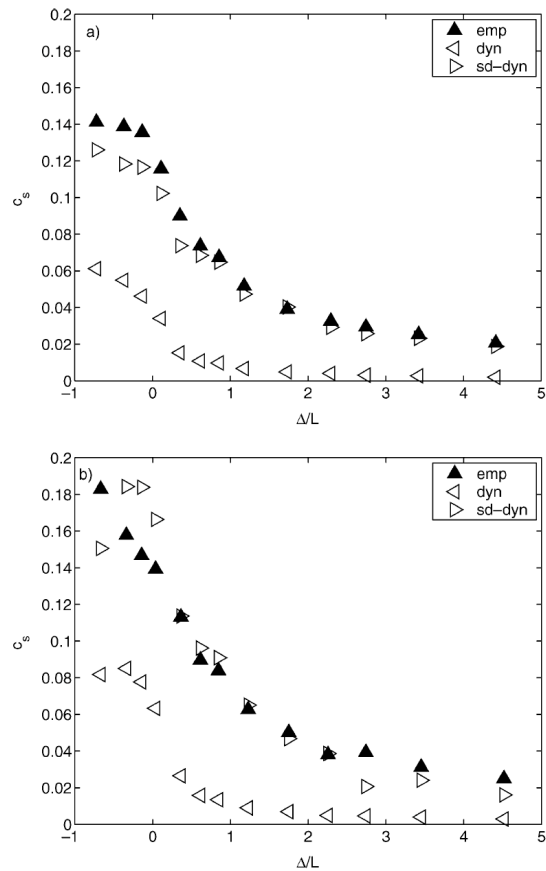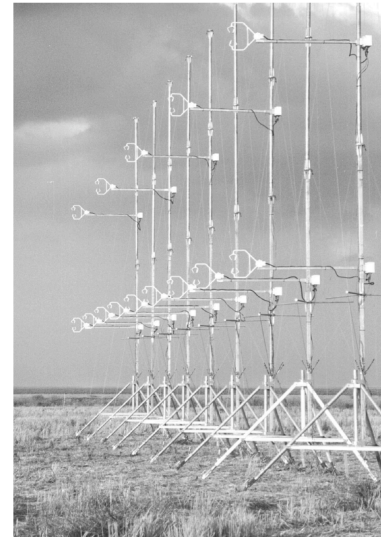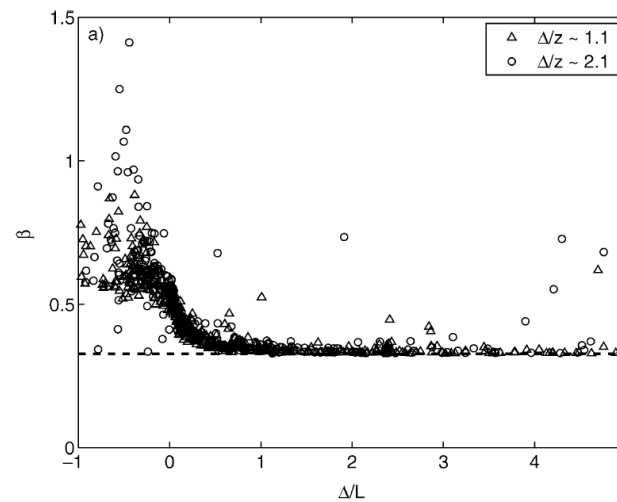Smagorinsky coefficients with stability (Kleissl et al, 2004)

FIG. 14. Smagorinsky coefficient $c_s^{(\Delta)}$ as a function of Δ/L for different SGS models. Variables are averaged over all segments in each stability bin. (a) Array 1, $\Delta/z \sim 2.1$ and (b) array 2, $\Delta/z \sim 1.1$.
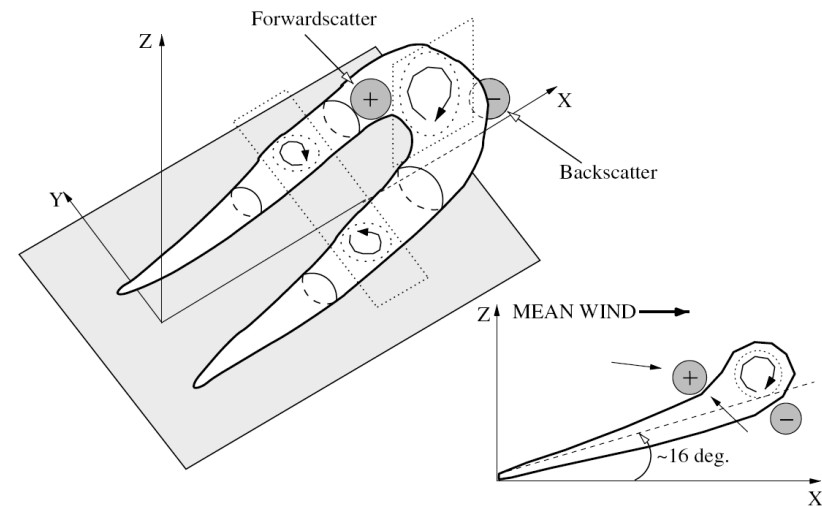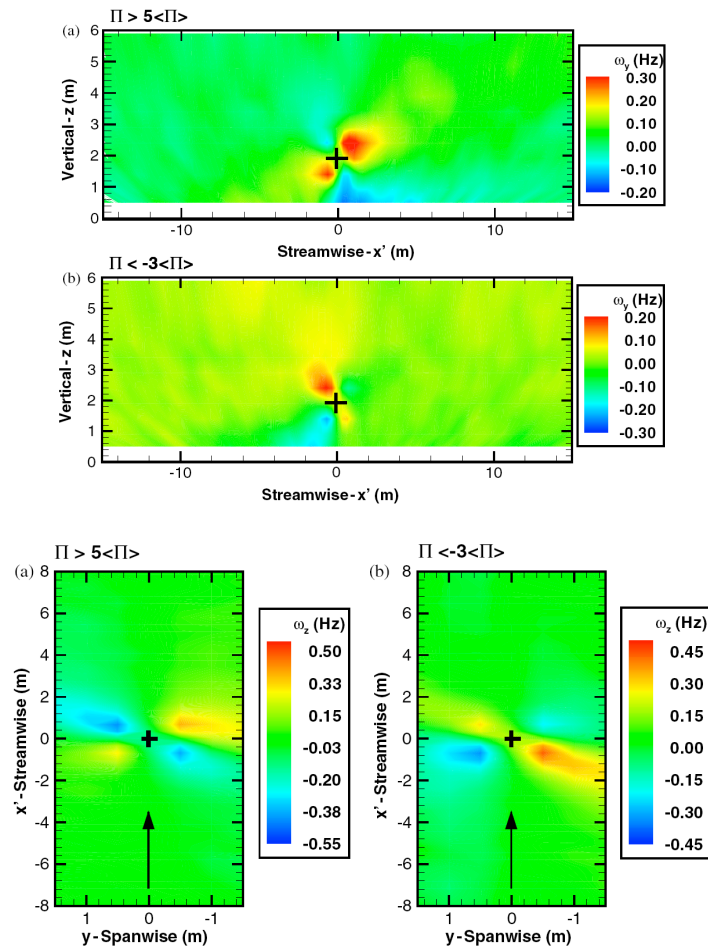
Experimental setup in Colorado

$$\frac{C_S^2(2\Delta)}{C_S^2(\Delta)} = \beta$$

THE UNIVERSITY OF UTAH

Coherent Structures and SGS models

- SGS and coherent structures in the Utah desert (Carper and Porté-Agel , 2004)

Figure 14. Conceptual model relating strong positive (+) and negative (−) SFS dissipation events to different regions (shaded) around a hairpin-like coherent structure. The solid lines outline an isosurface of vorticity with arrows indicating the direction of rotation. The dotted lines indicate the planes on which the conditionally averaged fields are reported with the key results shown within dashed circles.