LES of Turbulent Flows: Lecture 7
(ME EN 7960-008)

Prof. Rob Stoll
Department of Mechanical Engineering
University of Utah

Spring 2009

Turbulence modeling (alternative strategies)

• So far our discussion of turbulence modeling has centered around separating the flow into resolved and SFSs using a low-pass filtering operation with the goal of reducing the # of degrees of freedom in our numerical solution.

• This is not the only way to accomplish complexity reduction in a turbulent flow. Here we briefly review a few other methods

• **Coherent Vortex Simulations (CVS)**: Farge and Schneider, Flow Turb. Comb. 2001
The turbulent flow field is decomposed into coherent and random components using either a continuous or orthonormal wavelet filter (see the next page for a very brief review of wavelets)

-In the original formulation the separation between coherent and random motions is assumed to be complete with the random part mimicking viscous dissipation.
-Goldstein and Vasilyev (Phys of Fluids, 2004) introduced "**stochastic coherent adaptive LES**", a variation on CVS

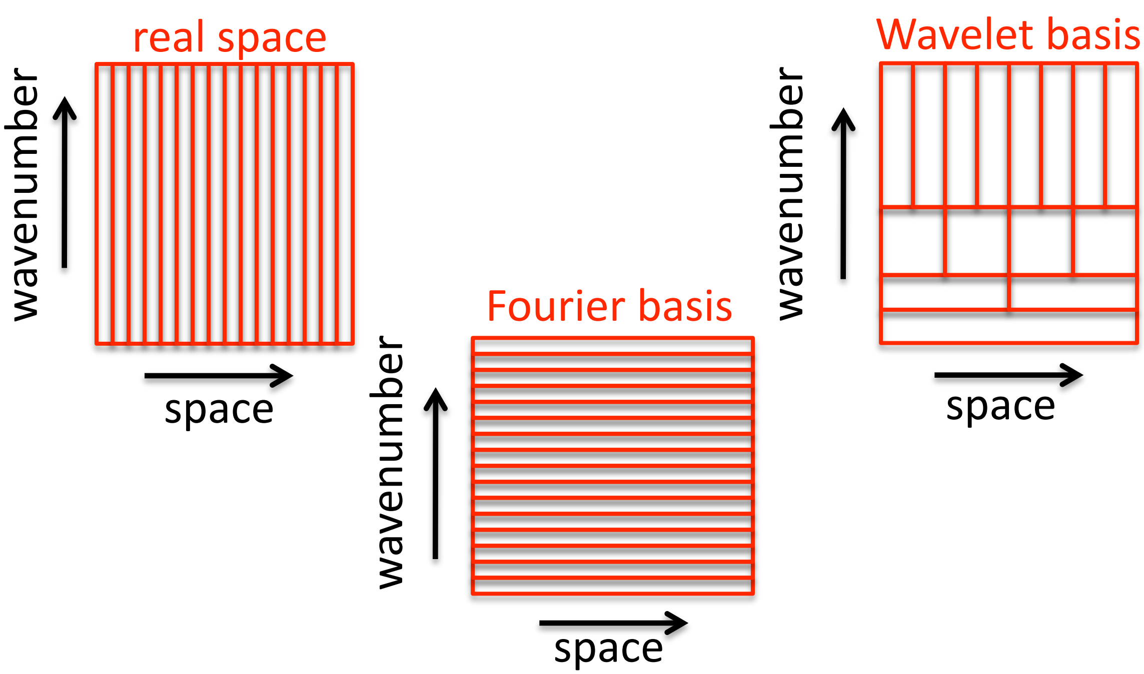
• they use the CVS wavelet decomposition but do not assume that the wavelet filter completely eliminates all the coherent motions from the SFSs => the SFS components themselves contain coherent and random components.

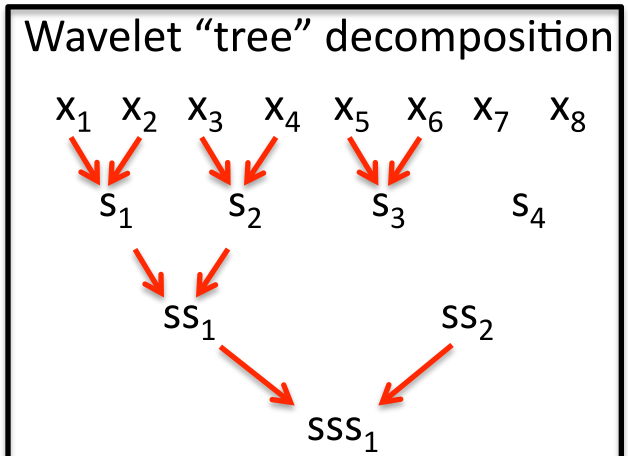
Wavelet decomposition (a brief overview)

- The CVS method uses **wavelet decomposition**. For general review see Daubechies (1992) or Mallat (2004). For wavelets in turbulence see Farge, (Annual Reviews in Fluid Mech, 1992).
- Wavelets offer an optimal space frequency decomposition, in 1D:

$$W_f(a,b) = |a|^{-\frac{1}{2}} \int f(x)\Psi\left(\frac{x-b}{a}\right) dx$$

where ***Ψ* is the basis function** (sometimes called the "Mother Wavelet"), ***b* translates** the basis function (location) and ***a* scales** the basis function (dilatation).





For the Haar wavelet

$s_1=(x_1+x_2)/2^{½}$

Turbulence modeling (alternative strategies)

- **Filtered Density Functions (FDF)**: Colucci et al., (Phys. of Fluids, 1998)

  -In this method, the **evolution of the filtered probability density functions** is solved for (i.e., we solve for the evolution of the SFS general moments)

  -Similar to general PDF transport methods 1st introduced by Lundgren (Phys. of Fluid, 1969) and outlined in detail in Pope chapter 12

  -Many applications use FDF for scalars in turbulent reacting flows while traditional (low-pass filtered N-S) equations are solved for momentum

  -This type of method is often employed for LES with Lagrangian particle models and for chemically reactive flows. In **Lagrangian particle models it leads to** a form of the **Langevin equations for SFS particle evolution** and in **chemically reactive flows** it has the advantage that the **reactions occur in closed form**

LES and numerical methods

- LES requires that the filtered equations of motion (see Lectures 5 and 6) be solved on a numerical grid.
- **In LES we need to accurately represent high wavenumber turbulent fluctuations** (small scale turbulence) this means either:
  - -we use high-order schemes (e.g., spectral methods)
  - -we use fine grids with low-order schemes (e.g., 2^{nd} order central differences)
- High-order schemes are more expensive but for a given mesh they are more accurate (see Pope 571-579 for a discussion of resolving filtered fields)
- Low-order finite difference (or volume) schemes provide flexibility of geometry but give rise to complications when modeling small scales motions
- For **low-order FD schemes truncation errors can be on the order of SGS contributions** unless Δ is considerably larger than the grid spacing (see Ghosal J. Comp. Phys, 1996)
- Note that although 2^{nd} order schemes may have undesirable truncation errors (with respect to SGS model terms), **even order schemes are non-dissipative while SGS models (on average) are purely dissipative**, therefore, <u>all hope is not lost!</u>
- **The same is not true for dissipative schemes** common in compressible flow solutions For example in upwind schemes and TVD or FLS schemes (see Leveque 1992 for a review of this type of numerics)

LES and numerical methods

- The filter applied in LES can be either **implicit** or **explicit**

  -Implicit filtering: The grid (or numerical basis) is assumed to be the LES low-pass filter

  - **Pro:** takes full advantage of the numerical grid resolution
  - **Cons**: for some methods it is helpful to know the shape of the LES filter (this can be difficult to determine for some numerical methods). Truncation error (see above) can also become an issue.

  -**Explicit filtering**: A filter (typically box or Gaussian) is applied to the numerically grid (i.e., explicitly to the discretized N-S equations)

  - **Pros**: truncation error is reduced and the filter shape is well defined
  - **Cons**: loss of resolution. The total simulation time goes up as Δ_g^4 (where Δ_g is the grid spacing) so maintaining the same space resolution as an implicit filter with $\Delta_g/\Delta=½$ will take 2^4=16 more grid points.

- For reviews of LES and numerics see Guerts chapter 5 and Sagaut chapter 8.2 and 8.3