LES of Turbulent Flows: Lecture 3
(ME EN 7960-008)

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Equations of Motion

• Turbulent flow (and fluid dynamics in general) can be mathematically described by the Navier-Stokes equations (see Bachelor, 1967 for a derivation of equations)

• **Conservation laws for incompressible flow:**

- Mass:

$$\frac{\partial u_i}{\partial x_i} = 0$$

- Momentum:

$$\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{1}{Re}\frac{\partial^2 u_i}{\partial x_j^2} + F_i$$

- Scalar concentration:

$$\frac{\partial \theta}{\partial t} + \frac{\partial u_j \theta}{\partial x_j} = \frac{1}{Sc}\frac{\partial^2 \theta}{\partial x_j^2} + Q$$

Equations of Motion

- **Conservation laws for incompressible flow:**

- Mass: $$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} = 0$$

- Momentum:

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\frac{\mu(T)}{Re}\left(\frac{\partial u_i}{\partial x_j}\frac{\partial u_j}{\partial x_i} - \frac{2}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij}\right)\right] + F_i$$

-energy: $e \equiv \text{total energy density} = \frac{p}{\gamma - 1} + \frac{1}{2}\rho u_i u_i$ where $\gamma = \frac{C_p}{C_v}$

$$\frac{\partial e}{\partial t} + \frac{\partial (e+p)u_j}{\partial x_j} = \frac{\partial}{\partial x_j}\left[u_i \frac{\mu(t)}{Re}\left(S_{ij} - \frac{2}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij}\right)\right] + \frac{\partial}{\partial x_j}\left(\lambda \frac{\partial T}{\partial x_j}\right)$$

In the heat flux vector (last term RHS) $\lambda(T) \equiv \text{thermal conductivity} = \rho C_p$

which is related to the molecular Prandtl number by: $Pr = \frac{\nu}{\kappa} = \frac{C_p \mu(T)}{\lambda(T)}$

To close the equation set we need a relation for viscosity: $\mu(T) = \frac{bT^{1/2}}{1 + S/T}$

and an equation of state. For an ideal gas we can use $p = \rho RT$

Equations of Motion

- **Boussinesq approximation:** (see Stull 1988 pg. 84)

  - Used to included the effects of density variations in the context of the incompressible flow equations with the stipulation that vertical motions are limited by buoyancy (used in most ABL studies).

  - Taking deviations from the averaged ideal gas law we derive a linearized version of the perturbation ideal gas law:

$$\frac{p'}{\langle p\rangle} = \frac{\rho'}{\langle \rho\rangle} + \frac{\theta'_p}{\langle \theta_p\rangle}$$

  - Making the assumption that $\frac{p'}{\langle p\rangle}$s small compared to other terms => $\frac{\rho'}{\langle \rho\rangle} = -\frac{\theta'_p}{\langle \theta_p\rangle}$

  - The above relation is used to neglect density variations in the inertial terms but keep them in the buoyancy term.

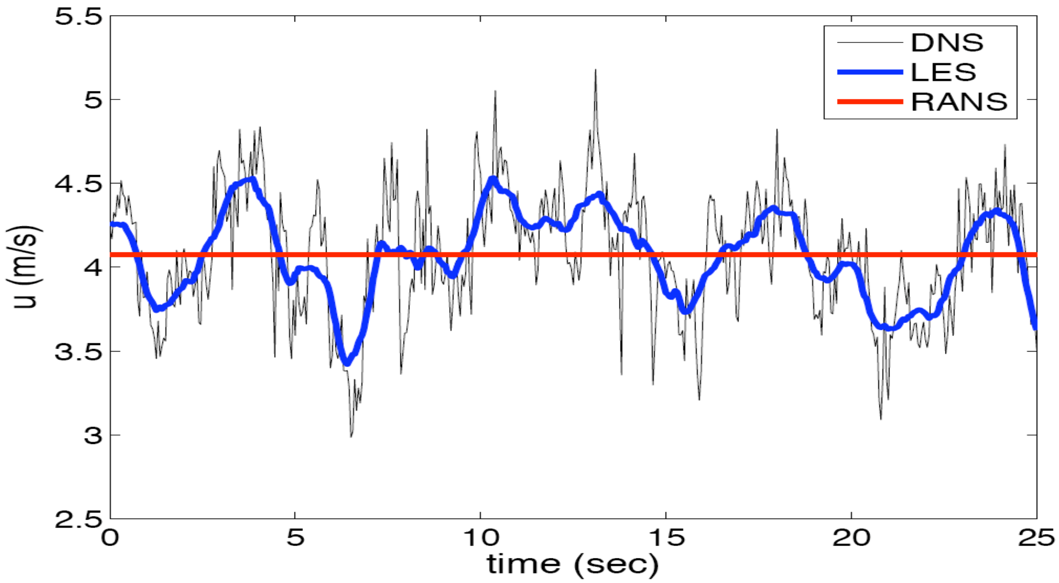
  - Practically, we add a buoyancy forcing term to the incompressible momentum equation (assuming dry air):

$$-\left[g - \left(\frac{\theta'_p}{\langle \theta_p\rangle}\right)\right]\delta_{i3}$$

Where θ_ps the potential temperature = $T\left(\frac{p_o}{p}\right)^{0.286}$ and p_o is a reference pressure

Approximating the equations of motion

- **In Numerical studies**, the equations of motion (incompressible, compressible or Boussinesq fluid) must be approximated on a computational grid
- **Three basic methodologies** are prevalent in turbulence application and research:
  - Direct Numerical Simulation (**DNS**)
    - resolve all eddies
  - Large-Eddy Simulation (**LES**)
    - resolve larger eddies, model smaller 'universal' ones
  - Reynolds-Averaged Navier-Stokes (RANS)
    - model just ensemble statistics



Some Pros and Cons of each Method

Direct Numerical Simulation (**DNS**):

- Pros
  - No turbulence model is required
  - Accuracy is only limited by computational capabilities
  - can provide reference data not available through experiments (i.e., unsteady 3D velocity and scalar fields)

- Cons
  - Restricted to low Re with relatively simple geometries
  - Very high cost in memory and computational time
  - typically "largest-possible" number of grid points is used without proper convergence evaluation.

Some Pros and Cons of each Method

Large-Eddy Simulation (**LES**):

- Pros
  - Only the small scales require modeling
  - Much cheaper computational cost than DNS
  - Unsteady predictions of flow are made => gain info about extreme events in addition to the mean
  - In principle, we can gain as much accuracy as desired by refining our numerical grid
- Cons
  - Basic assumption (small scales are universal) requires independence of small (unresolved) scales from boundary conditions (especially important for flow geometry).
  - Still very costly in practical engineering applications
  - Filtering and turbulence theory of small scales still needs development for complex geometry and highly anisotropic flows

Some Pros and Cons of each Method

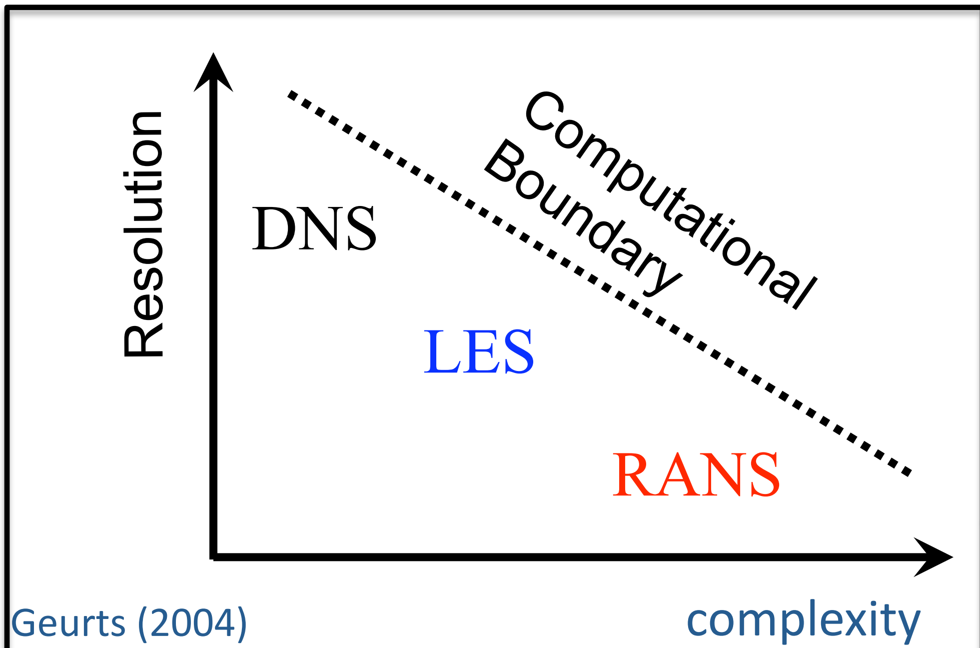
Reynolds Averaged Navier-Stokes (**RANS**):

• Pros

- Low computational demand
(can obtain mean stats in short time)
- can be used in highly complex geometry
- When combined with empirical information, can be highly useful for engineering applications

•Cons

- Only steady flow phenomena are can be explored when taking full advantage of computational reduction
- Models are not universal => usually pragmatic "tuning" is required for specific applications
- More accurate turbulence models result in highly complex equation sets



Capabilities of different simulation methods