LES of Turbulent Flows: Lecture 5
(ME EN 7960-008)

Prof. Rob Stoll
Department of Mechanical Engineering
University of Utah

Spring 2009

Typical LES filters

Common (or classic) LES filters:

- Box or top-hat filter: (equivalent to a local average)

$$G(x-\zeta) = \begin{cases} \frac{1}{\Delta} \text{ if } |x-\zeta| \leq \frac{\Delta}{2} \\ 0 \quad \text{otherwise} \end{cases}$$

transfer function

$$\hat{G}(k) = \frac{sin(k\Delta/2)}{k\Delta/2}$$

- Gaussian filter: (γ typically = 6)

$$G(x-\zeta) = \frac{\gamma}{\pi\Delta^2}^{\frac{1}{2}} exp\left(\frac{-\gamma|x-\zeta|^2}{\Delta^2}\right)$$

transfer function

$$\hat{G}(k) = exp\left(\frac{-\Delta^2 k^2}{4\gamma}\right)$$

- Spectral or sharp cutoff filter:

$$G(x-\zeta) = \frac{sin(k_c(x-\zeta))}{k_c(x-\zeta)}$$

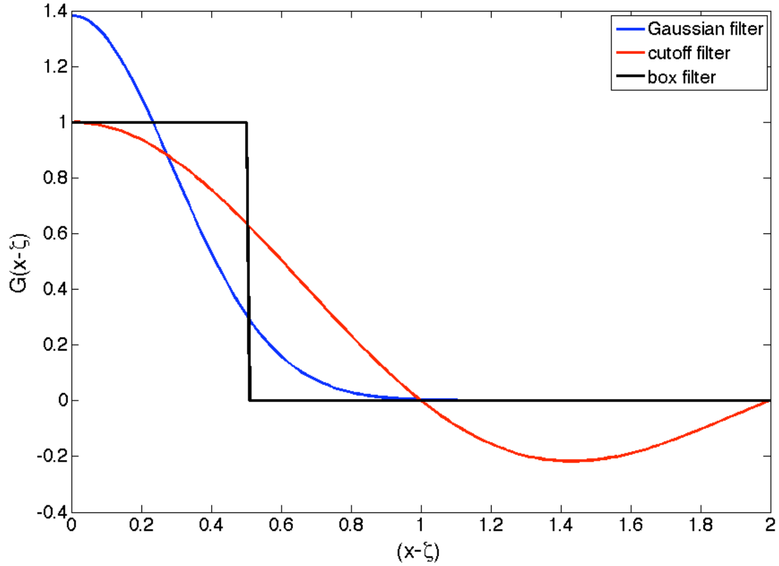
transfer function

$$\hat{G}(k) = \begin{cases} 1 \text{ if } |k| \leq k_c \\ 0 \quad \text{otherwise} \end{cases}$$

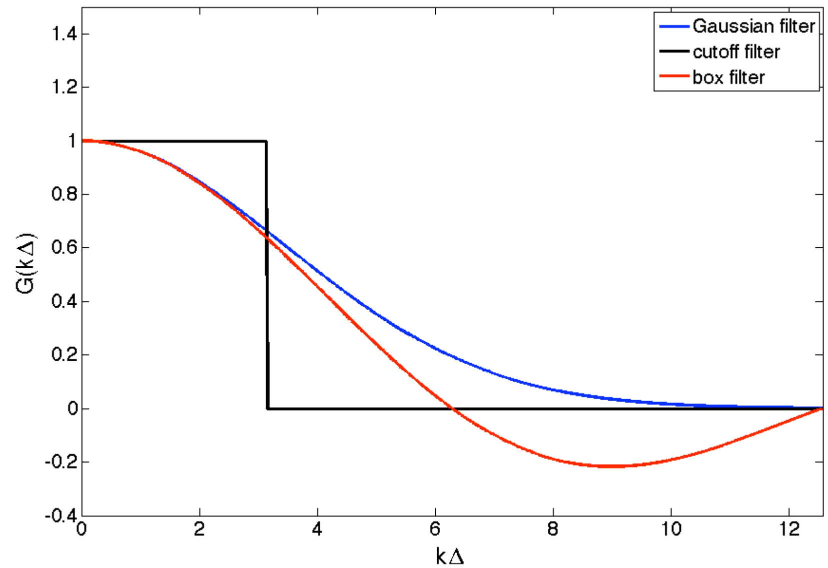
(recall that k_c is our characteristic wavenumber cutoff)

LES Filters and their transfer functions

Real Space Filters



Filter Transfer Function



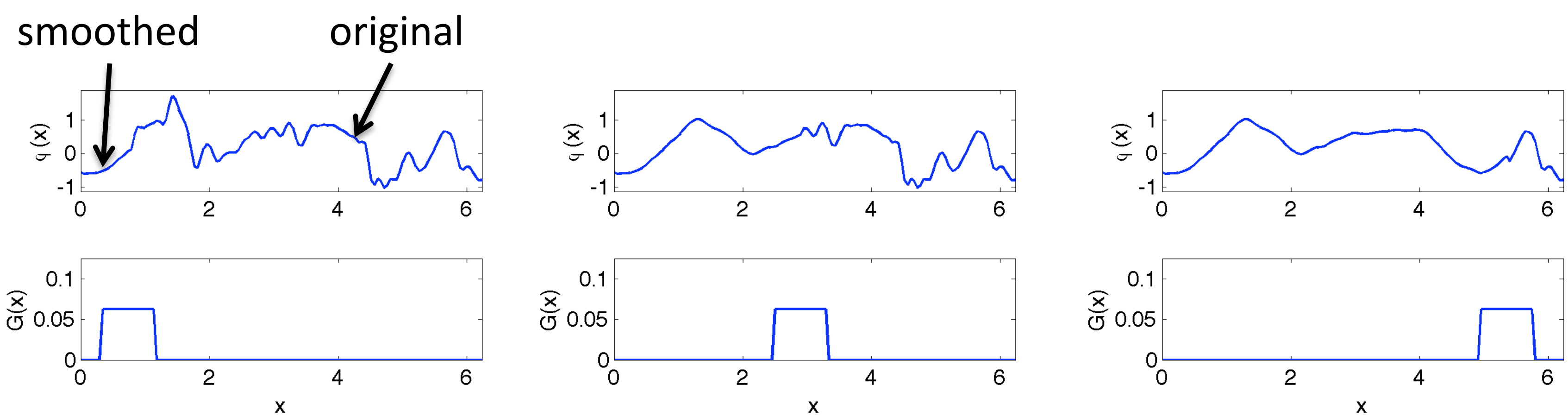
Only the Gaussian filter is local in both real and wave space

Convolution Example

- We defined convolution of two functions as: $\widetilde{\phi}(\vec{x},t) = \int\limits_{-\infty}^{\infty} \phi(\vec{x}-\vec{\zeta},t)G(\vec{\zeta})d\vec{\zeta}$
- How can we interpret this relation?

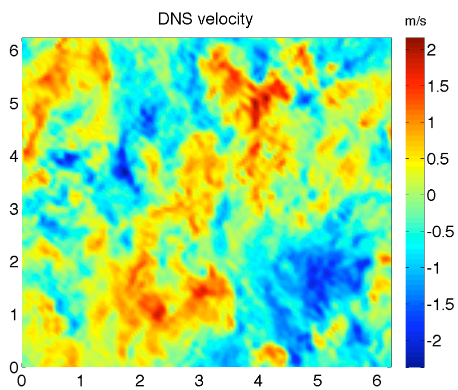
-*G*, our filter kernel 'moves' along our function ϕ smoothing it out (provided it is a low-pass filter):

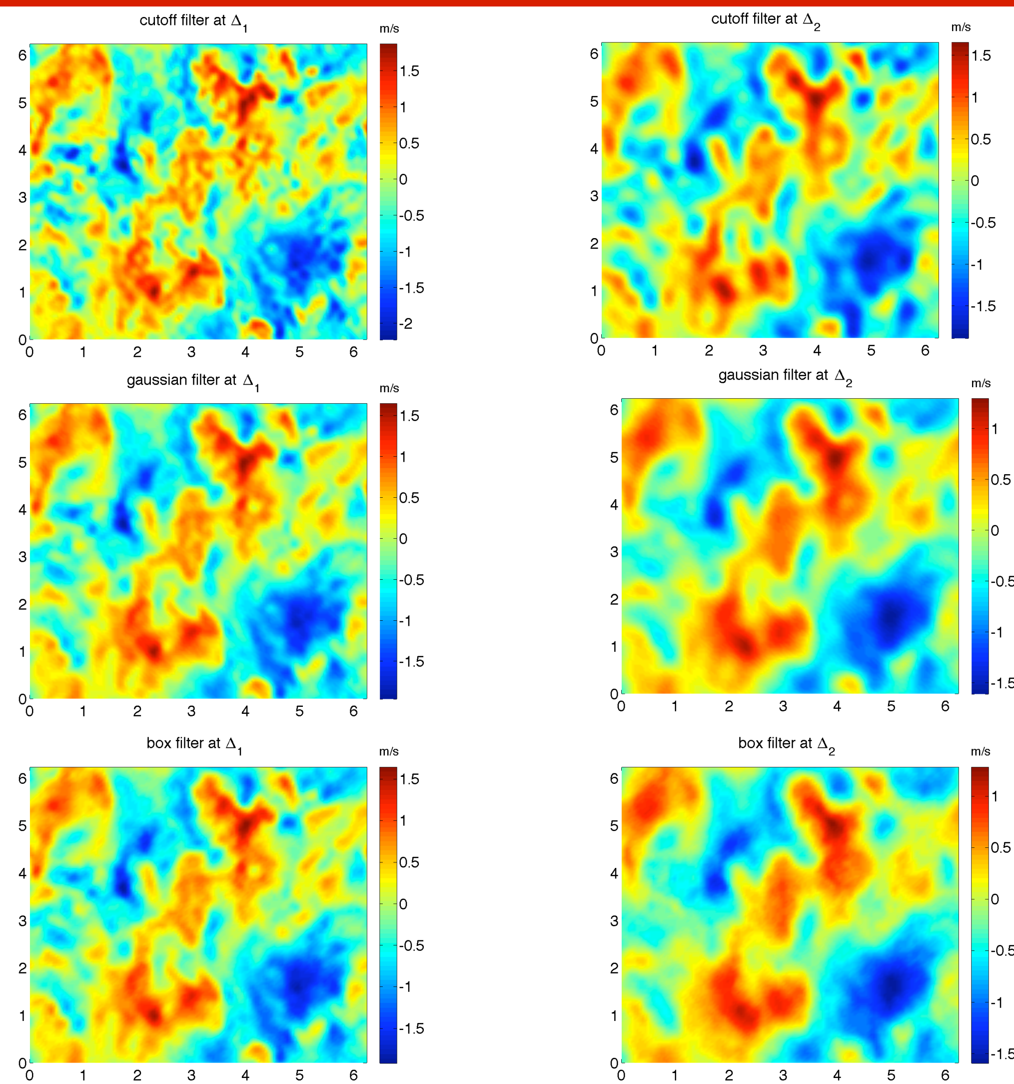
- Example using a box filter applied in real space (see mfile conv_example.m):



Filtering Turbulence (real space)

Note: here (and throughout the presentation) we are using **DNS data from Lu et al.** (International Journal of Modern Physics C, 2008).

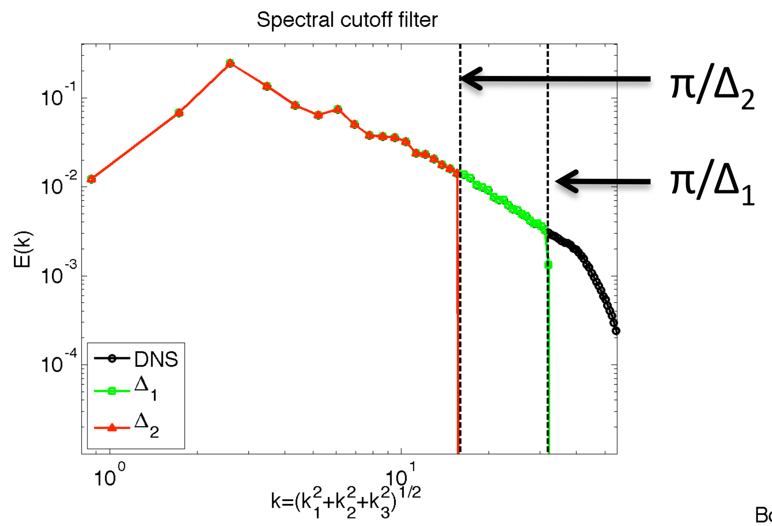


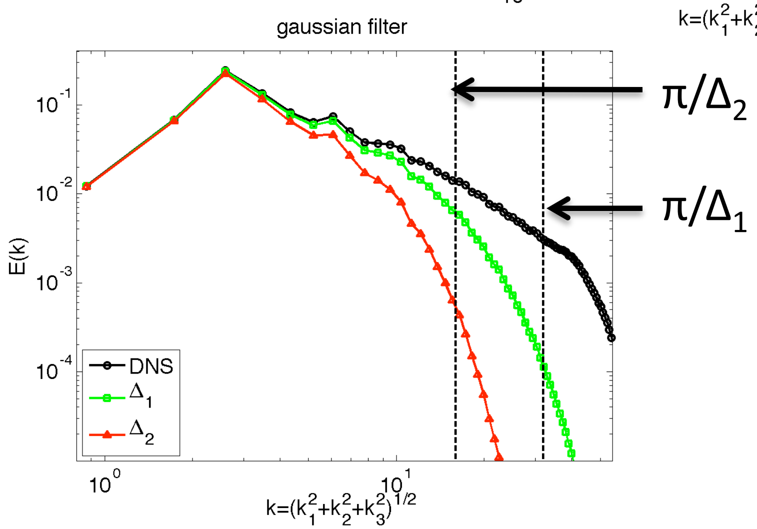


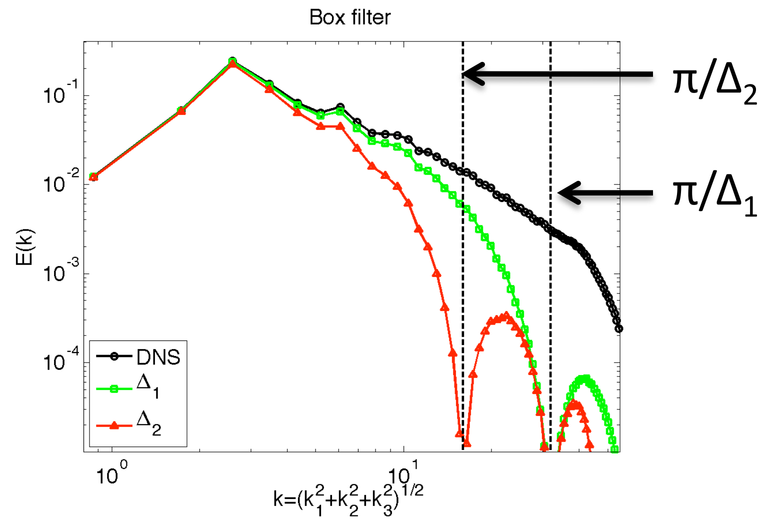
See mfile:
Spectra_comparison.m

Filtering Turbulence (wave space)

See mfile:
Spectra_comparison.m

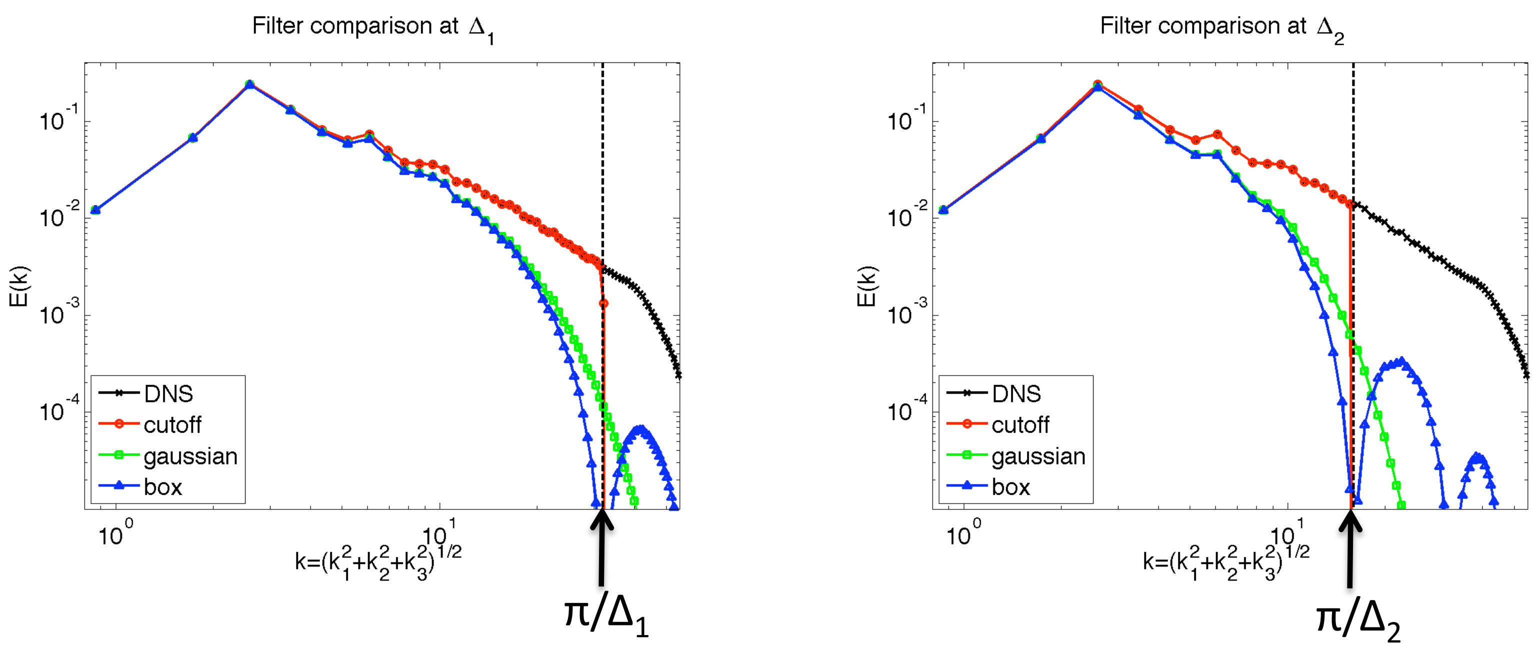






Filtering Turbulence (wave space)

Comparison between different filters

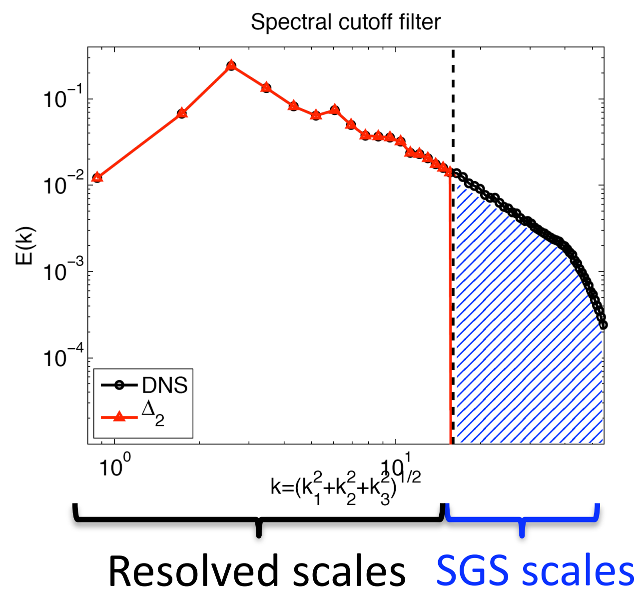


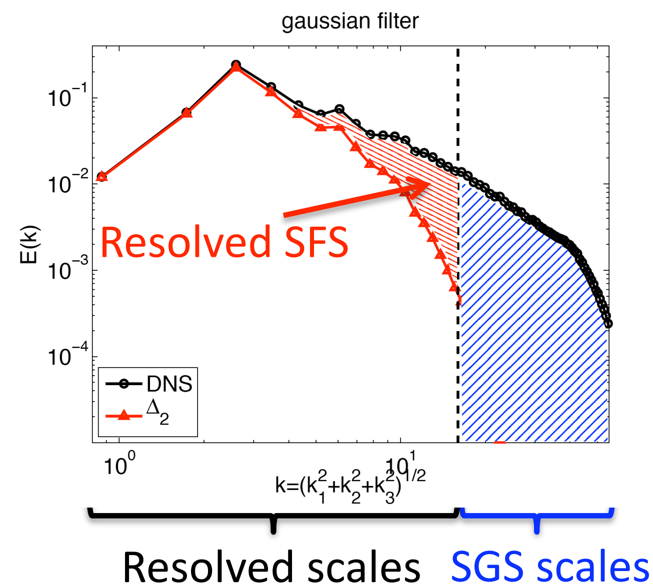
Decomposition of Turbulence for real filters

The LES filter can be used to decompose the velocity field into resolved and subfilter scale (SFS) components

$$\phi(\vec{x},t) = \widetilde{\phi}(\vec{x},t) + \phi'(\vec{x},t)$$

We can use our filtered DNS fields to look at how the choice of our filter kernel affects this separation in wavespace





The Gaussian filter (or box filter) does not have as compact of support in wavespace as the cutoff filter. This results in attenuation of energy at scales larger than the filter scale. The scales affected by this attenuation are referred to as **Resolved SFSs**.

Filtering the incompressible N-S equations

- What happens when we apply one of the above filters to the N-S equations?

-**Conservation of Mass:**

filtering both sides of the conservation of mass: $\widetilde{\dfrac{\partial u_i}{\partial x_i}} = 0 \Rightarrow \dfrac{\partial \tilde{u}_i}{\partial x_i} = 0$

where we have used the property of LES filters => $\widetilde{\dfrac{\partial \phi}{\partial \vec{x}}} = \dfrac{\partial \widetilde{\phi}}{\partial \vec{x}}$ and (~) denotes the filtering operation.

-Conservation of Momentum:

Using the filter properties $\widetilde{a} = a$, $\widetilde{\phi + \zeta} = \widetilde{\phi} + \widetilde{\zeta}$ and $\widetilde{\dfrac{\partial \phi}{\partial \vec{x}}} = \dfrac{\partial \widetilde{\phi}}{\partial \vec{x}}$ we can write the momentum equation as:

$$\frac{\partial \tilde{u}_i}{\partial t} + \frac{\partial \widetilde{u_i u_j}}{\partial x_j} = -\frac{\partial \tilde{p}}{\partial x_i} + \frac{1}{Re}\frac{\partial^2 \tilde{u}_i}{\partial x_j^2} + F_i$$

The 2nd term on the LHS (convective term) now contains the unknown $\widetilde{u_i u_j}$ we can rewrite this term to obtain the standard LES equations for incompressible flow

Filtering the incompressible N-S equations

We can add and subtract $\tilde{u}_i\tilde{u}_j$ from the convective term:

$$\frac{\partial \widetilde{u_i u_j}}{\partial x_j} = \frac{\partial \left(\widetilde{u_i u_j} + \tilde{u}_i\tilde{u}_j - \tilde{u}_i\tilde{u}_j\right)}{\partial x_j} = \frac{\partial \tilde{u}_i\tilde{u}_j}{\partial x_j} + \frac{\partial \left(\widetilde{u_i u_j} - \tilde{u}_i\tilde{u}_j\right)}{\partial x_j}$$

Putting this back in the momentum equation and rearranging we have

$$\frac{\partial \tilde{u}_i}{\partial t} + \frac{\partial \tilde{u}_i\tilde{u}_j}{\partial x_j} = -\frac{\partial \tilde{p}}{\partial x_i} + \frac{1}{Re}\frac{\partial^2 \tilde{u}_i}{\partial x_j^2} - \frac{\partial \tau_{ij}}{\partial x_j} + F_i$$

SFS force vector

where $\tau_{ij} = \widetilde{u_i u_j} - \tilde{u}_i\tilde{u}_j$ is the **subfilter scale (SFS) stress tensor**

- For the **scalar concentration** equation we can go through a similar process to obtain:

$$\frac{\partial \tilde{\theta}}{\partial t} + \frac{\partial \tilde{u}_j\tilde{\theta}}{\partial x_j} = \frac{1}{Sc}\frac{\partial^2 \tilde{\theta}}{\partial x_j^2} - \frac{\partial q_j}{\partial x_j} + Q$$

Where $q_j = \widetilde{u_j\theta} - \tilde{u}_j\tilde{\theta}$ is the **SFS flux**